



Plasma Theories Single Particle Model

Mohammed Shihab

Physics Department, Faculty of Science
Tanta University





What is plasma?

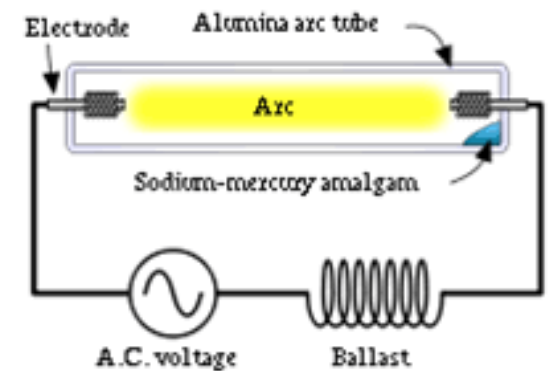
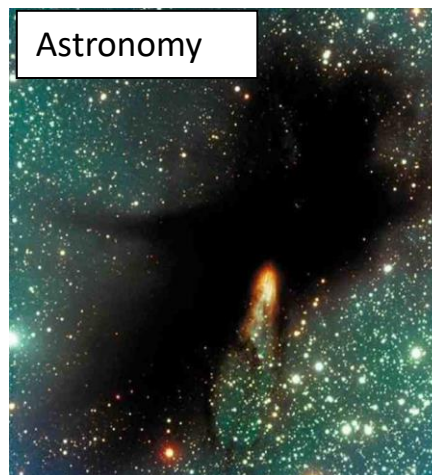
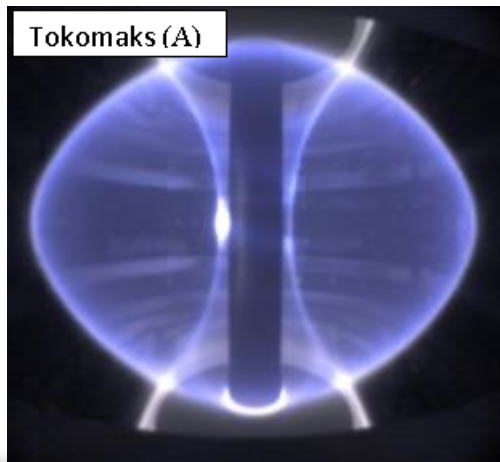
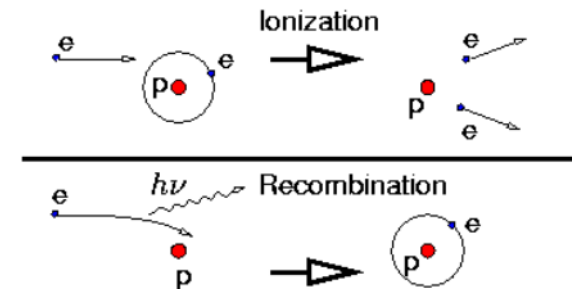
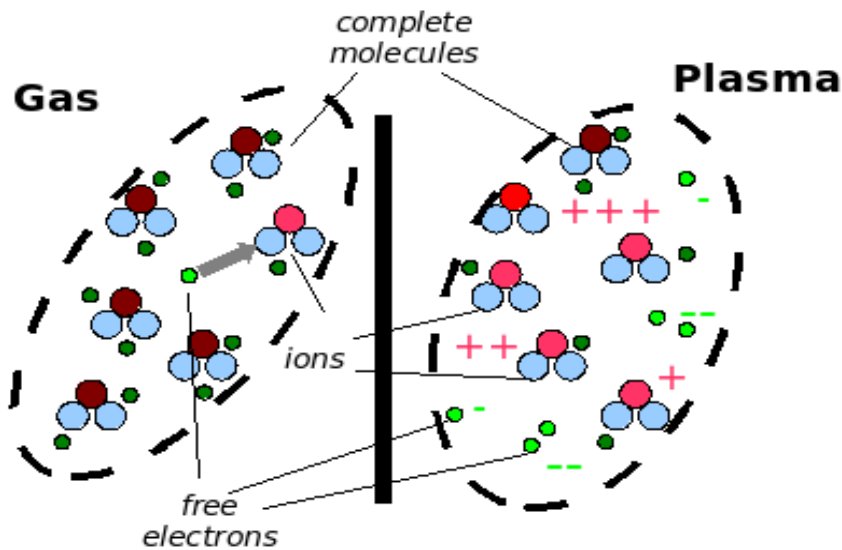
- Plasma is the dominant constituent of the universe.



- It is the **fourth state** of matter.
- The word of plasma seems to be a **misnomer**. It comes from a Greek word which means something **molded** or **fabricated** because of behavior, it does not tend to conform to external influences
- Plasma is simply a system of charged particles such as electrons and ions and excited neutral species.



What is plasma?





Plasma parameters

- Debye length is much smaller than the plasma dimensions

$$\lambda_D \ll L$$

- So the plasma consists of quasineutral spheres, therefore, the plasma is defined as quasineutral exhibits collective behavior.
- Number of particles in Debye Sphere is greater than 1.
- From the definition of plasma density

$$n = \frac{N}{V} = \frac{\text{number of particles}}{\text{Volume}}$$

- In a sphere

$$N = n * (4/3)\pi r^3$$



Plasma parameters

- The frequency of a process times the relaxation of this process must be greater than 1

$$\omega T \gg 1$$

- For example: the ionization rate must be greater than the recombination rate, otherwise, the plasma will die out.



Applications

- Plasma plays an essential role in different applications such as
- gas discharges
- controlled thermonuclear fusion
- space physics
- modern astrophysics
- energy conversion and ion propulsion
- Solid state plasma
- Gas laser
- Microelectronics
- Water treatment
- Cancer treatment
- Textile treatment



Modeling?

What is the volume of the cat?





Challenges of plasma modeling I

- Q** (i) How many ions are there in a cubic millimetre ($V = 10^{-9} \text{ m}^3$) of plasma of charged particle density $n = 10^{16} \text{ m}^{-3}$?
- (ii) How far will an electron travel in $t = 0.1 \mu\text{s}$ when accelerated in vacuum from rest by an electric field of $E = 10^2 \text{ V m}^{-1}$?
- (iii) In a typical low-pressure, electrical discharge plasma a large fraction of electrons have speeds around $v = 10^6 \text{ m s}^{-1}$ and collide with gas atoms typically every $\lambda \sim 10^{-1} \text{ m}$, depending on the pressure; what is the average time between successive collisions?
- A** (i) $N = n \times V = 10^7$.
- (ii) $s = \frac{1}{2}(eE/m)t^2 \approx 10^{-1} \text{ m}$.
- (iii) $\tau = \lambda/v \sim 10^{-7} \text{ s}$.



Challenges of plasma modeling II

It is challenging to solve the equation of motion of all plasma particles (not feasible): Because the number of particles is too much.



- The equation of motion must be solved in a self-consistent way with electric and magnetic fields.
- Maxwell's equations must be solved for each particle.



Plasma Theories

Models



Single Particle

Dielectric (electric) Model

Fluid Model

Kinetic model

Single Particle Model



Equation of motion

$$m \frac{d\vec{v}}{dt} = q\vec{E} + q\vec{v} \times \vec{B} + \vec{F}_g$$

- SI Units
 - mass (m) - kg
 - length (l) - m
 - time (t) - s
 - electric field (E) - V/m
 - magnetic field (B) - T
 - velocity (v) - m/s
 - F_g stands for non-electromagnetic forces (e.g. gravity) - usually ignorable.



Electric Field only

- Free electrons may eject from the cathode. The electric field will accelerate free electrons toward the anode. Ionization of neutral atoms and molecules may take place when the kinetic energy of electrons is greater than the ionization of neutral atoms and molecules.

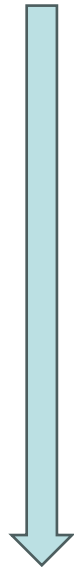


$$m \frac{dv}{dt} = -eE$$

$$E = -\frac{d\phi}{dx}$$

$$m \frac{dv}{dt} = e \frac{d\phi}{dx}$$

$$m \frac{dv}{dt} = e \frac{d\phi}{dt} \frac{dt}{dx}$$



$$m \frac{dv}{dt} = e \frac{d\phi}{dt} \frac{1}{v}$$

$$mv dv = e d\phi$$

$$\frac{1}{2}mv^2 = e\phi$$

The electric energy is converted into kinetic energy



Electron Current

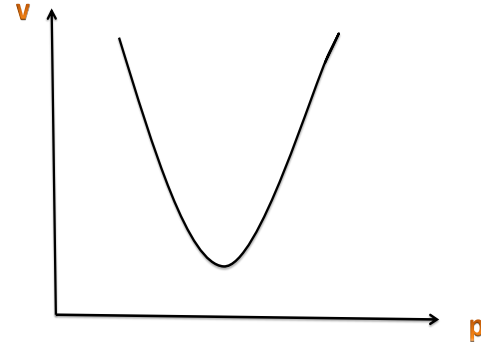
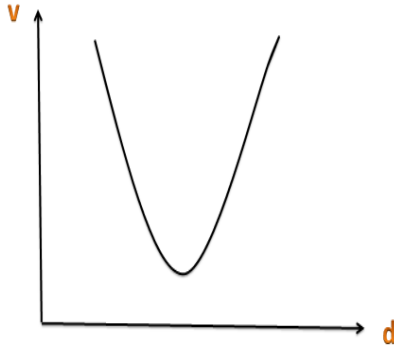
- Calculate the minimum current to light a hydrogen lamp. The ionization energy of hydrogen atom is 13.6 eV. The cross-section of the lamp is 3 cm² and plasma density of 10¹⁰ cm⁻³. *Google e and m!*
- *What is current density and its physical units!*

$$J = \frac{I}{A} = n_e e v = n_e e \sqrt{\frac{2e\phi}{m}}$$

$e\phi$ = ionization energy



Pachen's Law

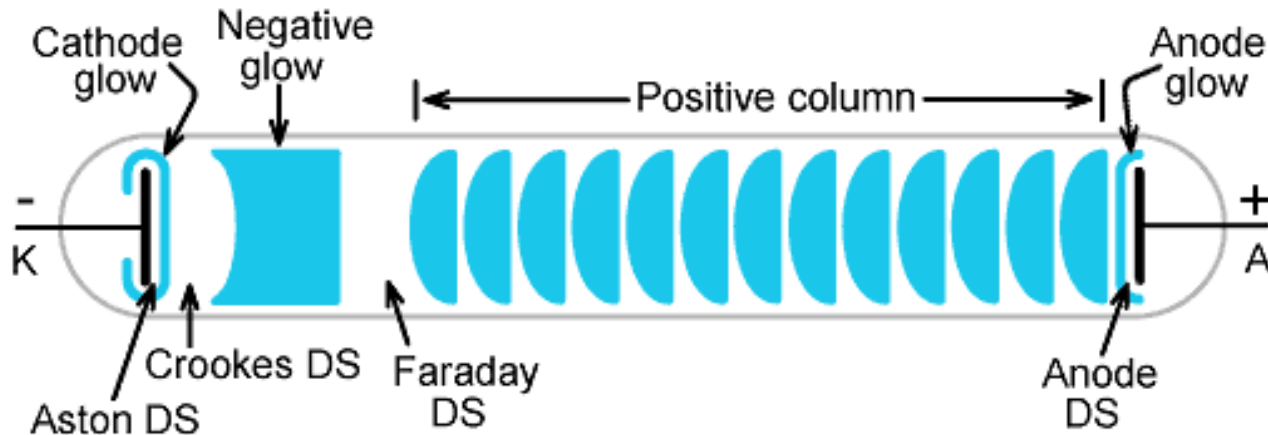


- The gas pressure and the distance between the electrodes affect the voltage required to achieve discharge.
- $v_{min} \propto pb = kpd$.
- Considering secondary electrons: $V_{BD} = Bpd / \ln\left[\frac{Apd}{\ln\left(\frac{1}{\gamma}\right)}\right]$
- Where A and B are gas dependent constants, and γ is the second Townsend ionization coefficient.

Collisions are necessary to produce plasmas



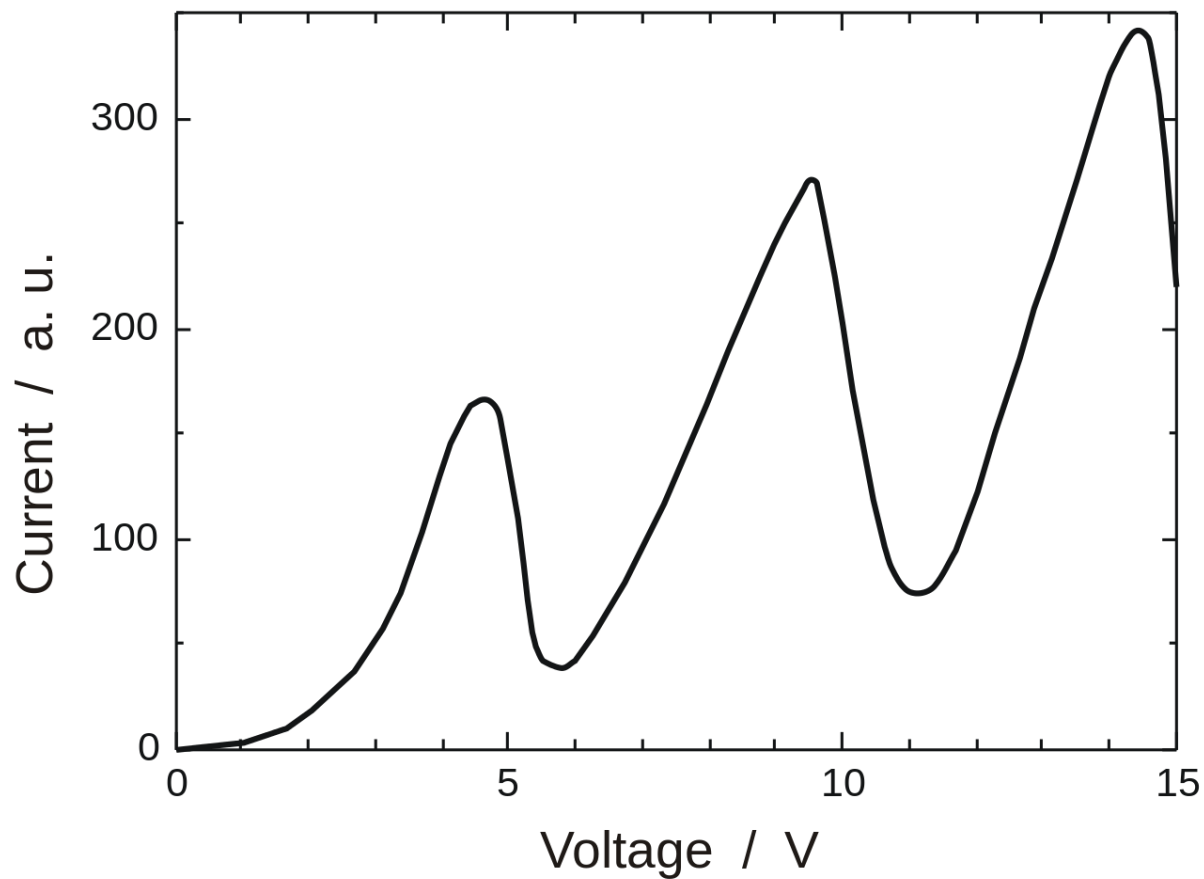
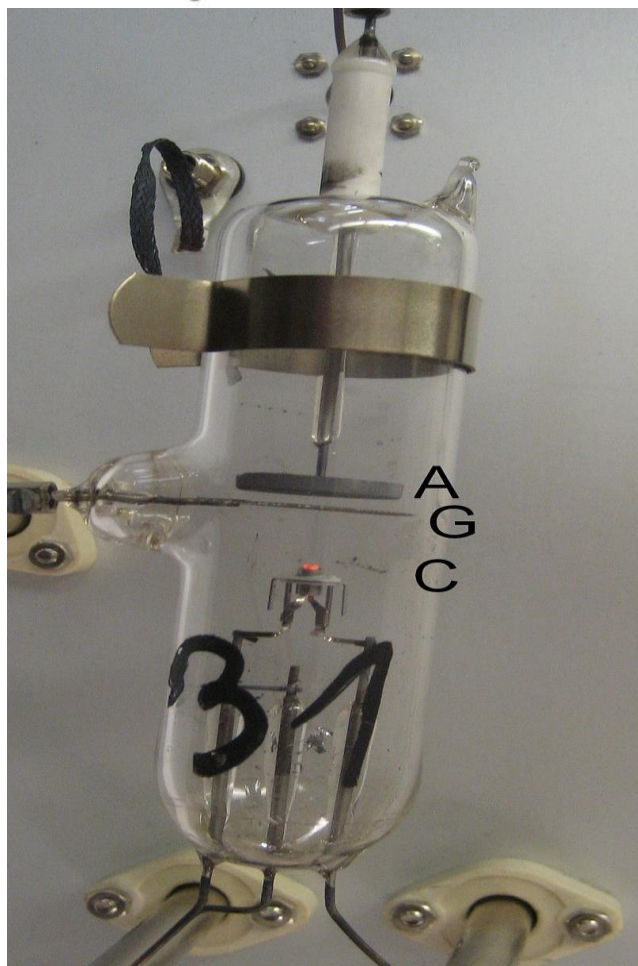
Normal and abnormal discharges



- Glow discharges are classified as normal or abnormal discharges
- The normal discharge has a constant current density at the cathode. When the power to the discharges varied, only the area over which they glow exists varies. After the entire cathode is enveloped within the glow, the current density increases. This is known as the abnormal or anomalous glow discharge. The abnormal discharge requires relatively large increases in the applied voltage to cause small increases in the current density. As the current density increases, it is accompanied by a decrease in cathode sheath thickness, therefore, leading to a higher electric field and voltage drop in the sheath, which increases the positive ion energy.



Franck–Hertz experiment



The Physics Nobel Prize for the year 1925.

Single Particle Model



Equation of motion

$$m \frac{d\vec{v}}{dt} = q\vec{E} + q\vec{v} \times \vec{B} + \vec{F}_g$$

- SI Units
 - mass (m) - kg
 - length (l) - m
 - time (t) - s
 - electric field (E) - V/m
 - magnetic field (B) - T
 - velocity (v) - m/s
 - F_g stands for non-electromagnetic forces (e.g. gravity) - usually ignorable.



Magnetic Field

- **B acts to change the motion of a charged particle only in directions perpendicular to the motion.**
 - Set $E = 0$, assume B along z-direction.

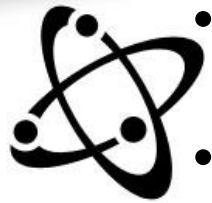
$$m\dot{v}_x = qv_y B$$

$$m\dot{v}_y = -qv_x B$$

$$\ddot{v}_x = \frac{q\dot{v}_y B}{m} = -\frac{q^2 v_x B^2}{m^2} = -\Omega_c^2 v_x$$

$$\ddot{v}_y = -\frac{q^2 v_y B^2}{m^2} = -\Omega_c^2 v_y$$

$$\Omega_c = \frac{|q| B}{m}$$



- Solution is circular motion dependent on initial conditions.
- Assuming at $t=0$: $v_x = 0; v_y = v_{\perp}$

and

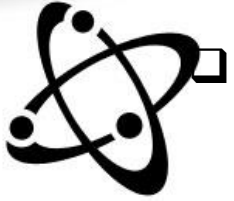
$$v_x = v_{\perp} \sin(\pm \Omega_c t)$$

$$v_y = v_{\perp} \cos(\pm \Omega_c t)$$

$$x - x_0 = \mp \frac{v_{\perp}}{\Omega_c} \cos(\Omega_c t)$$

$$y - y_0 = \pm \frac{v_{\perp}}{\Omega_c} \sin(\Omega_c t)$$

- Equations of circular (Helical) motion with angular frequency Ω_c (cyclotron frequency or gyro frequency). Above signs are for positive charge, below signs are for negative charge.



A simpler way

- When the magnetic field is perpendicular to the particle velocity
 $v \perp B$
- At balance: the centripetal force=magnetic force

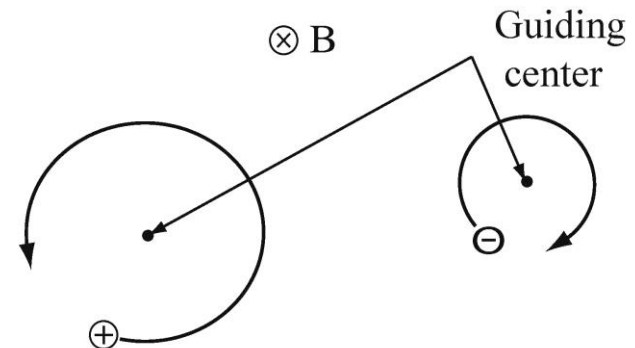
$$\frac{mv_{\perp}^2}{r} = qv_{\perp}B \quad \longrightarrow \quad r = \frac{mv_{\perp}}{qB}$$

- Periodic time $T = \frac{\text{Distance}}{\text{Velocity}} = \frac{2\pi r}{v_{\perp}} \quad \longrightarrow \quad T = \frac{2\pi m}{qB}$

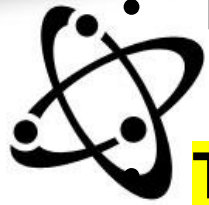
- Angular frequency

$$\Omega_c = 2\pi f = 2\pi/T$$

$$\Omega_c = \frac{qB}{m}$$



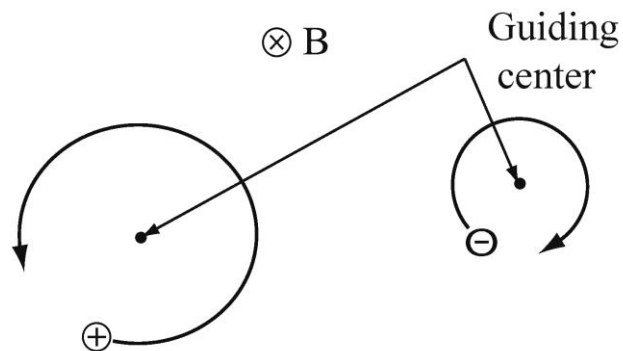
- If q is positive particle gyrates in left-handed sense
- If q is negative particle gyrates in a right-handed sense



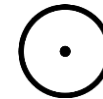
- The rotation direction (Hint)

The plasma is a diamagnetic material.

- Diamagnetic materials are those materials that are freely magnetized when placed in a magnetic field. However, the magnetization is in the direction opposite to that of the external magnetic field.



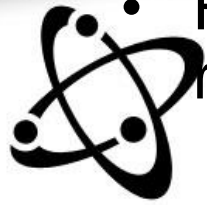
out



in



- The rotation direction could be known from Lenz's law. Charged particles in the plasma rotate in directions to cancel the external magnetic field.



- Radius of circle (r) - cyclotron radius or Larmor radius or gyro radius.

$$r = \frac{mv_{\perp}}{qB}$$

- The gyro radius is a function of energy.
 - Energy of charged particles is usually given in electron volts (eV)
 - Energy that a particle with the charge of an electron gets in falling through a potential drop of 1 Volt- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules (J)}$.
- The circular motion does no work on a particle

$$\vec{F} \cdot \vec{v} = m \frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{d(\frac{1}{2}mv^2)}{dt} = q\vec{v} \cdot (\vec{v} \times \vec{B}) = 0$$

Only the electric field can energize particles!
Particle energy remains constant in absence of E !

Conclusion: Important facts!



- ❑ Only the electric field can energize particles!
- ❑ Charged particles may rotate around the magnetic field lines.
- ❑ Magnetic force is zero when charged particles move parallel to the field lines.
- ❑ Magnetized plasma has at least two temperatures.

Thank You!

