Types of plasmas

- **(I) Classical plasma**

  +ve ions / electrons / -ve ions / positrons

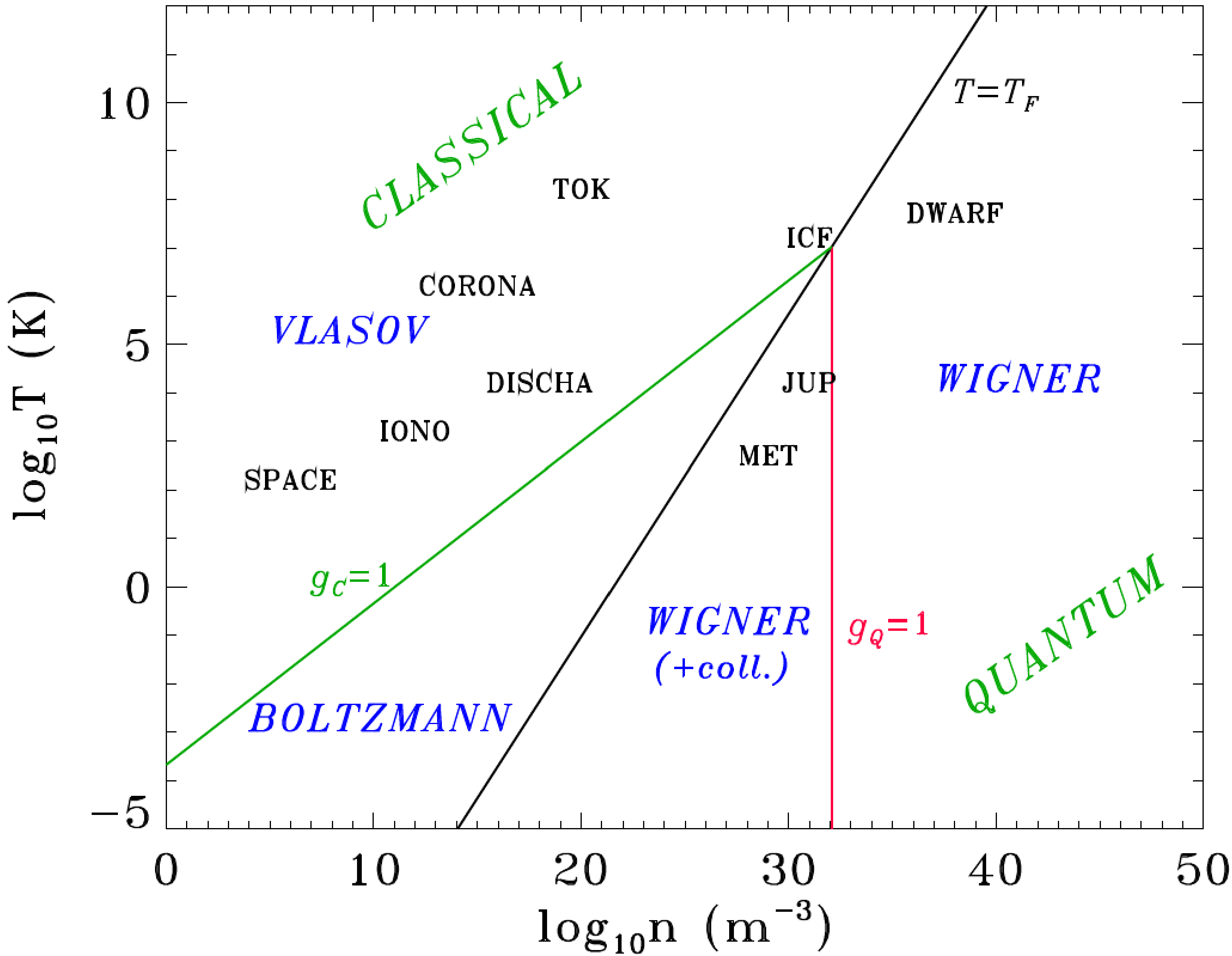
- **(II) Dusty (complex) plasma**

  +ve dust / -ve dust / +ve ions / electrons / -ve ions

- **(III) Quantum plasma**

  Electrons / positrons / holes / +ve ions

Plasma applications & observations



Forces in plasma

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force
- **15 Forces**

Types & Forces

- Classical
- Dusty
- Quantum

Experiment

OR

Application

OR

Observation

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force

Types & Forces, cont.

What is the criteria to decide the leading force?

- ✔ **Understanding each force → 15 forces**
- ✔ **Knowing the physics of the Exp. / App. / Obs.**
- ✔ **Select a suitable plasma type → 3 types**

- **Why waves is important in plasma?**
- **Plasma models/theories**
- **Plasma components & Waves**
- **ESWs & EMWs**
- **Linear & Nonlinear theory**
- **Idea of perturbation**
- **Electrons oscillation & wave**
- **Ion wave**

Fig. 4.1 Spatial variation of the electric field of two waves with a frequency difference

This is a sinusoidally modulated wave (Fig. 4.1). The envelope of the wave, given by cos $[(\Delta k)x - (\Delta\omega)t]$, is what carries information; it travels at velocity $\Delta\omega/\Delta k$. Taking the limit $\Delta\omega \to 0$, we define the *group velocity* to be

$$\boxed{v_g = d\omega/dk} \tag{4.10}$$

It is *this* quantity that cannot exceed c.

4.3 Plasma Oscillations

If the electrons in a plasma are displaced from a uniform background of ions, electric fields will be built up in such a direction as to restore the neutrality of the plasma by pulling the electrons back to their original positions. Because of their inertia, the electrons will overshoot and oscillate around their equilibrium positions with a characteristic frequency known as the *plasma frequency*. This oscillation is so fast that the massive ions do not have time to respond to the oscillating field and may be considered as fixed. In Fig. 4.2, the open rectangles represent typical elements of the ion fluid, and the darkened rectangles the alternately displaced elements of the electron fluid. The resulting charge bunching causes a spatially periodic **E** field, which tends to restore the electrons to their neutral positions.

We shall derive an expression for the plasma frequency ω_p in the simplest case, making the following assumptions: (1) There is no magnetic field; (2) there are no thermal motions ($KT = 0$); (3) the ions are fixed in space in a uniform distribution; (4) the plasma is infinite in extent; and (5) the electron motions occur only in the x direction. As a consequence of the last assumption, we have

$$\nabla = \hat{\mathbf{x}}\,\partial/\partial x \quad \mathbf{E} = E\hat{\mathbf{x}} \quad \nabla \times \mathbf{E} = 0 \quad \mathbf{E} = -\nabla\phi \tag{4.11}$$

There is, therefore, no fluctuating magnetic field; this is an electrostatic oscillation.

The electron equations of motion and continuity are

$$mn_e\left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla)\mathbf{v}_e\right] = -en_e\mathbf{E} \tag{4.12}$$

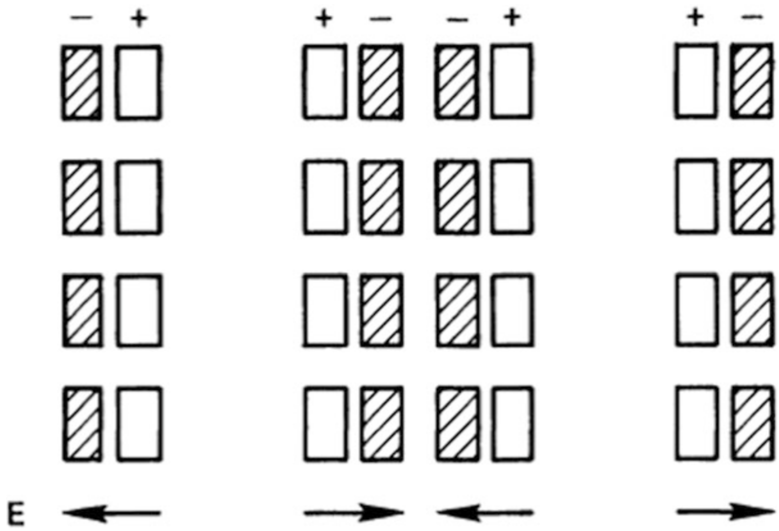


Fig. 4.2 Mechanism of plasma oscillations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0 \tag{4.13}$$

The only Maxwell equation we shall need is the one that does not involve **B**: Poisson's equation. This case is an exception to the general rule of Sect. 3.6 that Poisson's equation cannot be used to find **E**. This is a high-frequency oscillation; electron inertia is important, and the deviation from neutrality is the main effect in this particular case. Consequently, we write

$$\varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \partial \mathbf{E}/\partial \mathbf{x} = e(n_i - n_e) \tag{4.14}$$

Equations (4.12)–(4.14) can easily be solved by the procedure of *linearization*. By this we mean that the amplitude of oscillation is small, and terms containing higher powers of amplitude factors can be neglected. We first separate the dependent variables into two parts: an "equilibrium" part indicated by a subscript 0, and a "perturbation" part indicated by a subscript 1:

$$n_e = n_0 + n_1 \quad \mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_1 \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 \tag{4.15}$$

The equilibrium quantities express the state of the plasma in the absence of the oscillation. Since we have assumed a uniform neutral plasma at rest before the electrons are displaced, we have

$$\nabla n_0 = \mathbf{v}_0 = \mathbf{E}_0 = 0 \tag{4.16}$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial \mathbf{v}_0}{\partial t} = \frac{\partial \mathbf{E}_0}{\partial t} = 0$$

Equation (4.12) now becomes

$$m\left[\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \cdot \overset{0}{\nearrow} \nabla)\mathbf{v}_1\right] = -e\mathbf{E}_1 \tag{4.17}$$

The term $(\mathbf{v}_1 \cdot \nabla)\mathbf{v}_1$ is seen to be quadratic in an amplitude quantity, and we shall linearize by neglecting it. The *linear theory* is valid as long as $|v_1|$ is small enough that such quadratic terms are indeed negligible. Similarly, Eq. (4.13) becomes

$$\begin{aligned} &\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1 + \overset{0}{\nearrow} n_1 \mathbf{v}_1) = 0 \\ &\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 + \mathbf{v}_1 \cdot \overset{0}{\nearrow} \nabla n_0 = 0 \end{aligned} \tag{4.18}$$

In Poisson's equation (4.14), we note that $n_{i0} = n_{e0}$ in equilibrium and that $n_{i1} = 0$ by the assumption of fixed ions, so we have

$$\varepsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1 \tag{4.19}$$

The oscillating quantities are assumed to behave sinusoidally:

$$\begin{aligned} \mathbf{v}_1 &= v_1 e^{i(kx-\omega t)} \hat{\mathbf{x}} \\ n_1 &= n_1 e^{i(kx-\omega t)} \\ \mathbf{E} &= E_1 e^{i(kx-\omega t)} \hat{\mathbf{x}} \end{aligned} \tag{4.20}$$

The time derivative $\partial/\partial t$ can therefore be replaced by $-i\omega$, and the gradient ∇ by $ik\hat{\mathbf{x}}$. Equations (4.17)–(4.19) now become

$$-im\omega v_1 = -eE_1 \tag{4.21}$$

$$-i\omega n_1 = -n_0 ikv_1 \tag{4.22}$$

$$ik\varepsilon_0 E_1 = -en_1 \tag{4.23}$$

Eliminating n_1 and E_1, we have for Eq. (4.21)

$$-im\omega v_1 = -e\frac{-e}{ik\varepsilon_0}\frac{-n_0 ikv_1}{-i\omega} = -i\frac{n_0 e^2}{\varepsilon_0 \omega} v_1 \tag{4.24}$$

If v_1 does not vanish, we must have

$$\omega^2 = n_0 e^2 / m\varepsilon_0$$

The *plasma frequency* is therefore

$$\boxed{\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0 m}\right)^{1/2} \quad \text{rad/sec}} \tag{4.25}$$

Numerically, one can use the approximate formula

$$\omega_p/2\pi = f_p \approx 9\sqrt{n} \ \left(n \text{ in } m^{-3}\right) \tag{4.26}$$

This frequency, depending only on the plasma density, is one of the fundamental parameters of a plasma. Because of the smallness of m, the plasma frequency is usually very high. For instance, in a plasma of density $n = 10^{18}\ \text{m}^{-3}$, we have

$$f_p \approx 9\left(10^{18}\right)^{1/2} = 9 \times 10^9\ \text{sec}^{-1} = 9\,\text{GHz}$$

Radiation at f_p normally lies in the microwave range. We can compare this with another electron frequency: ω_c. A useful numerical formula is

$$f_{ce} \simeq 28\,\text{GHz}/\text{Tesla} \tag{4.27}$$

Thus if $B \approx 0.32$ T and $n \approx 10^{18}\ \text{m}^{-3}$, the cyclotron frequency is approximately equal to the plasma frequency for electrons.

Equation (4.25) tells us that if a plasma oscillation is to occur at all, it must have a frequency depending only on n. In particular, ω does not depend on k, so the group velocity $d\omega/dk$ is zero. The disturbance does not propagate. How this can happen can be made clear with a mechanical analogy (Fig. 4.3). Imagine a number of heavy balls suspended by springs equally spaced in a line. If all the springs are identical, each ball will oscillate vertically with the same frequency. If the balls are started in the proper phases relative to one another, they can be made to form a wave propagating in either direction. The frequency will be fixed by the springs, but the wavelength can be chosen arbitrarily. The two undisturbed balls at the ends will not be affected, and the initial disturbance does not propagate. Either traveling waves or standing waves can be created, as in the case of a stretched rope. Waves on a rope, however, must propagate because each segment is connected to neighboring segments.

Fig. 4.3 Synthesis of a wave from an assembly of independent oscillators

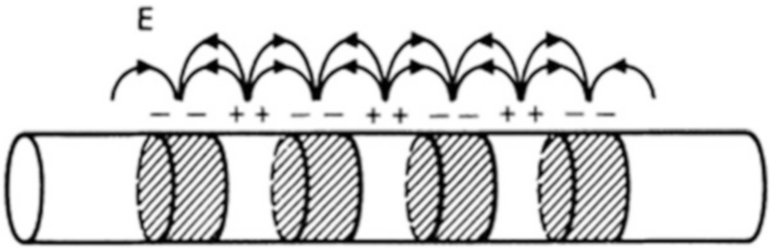


Fig. 4.4 Plasma oscillations propagate in a finite medium because of fringing fields

This analogy is not quite accurate, because plasma oscillations have motions in the direction of **k** rather than transverse to **k**. However, as long as electrons do not collide with ions or with each other, they can still be pictured as independent oscillators moving horizontally (in Fig. 4.3). But what about the electric field? Won't that extend past the region of initial disturbance and set neighboring layers of plasma into oscillation? In our simple example, it will not, because the electric field due to equal numbers of positive and negative infinite plane charge sheets is zero. In any finite system, however, plasma oscillations will propagate. In Fig. 4.4, the positive and negative (shaded) regions of a plane plasma oscillation are confined in a cylindrical tube. The fringing electric field causes a coupling of the disturbance to adjacent layers, and the oscillation does not stay localized.

Problems

4.2 The plasma density in the lower ionosphere has been measured during satellite re-entry to be about $10^{18}\ \mathrm{m}^{-3}$ at 50 km altitude, 10^{17} at 70 km, ad 10^{14} at 85 km. What are the plasma frequencies there?

4.3 Calculate the plasma frequency with the ion motions included, thus justifying our assumption that the ions are essentially fixed. (Hint: include the term n_{1i} in Poisson's equation and use the ion equations of motion and continuity.)

4.4 For a simple plasma oscillation with fixed ions and a space-time behavior exp[i $(kx - \omega t)$], calculate the phase δ for ϕ_1, E_1, and v_1 if the phase of n_1, is zero. Illustrate the relative phases by drawing sine waves representing n_1, ϕ_1, E_1, and v_1 (a) as a function of x at $t = 0$, (b) as a function of t at $x = 0$ for $\omega/k > 0$, and (c) as a function of t at $x = 0$ for $\omega/k < 0$. Note that the time patterns can be obtained by translating the x patterns in the proper direction, as if the wave were passing by a fixed observer.

4.5 By writing the linearized Poisson's equation used in the derivation of simple plasma oscillations in the form

$$\nabla \cdot (\varepsilon \mathbf{E}) = 0$$

derive an expression for the dielectric constant ε applicable to high-frequency longitudinal motions.

4.4 Electron Plasma Waves

There is another effect that can cause plasma oscillations to propagate, and that is thermal motion. Electrons streaming into adjacent layers of plasma with their thermal velocities will carry information about what is happening in the oscillating region. The plasma *oscillation* can then properly be called a plasma *wave*. We can easily treat this effect by adding a term $-\nabla p_e$ to the equation of motion Eq. (4.12). In the one-dimensional problem, γ will be three, according to Eq. (3.53). Hence,

$$\nabla p_e = 3KT_e \nabla n_e = 3KT_e \nabla (n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x} \hat{\mathbf{x}}$$

and the linearized equation of motion is

$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x} \tag{4.28}$$

Note that in linearizing we have neglected the terms $n_1 \ \partial v_1/\partial t$ and $n_1 E_1$ as well as the $(\mathbf{v}_1 \cdot \nabla)\mathbf{v}_1$ term. With Eq. (4.20), Eq. (4.28) becomes

$$-im\omega n_0 v_1 = -en_0 E_1 - 3KT_e ikn_1 \tag{4.29}$$

E_1 and n_1 are still given by Eqs. (4.23) and (4.22), and we have

$$im\omega n_0 v_1 = \left[en_0 \left(\frac{-e}{ik\varepsilon_0} \right) + 3KT_e ik \right] \frac{n_0 ik}{i\omega} v_1$$

$$\omega^2 v_1 = \left(\frac{n_0 e^2}{\varepsilon_0 m} + \frac{3KT_e}{m} k^2 \right) v_1$$

$$\boxed{\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{\mathrm{th}}^2} \tag{4.30}$$

where $v_{\mathrm{th}}^2 \equiv 2KT_e/m$. The frequency now depends on k, and the group velocity is finite:

$$2\omega\, d\omega = \frac{3}{2} v_{\mathrm{th}}^2 2k\, dk$$

$$v_g = \frac{d\omega}{dk} = \frac{3}{2} \frac{k}{\omega} v_{\mathrm{th}}^2 = \frac{3}{2} \frac{v_{\mathrm{th}}^2}{v_\phi} \tag{4.31}$$

That v_g is always less than c can easily be seen from a graph of Eq. (4.30). Figure 4.5 is a plot of the *dispersion relation* $\omega(k)$ as given by Eq. (4.30). At any point P on this curve, the slope of a line drawn from the origin gives the phase velocity ω/k.

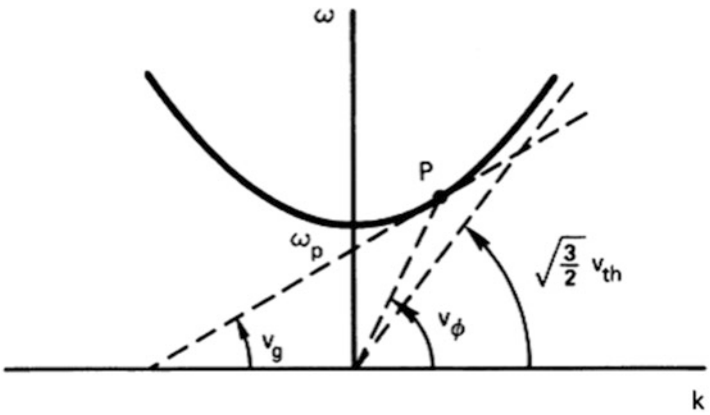


Fig. 4.5 Dispersion relation for electron plasma waves (Bohm–Gross waves)

The slope of the curve at P gives the group velocity. This is clearly always less than $(3/2)^{1/2}\ v_{\mathrm{th}}$, which, in our nonrelativistic theory, is much less than c. Note that at large k (small λ), information travels essentially at the thermal velocity. At small k (large λ), information travels more slowly than v_{th} even though v_ϕ is greater than v_{th}. This is because the density gradient is small at large λ, and thermal motions carry very little net momentum into adjacent layers.

The existence of plasma oscillations has been known since the days of Langmuir in the 1920s. It was not until 1949 that Bohm and Gross worked out a detailed theory telling how the waves would propagate and how they could be excited. A simple way to excite plasma waves would be to apply an oscillating potential to a grid or a series of grids in a plasma; however, oscillators in the GHz range were not generally available in those days. Instead, one had to use an electron beam to excite plasma waves. If the electrons in the beam were bunched so that they passed by any fixed point at a frequency f_p, they would generate an electric field at that frequency and excite plasma oscillations. It is not necessary to form the electron bunches beforehand; once the plasma oscillations arise, they will bunch the electrons, and the oscillations will grow by a positive feedback mechanism. An experiment to test this theory was first performed by Looney and Brown in 1954. Their apparatus was entirely contained in a glass bulb about 10 cm in diameter (Fig. 4.6). A plasma filling the bulb was formed by an electrical discharge between the cathodes K and an anode ring A under a low pressure (3×10^{-3} Torr) of mercury vapor. An electron beam was created in a side arm containing a negatively biased filament. The emitted electrons were accelerated to 200 V and shot into the plasma through a small hole. A thin, movable probe wire connected to a radio receiver was used to pick up the oscillations. Figure 4.7 shows their experimental results for f^2 vs. discharge current, which is generally proportional to density. The points show a linear dependence, in rough agreement with Eq. (4.26). Deviations from the straight line could be attributed to the $k^2 v_{\mathrm{th}}^2$ term in Eq. (4.30). However, not all frequencies were observed; k had to be such that an integral number of half wavelengths fit along