Plasma Regimes and Plasma Approximations

Waleed Moslem
Professor of Theoretical Plasma Physics

Problem of plasma science

Plasma physics is usually not a precise science. It is rather a web of overlapping points of view, each modeling a limited range of behavior. Understanding of plasmas is developed by studying these various points of view, all the while keeping in mind the linkages between the points of view.

علم البلازما ليس علم محدد المعالم ، إنها شبكة من وجهات النظر المتداخلة ، كل منها يمثل نطاقًا محدودًا من السلوك. يتم تطوير فهم البلازما من خلال دراسة وجهات النظر المختلفة هذه ، مع مراعاة الروابط بين وجهات النظر.

Aim of the lecture

You should learn

- Types of plasma & Plasma components
- Forces in plasma
- Plasma applications and observations
- Plasma approximations
- Examples

Types of plasmas

- **(I) Classical plasma**

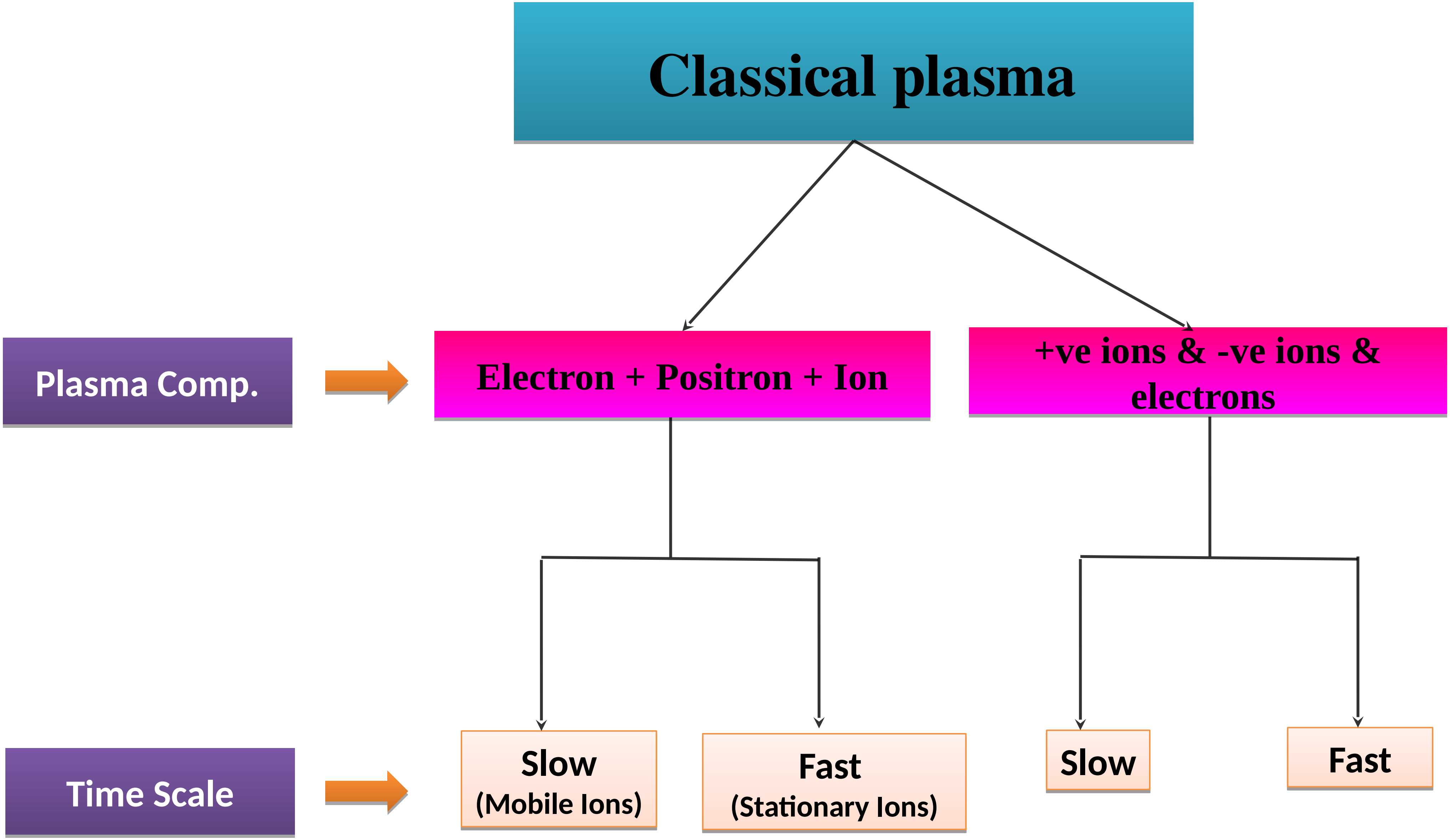
  +ve ions / electrons / -ve ions / positrons

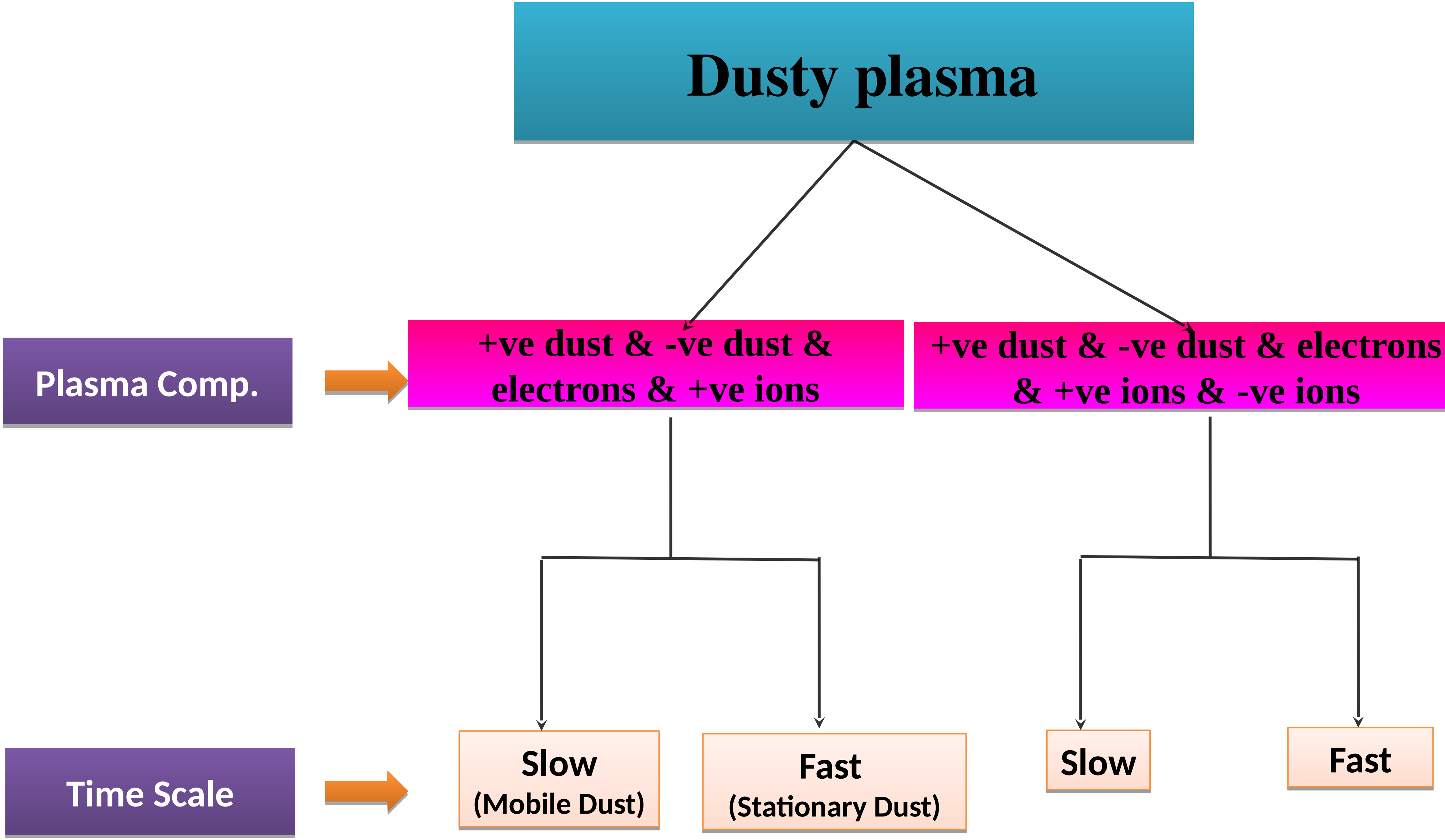
- **(II) Dusty (complex) plasma**

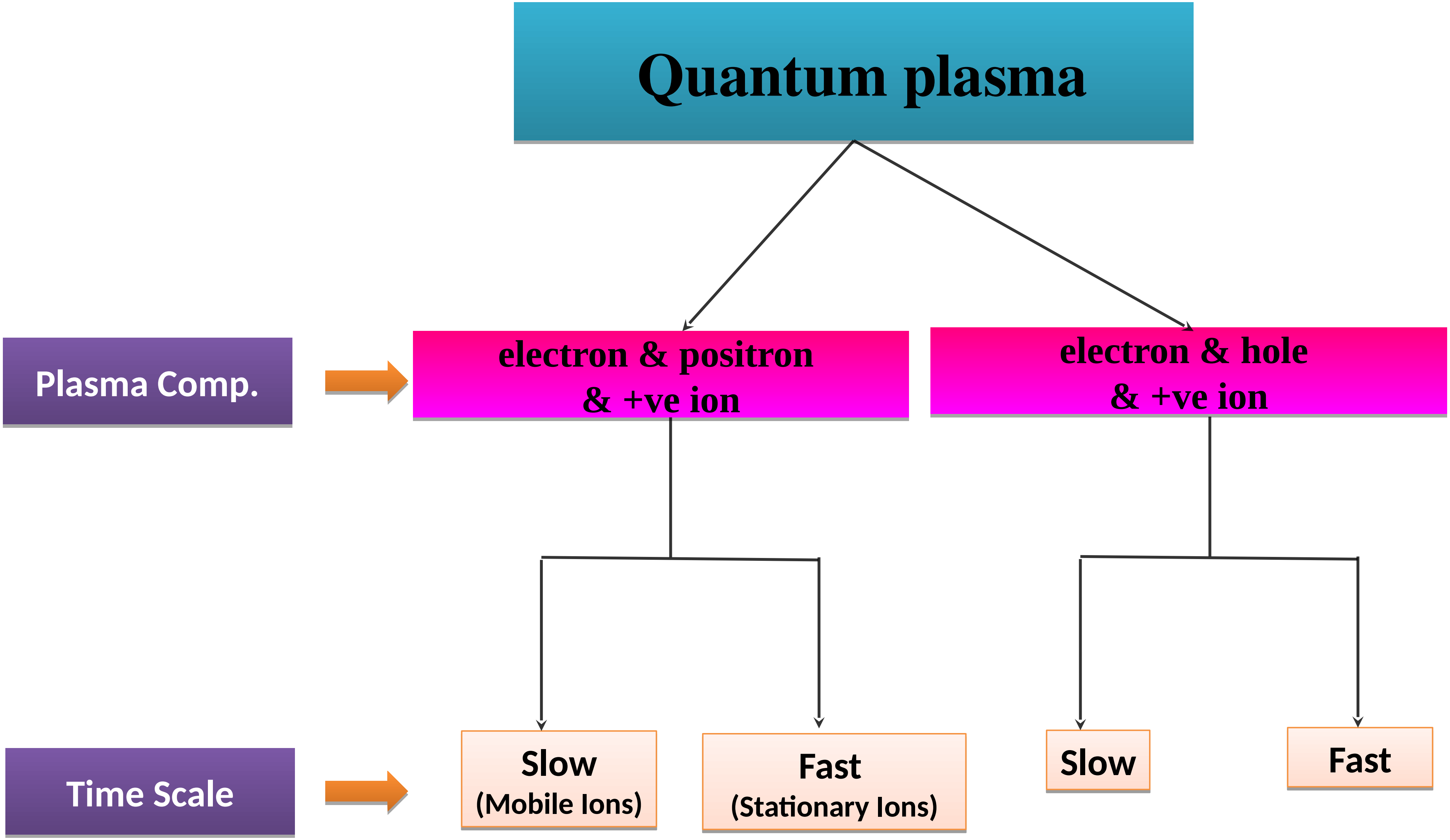
  +ve dust / -ve dust / +ve ions / electrons / -ve ions

- **(III) Quantum plasma**

  Electrons / positrons / holes / +ve ions


Classical plasma
Plasma Comp.
Electron + Positron + Ion
+ve ions & -ve ions & electrons
Time Scale
Slow
(Mobile Ions)
Fast
(Stationary Ions)
Slow
Fast


Dusty plasma
Plasma Comp.
+ve dust & -ve dust & electrons & +ve ions
+ve dust & -ve dust & electrons & +ve ions & -ve ions
Time Scale
Slow (Mobile Dust)
Fast (Stationary Dust)
Slow
Fast


Quantum plasma
Plasma Comp.
electron & positron
& +ve ion
electron & hole
& +ve ion
Time Scale
Slow
(Mobile Ions)
Fast
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Fast

Forces in plasma

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force
- **15 Forces**

Types & Forces

- Classical
- Dusty
- Quantum

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force

Types & Forces, cont.

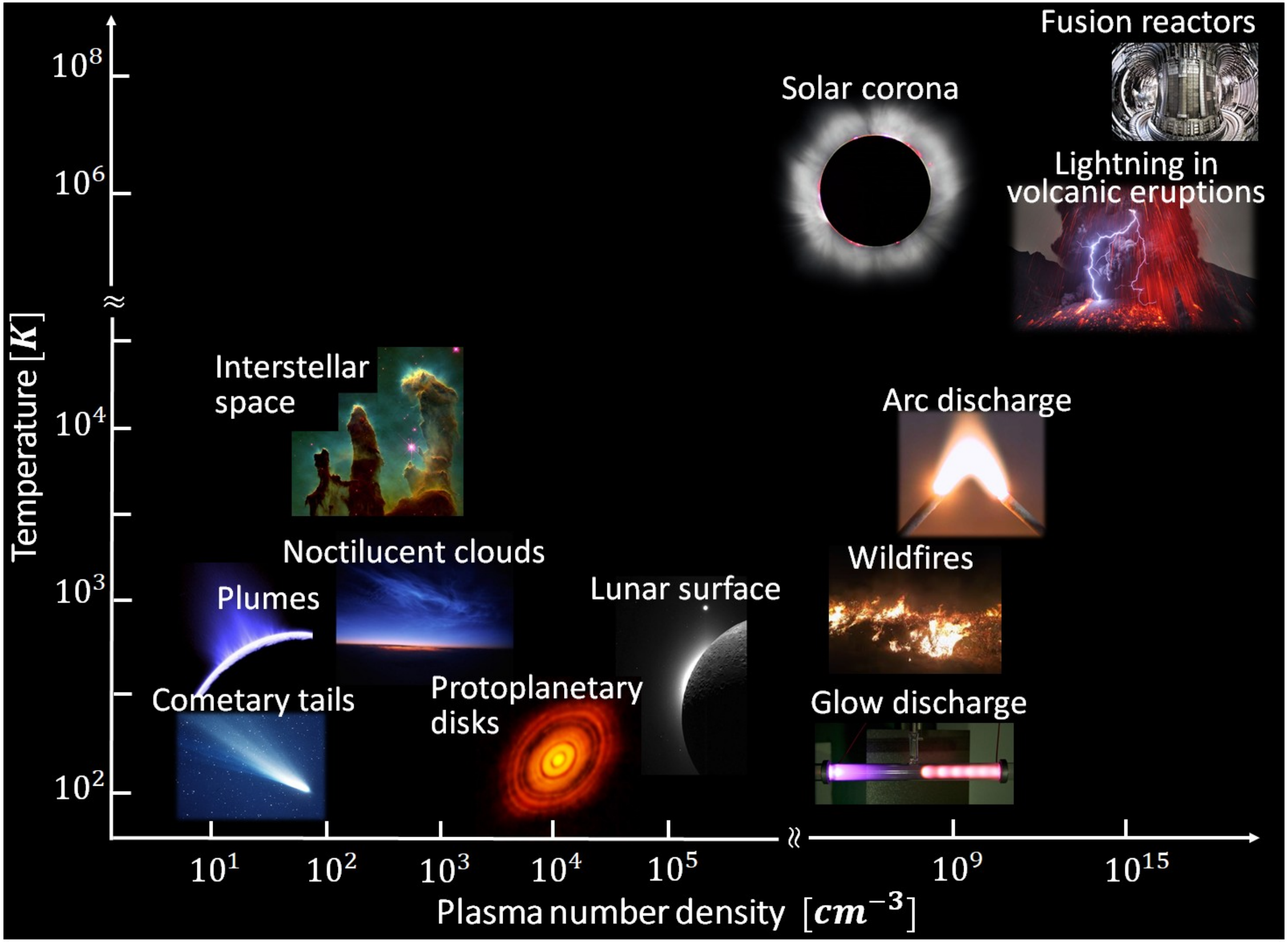
What are the criteria to study plasma?

- ✔ **Understanding each force**
- ✔ **Select a suitable plasma type**
- ✔ **Select a suitable plasma model**

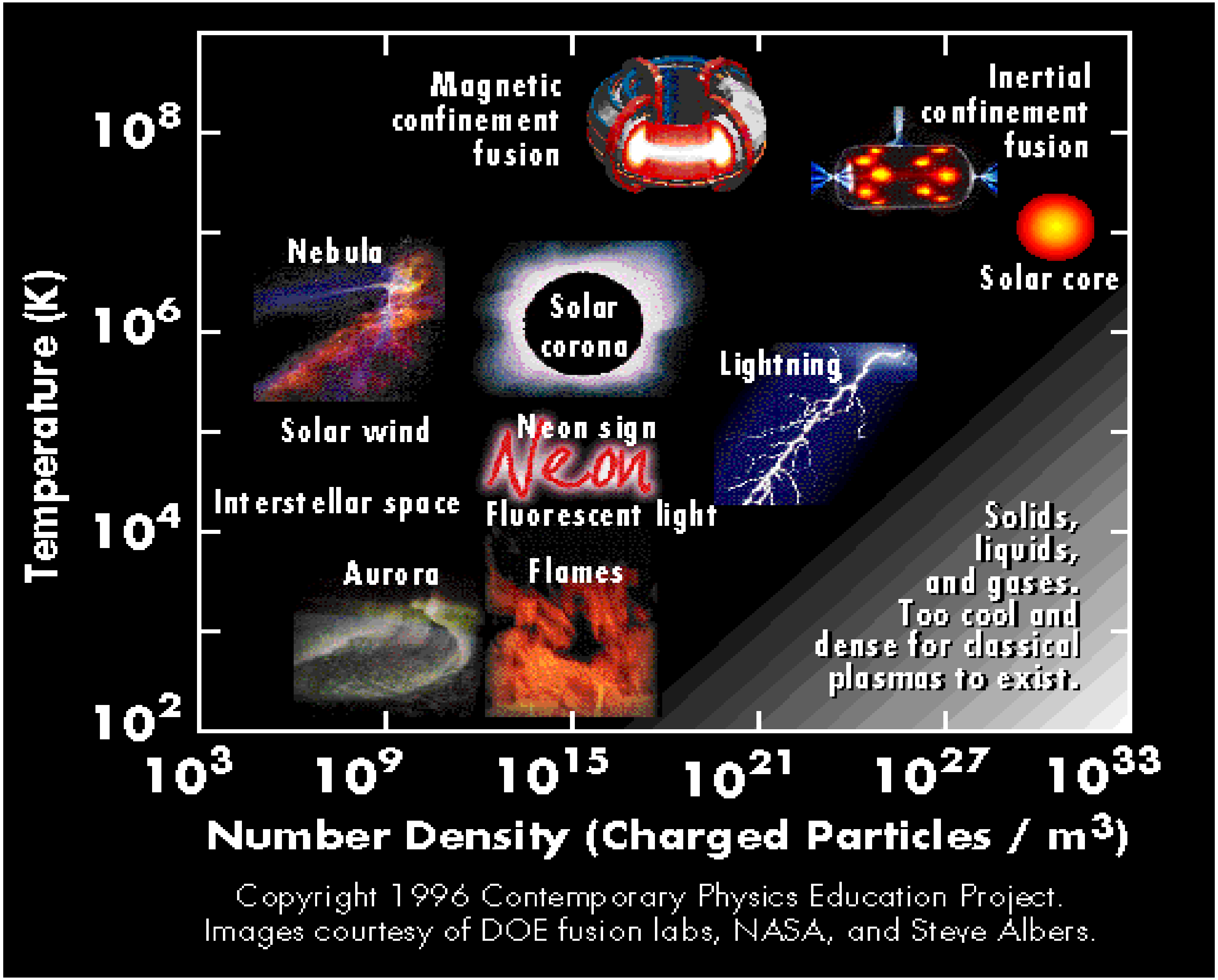
What else do you need

- ✔ **Knowing the physics of the Exp. / Appl. / Obs.**

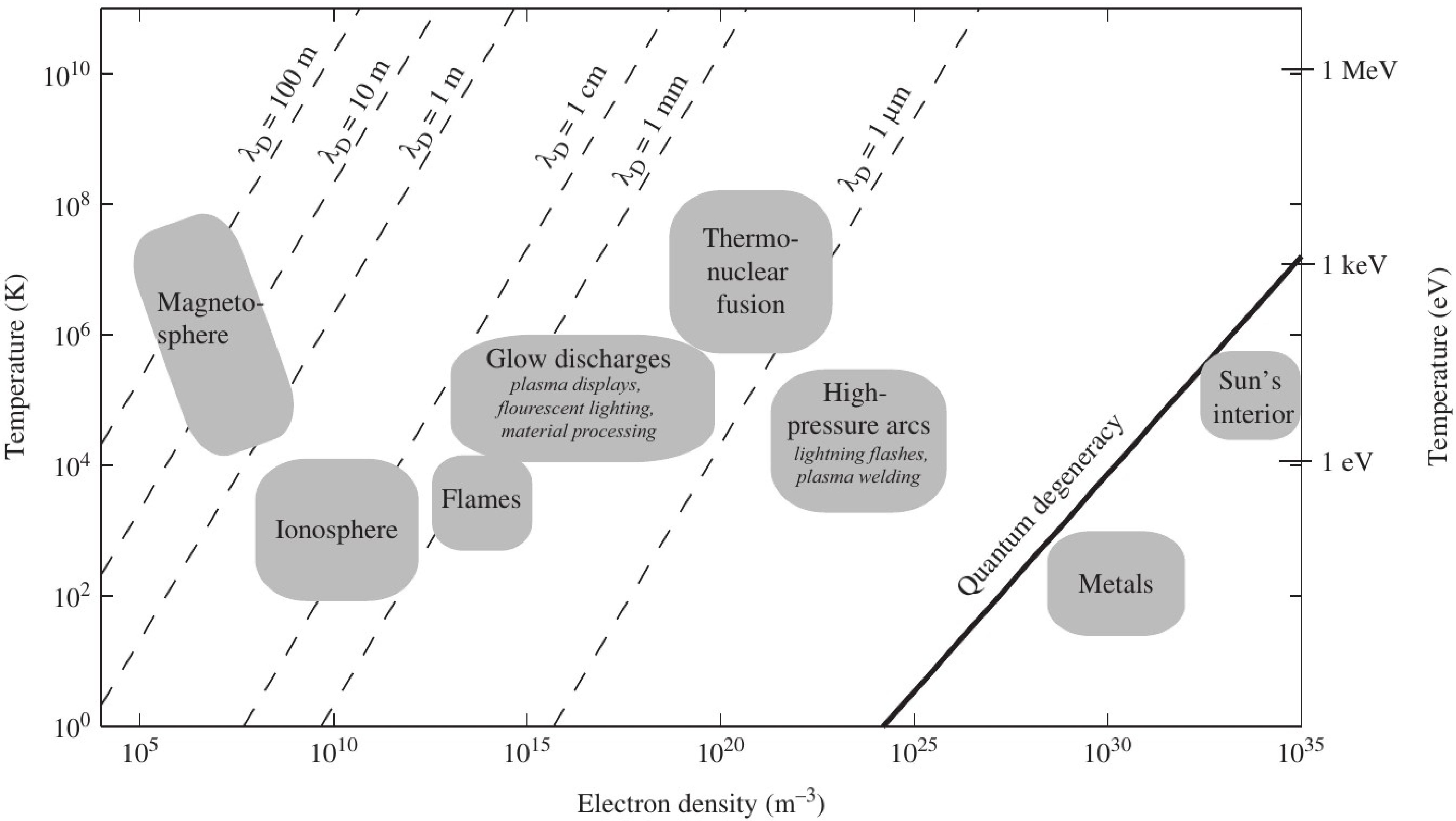
Plasma applications & observations



Plasma applications & observations, cont.



Plasma applications & observations, cont.



Types & Forces

- Classical
- Dusty
- Quantum

Experiment

OR

Application

OR

Observation

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force

Again: what are the criteria to study plasma?

- ✔ Understanding each force
- ✔ Select a suitable plasma type
- ✔ Select a suitable plasma model
- ✔ Plasma applications and observations

What else do you need

- ✔ Learning the plasma approximations

Plasma approximations

- **All approximations are wrong**
- **''The plasma approximation'' is, in the words of Pauli, ''not even wrong''**

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$(1+x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 - \tfrac{5}{128}x^4 + \tfrac{7}{256}x^5 - \dots$$

$$(1+x)^{-\frac{1}{2}} = 1 - \tfrac{1}{2}x + \tfrac{3}{8}x^2 - \tfrac{5}{16}x^3 + \tfrac{35}{128}x^4 - \tfrac{63}{256}x^5 + \dots$$

Plasma approximations

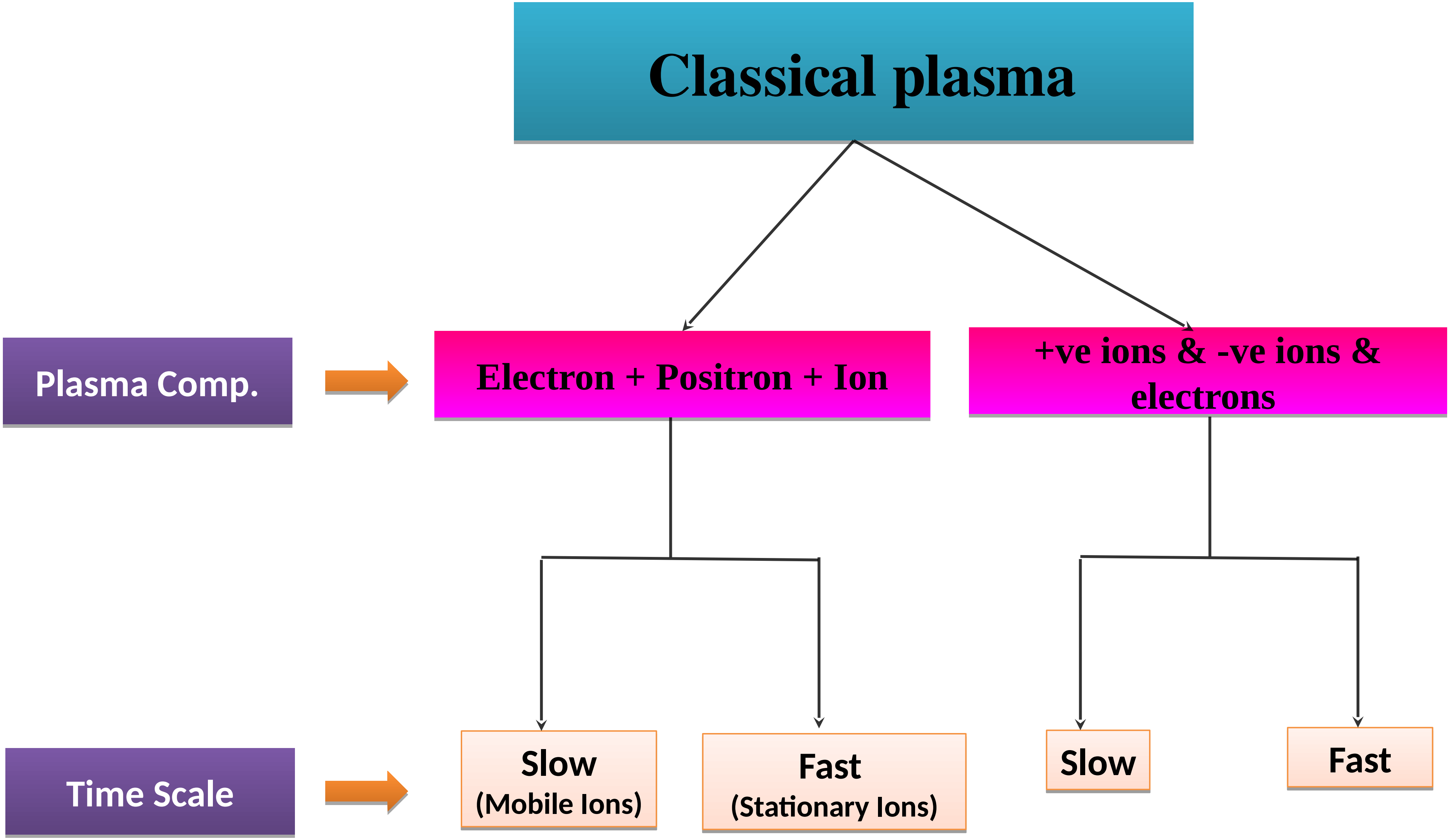
Useful expressions

- **What are the scale size in plasma?**

  Space or Laboratory

- **What are the characteristics time scale in plasma?**

  Mass of moving species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(Mobile Ions)
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(Stationary Ions)
Slow
Fast

Plasma approximations

Useful expressions

- **What are the scale size in plasma?**

Electron Debye radius $\lambda_{De} = \left(\frac{T_e}{4\pi n_e e^2}\right)^{1/2}$

Electron gyroradius $\rho_e = \frac{V_{te}}{\omega_{ce}}$, where $V_{te} = \left(\frac{T_e}{m_e}\right)^{1/2}$, $\omega_{ce} = \frac{eB_0}{m_e c}$.

Ion gyroradius $\rho_i = \frac{V_{ti}}{\omega_{ci}}$, where $V_{ti} = \left(\frac{T_i}{m_i}\right)^{1/2}$, $\omega_{ci} = \frac{eB_0}{m_i c}$.

Ion sound gyroradius $\rho_s = \frac{C_s}{\omega_{ci}}$.

Ion sound speed $C_s = \frac{T_e}{m_i}$.

Electron skin depth $\lambda_e = \frac{c}{\omega_{pe}}$.

Ion skin depth $\lambda_i = \frac{c}{\omega_{pi}}$.

Plasma approximations

Useful expressions

- **Importance of the scale size in plasma**

Electron gyroradius $\ll$ Ion gyroradius

$$\rho_e = \frac{V_{te}}{\omega_{ce}} \ll \rho_i = \frac{V_{ti}}{\omega_{ci}}$$

Compare with plasma system size (L)

- Magnetized
- Unmagnetized

Electron Debye radius $\lambda_{De} = \left(\frac{T_e}{4\pi n_e e^2}\right)^{1/2}$

When the Debye length being larger than the system implies that quasi-neutrality cannot be assumed → Sheath

$\lambda_D < L < l_s$

- Skin depth & Debye lenth
- Thermally non-relativistic & Low-temperature
- Some of the wave physics are not accessible
- To observe it, the wave should have fast time scale

$$\frac{l_s}{\lambda_D} = \sqrt{\frac{m_e c^2}{\kappa T_e}}.$$

$l_s < L < \lambda_D$ Thermally relativistic & High temperature

Plasma approximations

Useful expressions

- **What are the characteristic time scale in plasma?**

Inverse plasma periods ω_{pe}^{-1}, ω_{pi}^{-1}

$$\omega_{pe} = \left(\frac{4\pi n_0 e^2}{m_e}\right)^{1/2} \qquad \omega_{pi} = \left(\frac{4\pi n_0 e^2}{m_i}\right)^{1/2}$$

Inverse gyro periods ω_{ce}^{-1}, ω_{ci}^{-1}

$$\omega_{ci} = eB_0/m_i c.$$

Inverse collision frequency (collision period) $\nu_{i\alpha}^{-1}$, $\nu_{e\alpha}^{-1}$

Plasma approximations

Useful expressions

- **Importance of characteristic time scale in plasma**

$\omega \ll \omega_{ci}$ Low frequency

$\omega_{pi} \gg \omega_{ci}$ Dense magnetized plasma

$|\partial/\partial t| \ll \nu_{ei}$ = electron ion collision frequency

$|D/Dt| \gg \omega_{pi}, \omega_{ci}$ The ions do not participate in the wave dynamics

$\nu_{in} >> \omega_{ci}$ the ions are unmagnetized

$\nu_{e} << \omega_{ce}$ the electrons are magnetized

Plasma approximations

Importance of study plasma waves

1) **Omnipresent in space & laboratory**

2) **Better understanding of the ionosphere or magnetosphere ---> energy transport.**

3) **Provide the information on the generation of planetary radio emissions from magnetospheres**

4) **Plasma waves transport particles ---> loss of planetary atmosphere ---> accelerate particles to high energies.**

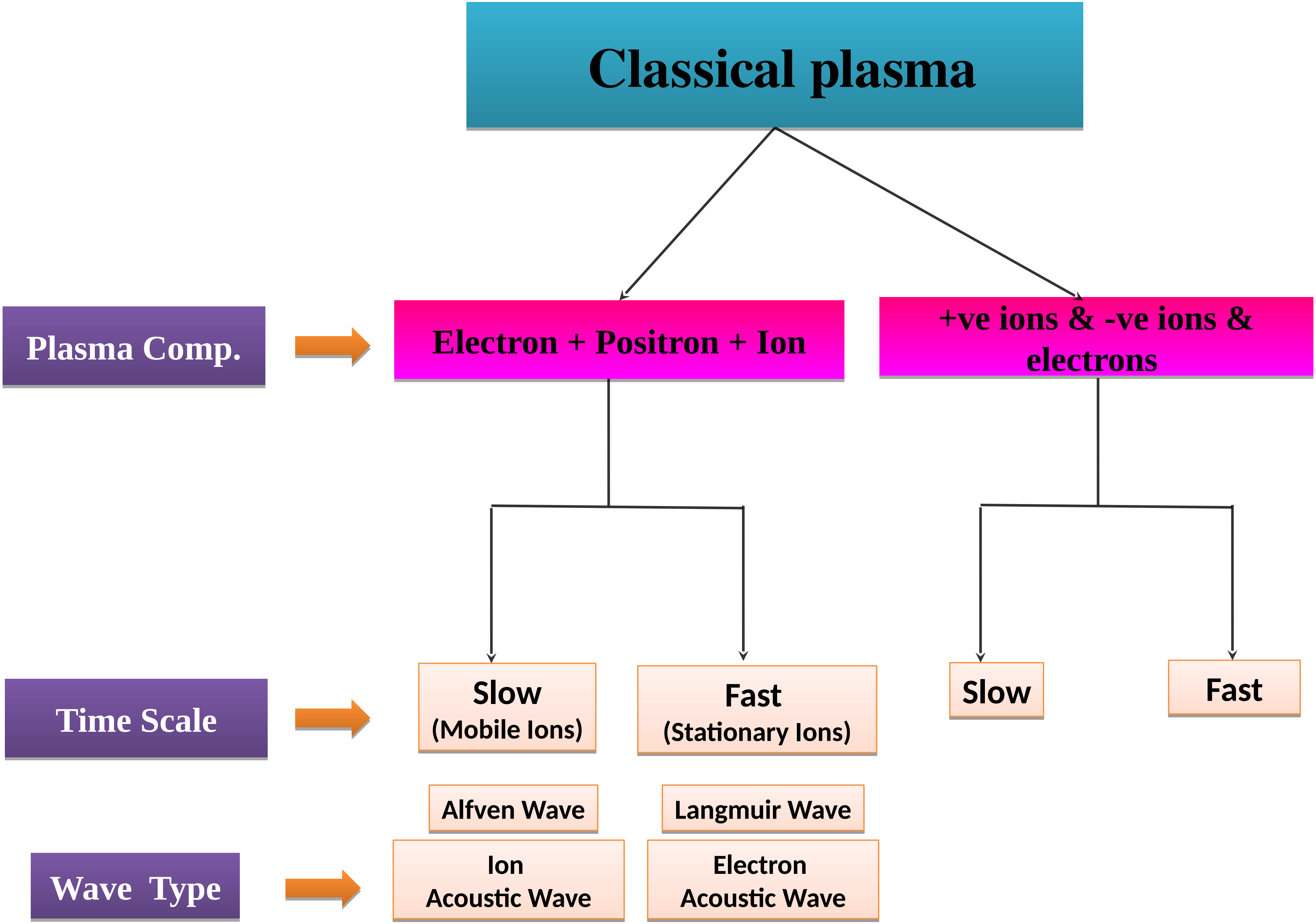
5) **Diagnostic tool ---> gives information about the local plasma parameters and plasma properties**

Plasma approximations

“HOW” would a wave be generated in a plasma?

Or “WHY” would a plasma give rise to a wave?

Perturbation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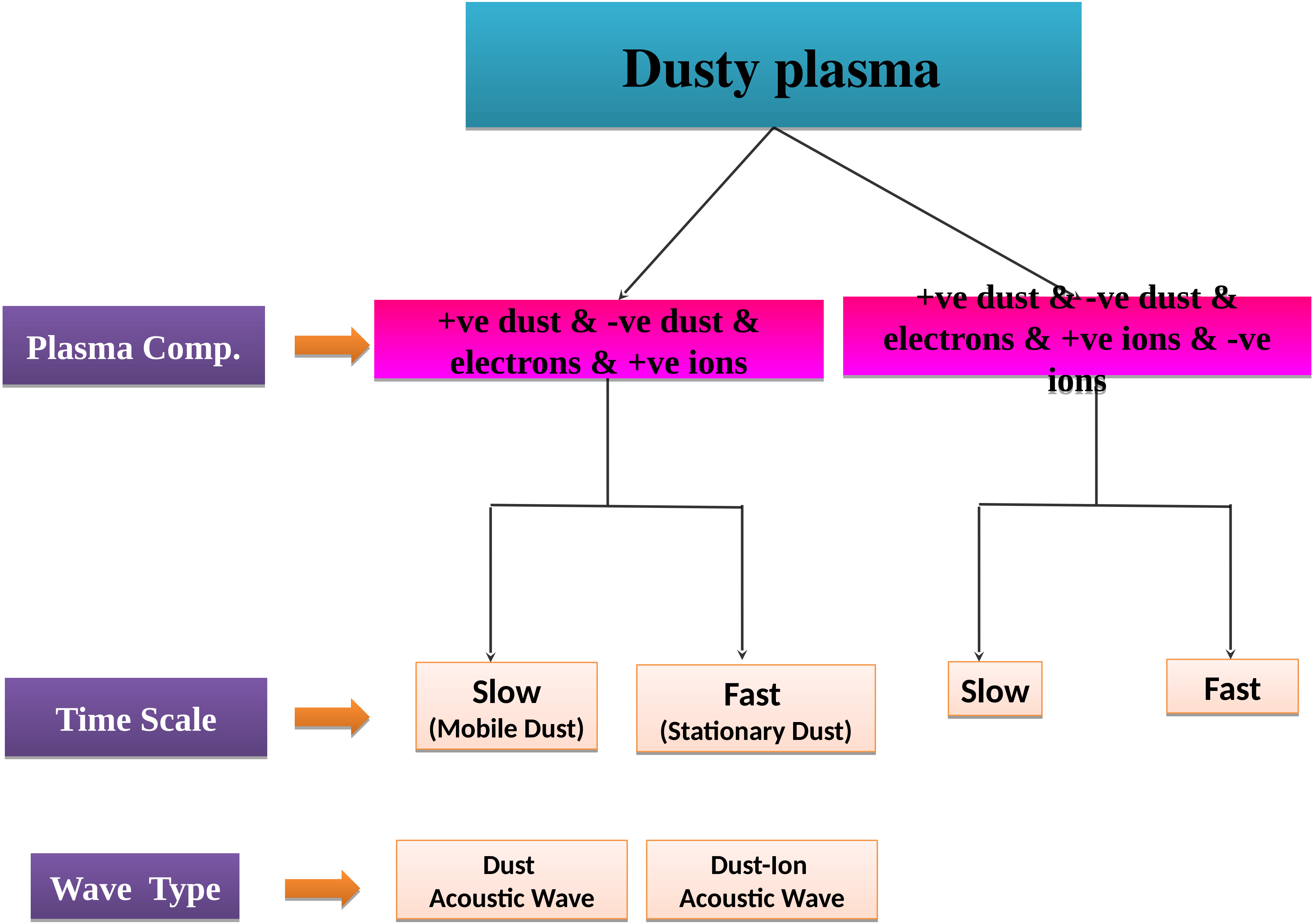
Fast
Wave Type
Alfven Wave
Ion Acoustic Wave
Langmuir Wave
Electron Acoustic Wave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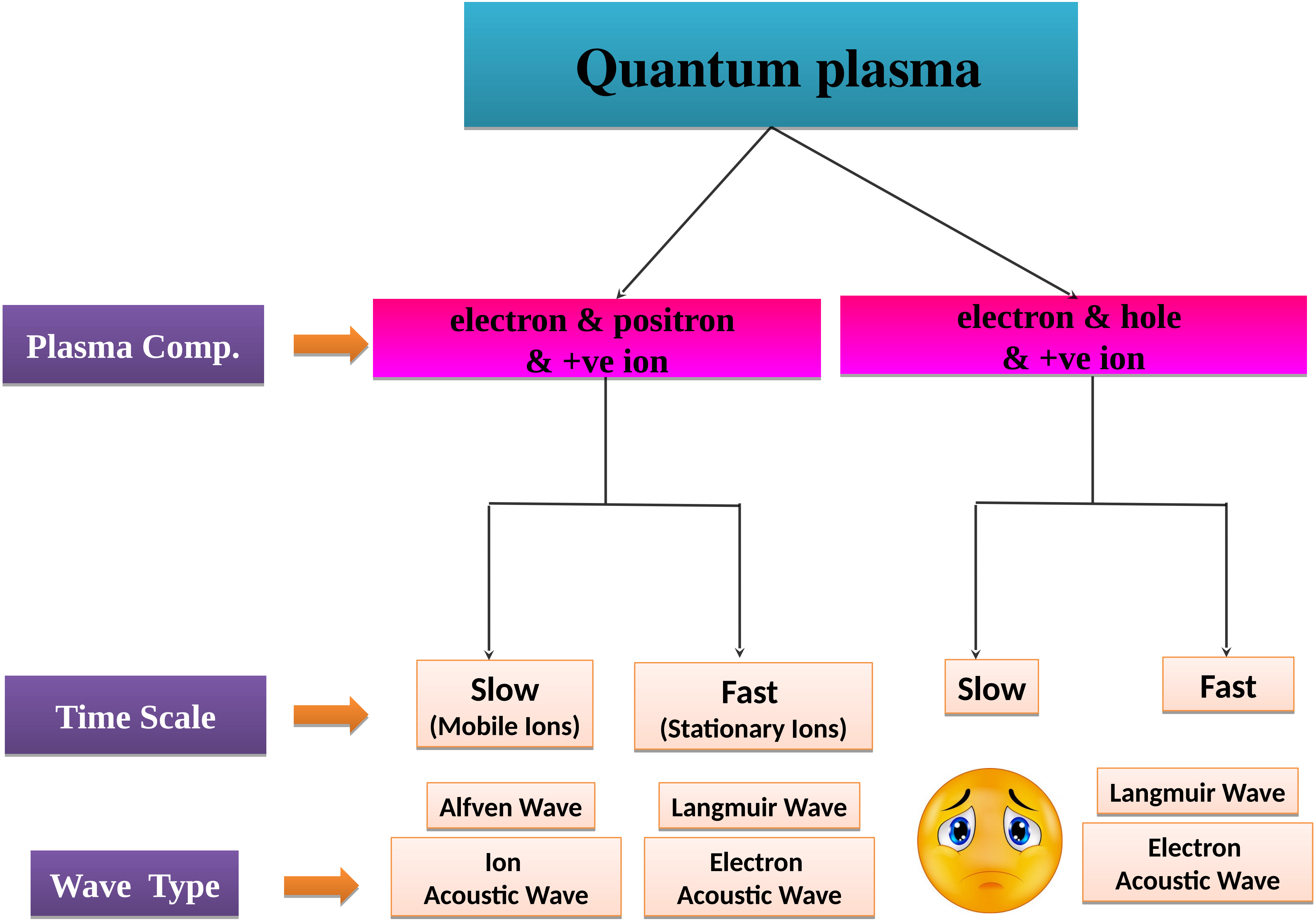
Slow
Fast
Wave Type
Dust
Acoustic Wave
Dust-Ion
Acoustic Wave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Wave Type
Alfven Wave
Ion Acoustic Wave
Langmuir Wave
Electron Acoustic Wave
Langmuir Wave
Electron Acoustic Wave

Plasma approximations

How to study waves

- **Dispersion relation ---> small wave amplitude --->**

  time & wavelength ---> time^{-1} & wavelength^{-1}
- **Perturbation methods ---> small -but finite- amplitude**
- **Exact solution ---> large amplitude**
- **Numerical solutions**

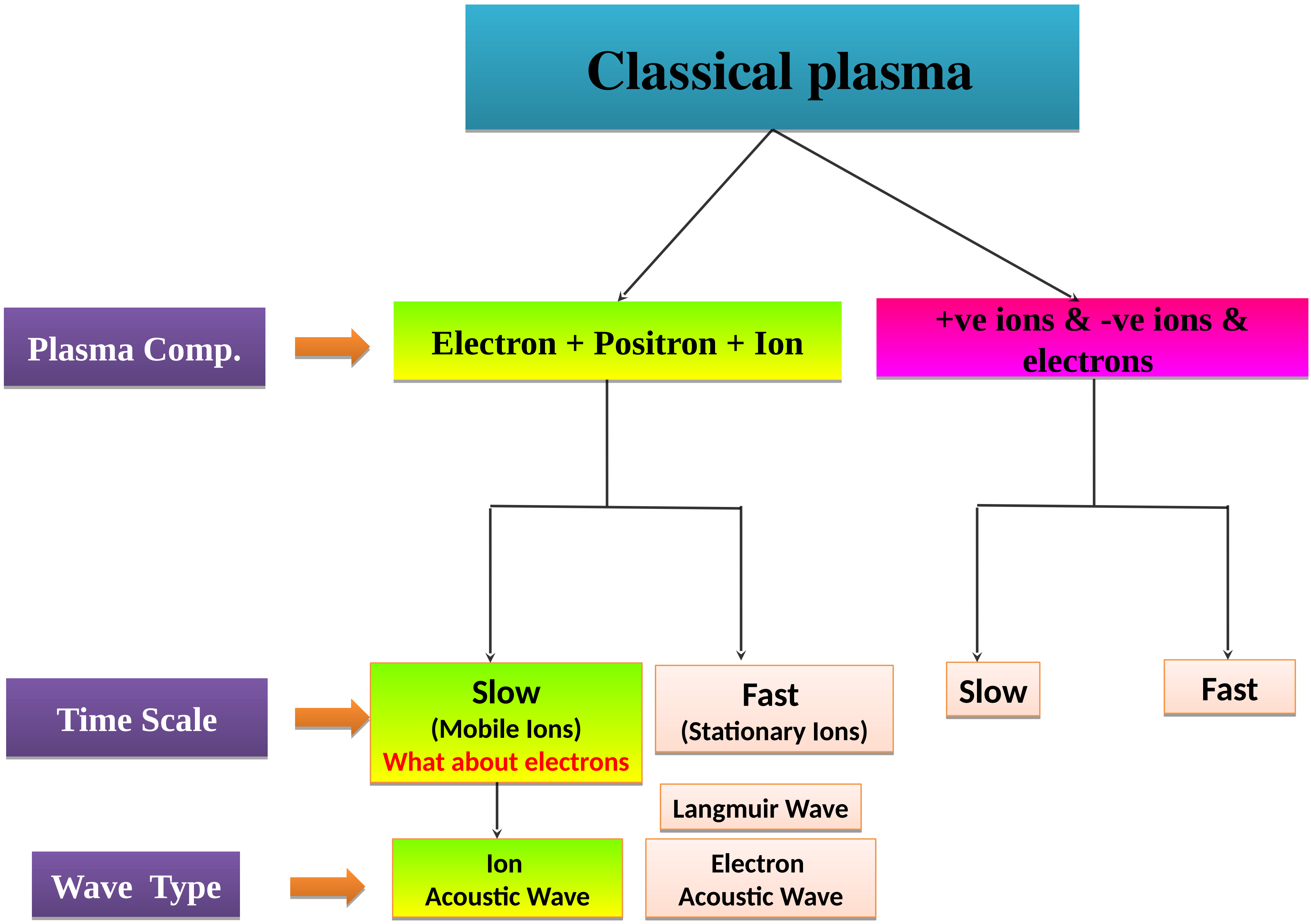
Plasma approximations

Example of approximation

---> Plasma component --->

Electron + Positron + Ion ---> Mass

---> Time scale --->Wave type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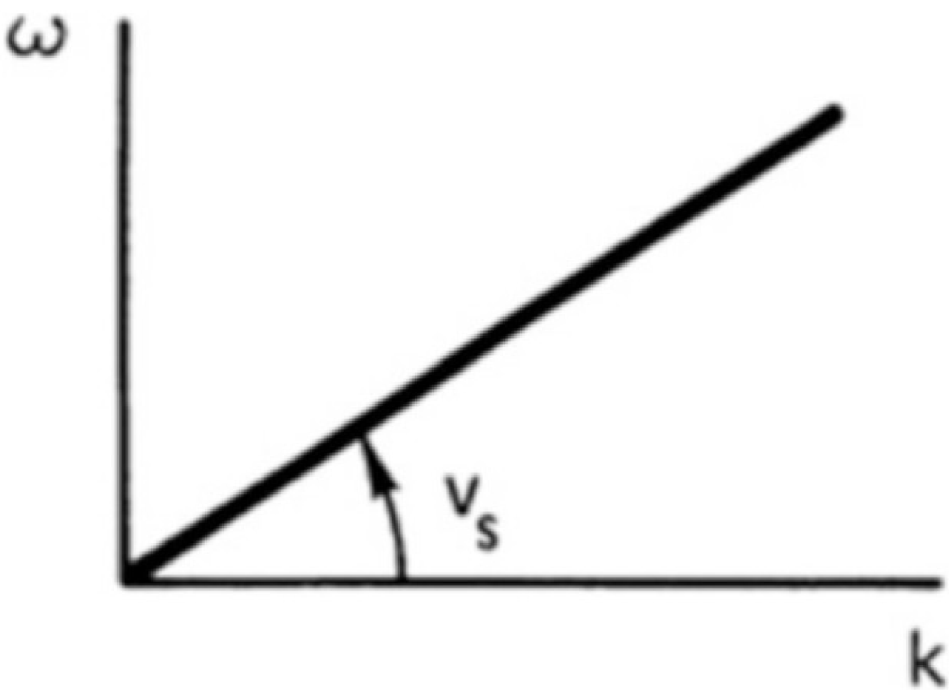
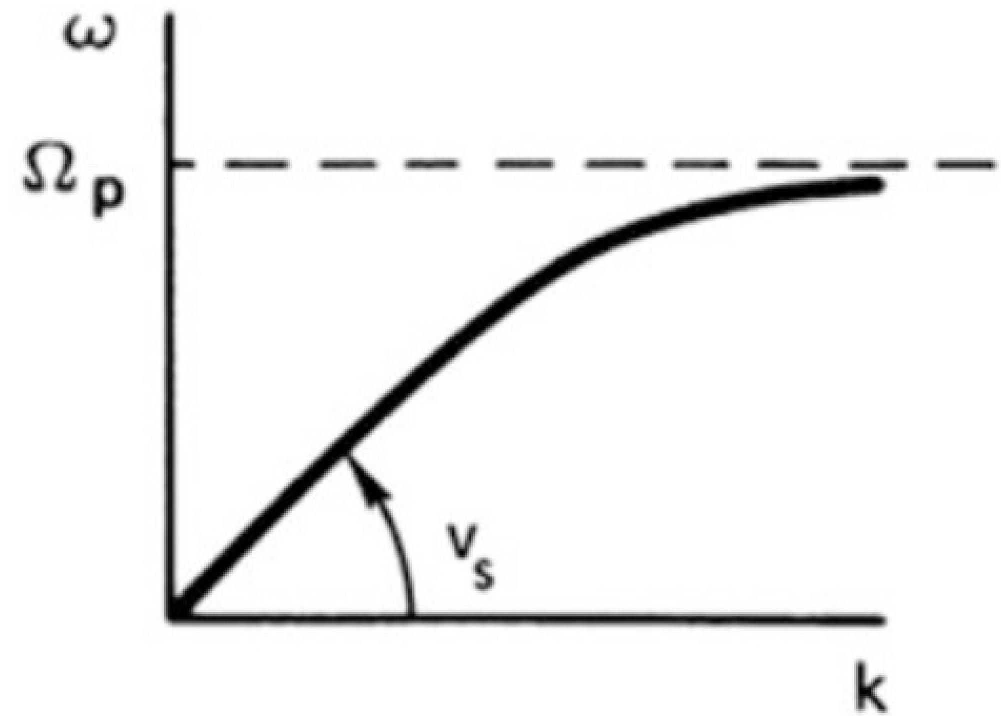
Time Scale
Slow
(Mobile Ions)
What about electrons
Fast
(Stationary Ions)
Slow
Fast
Langmuir Wave
Wave Type
Ion
Acoustic Wave
Electron
Acoustic Wave

Plasma approximations

- **Ion acoustic waves**





$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M}\right)^{1/2}$$

$$\frac{\omega}{k} = \left(\frac{KT_e}{M}\frac{1}{1 + k^2\lambda_D^2} + \frac{\gamma_i KT_i}{M}\right)^{1/2}$$

Plasma approximations

- **Validity of the Plasma Approximation**

$$\varepsilon_0 \nabla \cdot \mathbf{E}_1 = \varepsilon_0 k^2 \phi_1 = e(n_{i1} - n_{e1})$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M} \right)^{1/2} \qquad \frac{\omega}{k} = \left(\frac{KT_e}{M} \frac{1}{1 + k^2 \lambda_\mathrm{D}^2} + \frac{\gamma_i KT_i}{M} \right)^{1/2}$$

Plasma approximations

- **Importance of dispersion relation**

$$E_x = E_0 \exp\left(ikx - i\omega t\right)$$

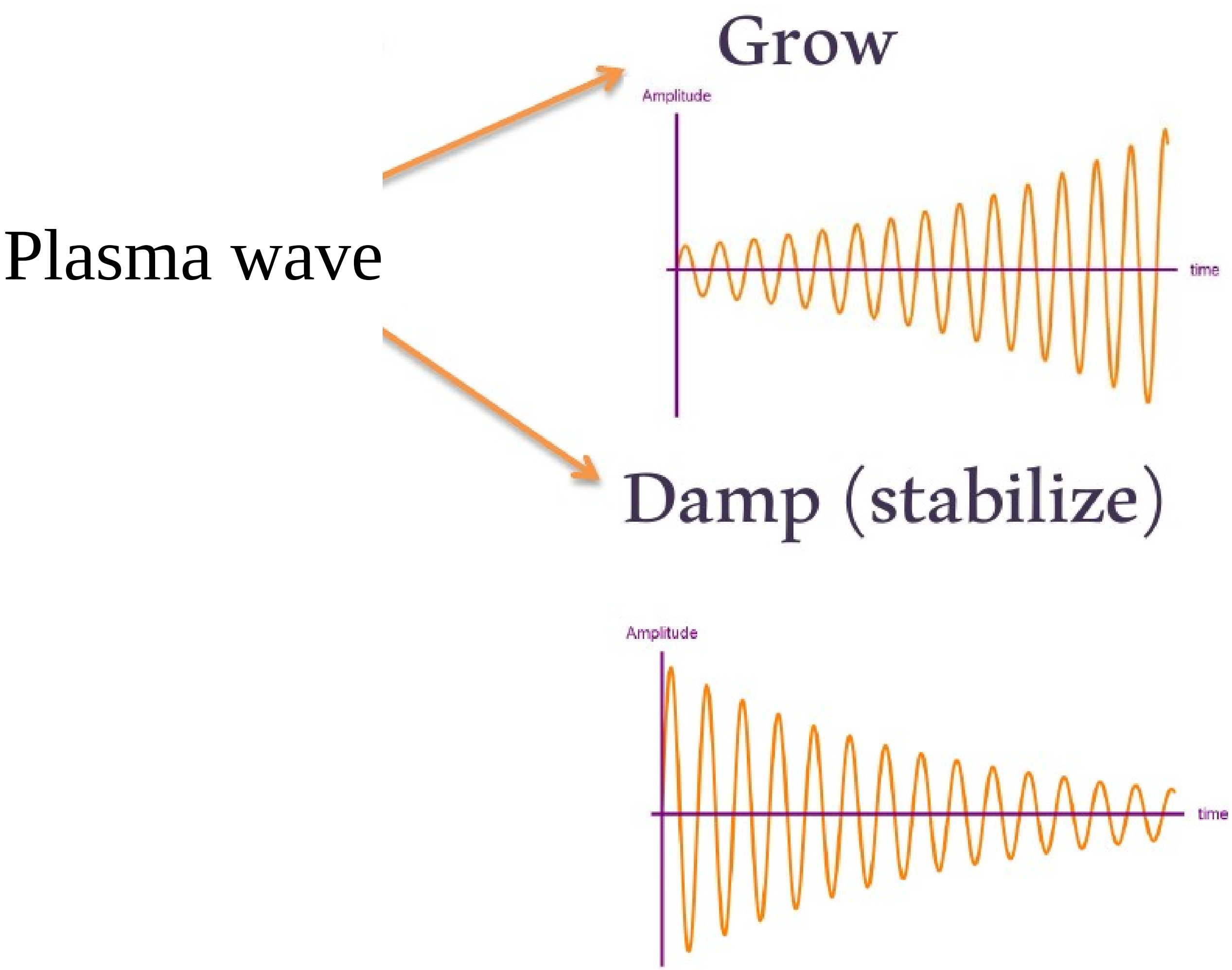
$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M}\right)^{1/2}$$

$$\omega = \omega_r + i\omega_i$$

- **Aim: study the evolution of wave with time**
- **Possibilities of real frequency (+ or –) ---> what happen?**
- **Possibilities of imaginary frequeny (+ or –) ---> what happen?**

Plasma approximations

- **Importance of dispersion relation**



Growing amplitudes, as a result of instabilities, which transfer energy from the plasma particles to the wave field.

Q: What happen if the imaginary frequency equal zero?

A: Wave has only real solutions & there is neither temporal damping nor instability

Plasma approximations

- **Boltzmann relation for electrons**

$$mn\left[\frac{\partial v_z}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})v_z\right] = qnE_z - \frac{\partial p}{\partial z}$$

$$\frac{\partial v_z}{\partial t} = \frac{q}{m}E_z - \frac{\gamma KT}{mn}\frac{\partial n}{\partial z}$$

$$\mathbf{E} = -\nabla\phi$$

$$m \rightarrow 0 \quad q = -e$$

$$qE_z = e\frac{\partial \phi}{\partial z} = \frac{\gamma KT_e}{n}\frac{\partial n}{\partial z}$$

$$n = n_0 \exp\left(e\phi/KT_e\right)$$

Plasma approximations

- **Boltzmann relation for electrons**

$$n = n_0 \exp\left(e\phi / KT_e\right)$$

What are the losing information/effects due to the approximation?

- **Velocity with time & space**
- **Mass**
- **Magnetic field**
- **Other forces**
- **What is the final decision ---> Obs. or Exp.**

Plasma approximations

- **Boltzmann relation for electrons**

$$n = n_0 \exp\left(e\phi / KT_e\right)$$

$$n_e = n = n_0 \exp\left(\frac{e\phi_1}{KT_e}\right) = n_0\left(1 + \frac{e\phi_1}{KT_e} + \cdots\right)$$

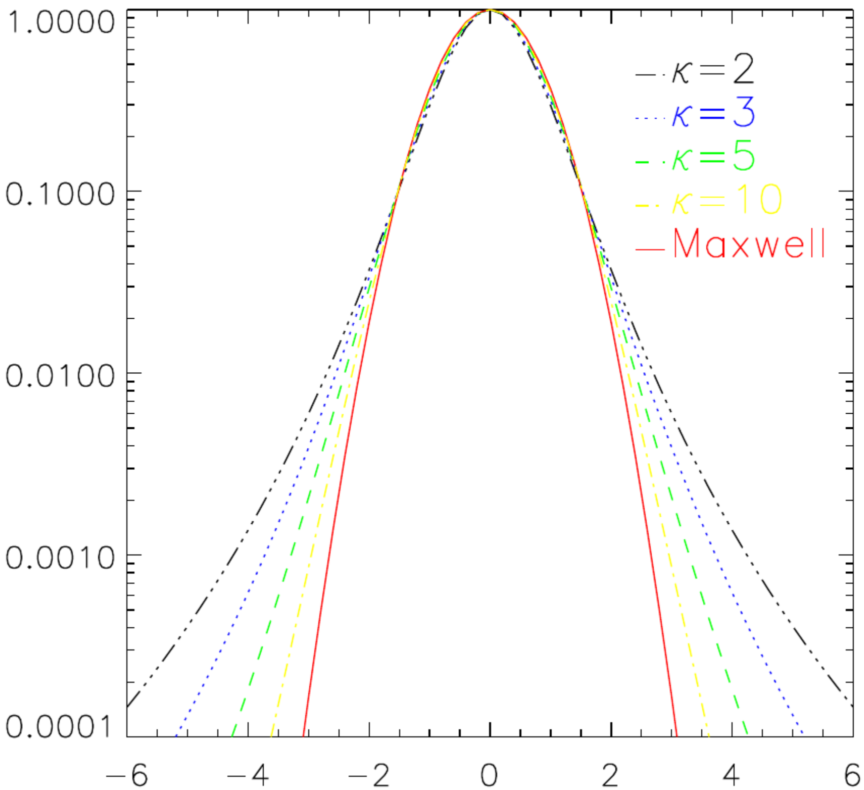
Define from the above expression the following:

- **Small amplitude limit?**
- **Small -but finit- amplitude limit?**
- **Finite amplitude limit?**

Plasma approximations

- **Superthermal electrons**

$$n_e = n_{e0}\left[1 - \frac{e\Phi}{\left(\kappa - \frac{3}{2}\right)k_B T_e}\right]^{-\kappa+1/2}$$

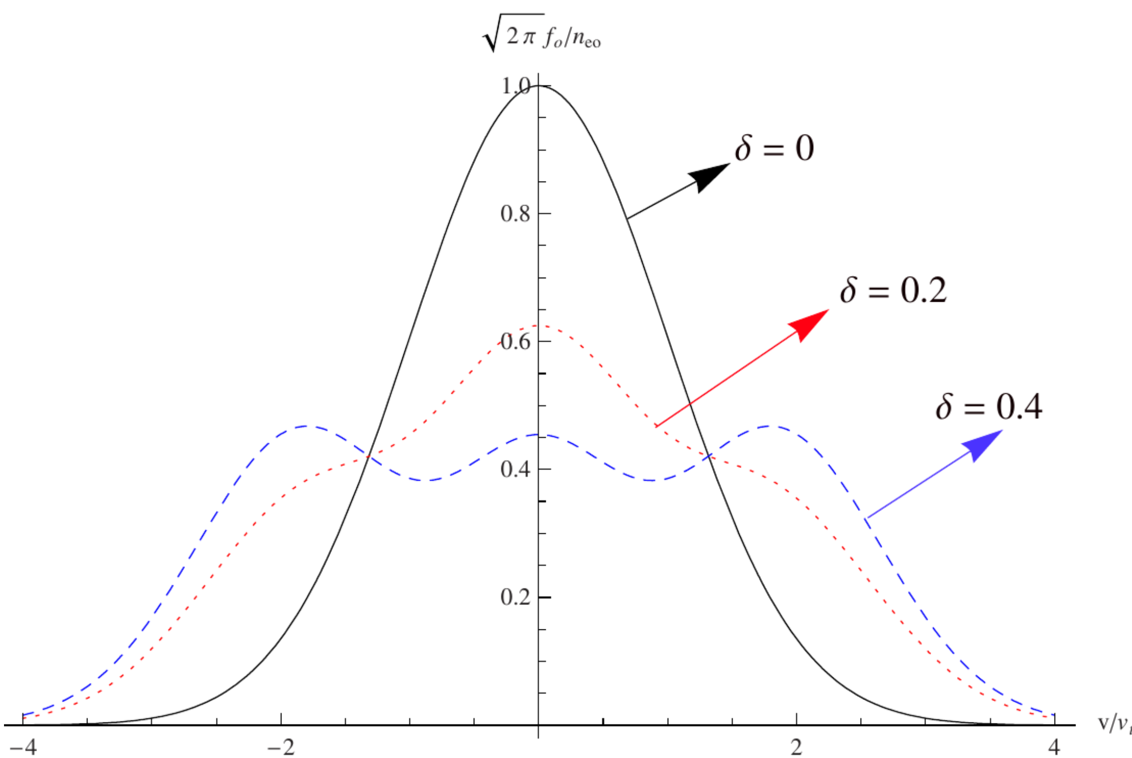


- Excess energetic electrons
- Characterized by a long tail in the high energy region
- Earth's foreshock $3 < \kappa_e < 6$
- Solar wind $2 < \kappa_e < 6$

Plasma approximations

- **Nonthermal electrons**

$$n_e = n_o \exp(\Phi)\frac{4\alpha\Phi^2 - 4\alpha\Phi + 3\alpha + 1}{3\alpha + 1}$$



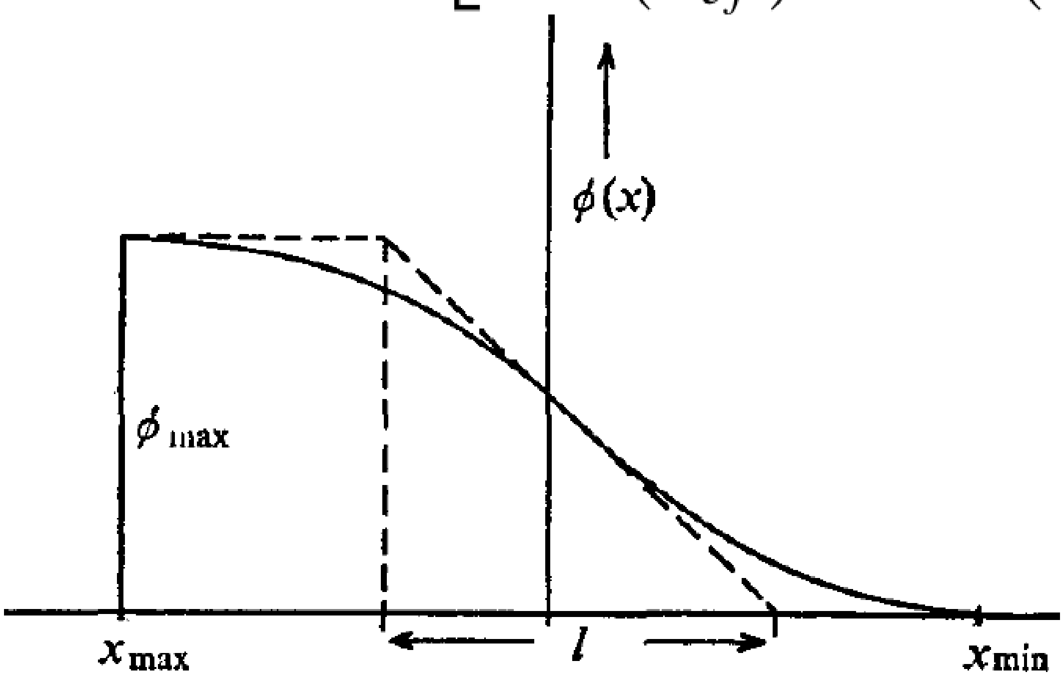
- Excess energetic electrons
- Characterized by a long wings in the high energy region
- Upper earth ionosphere
- Freja satellite

Plasma approximations

- **Trapped/Nonisothermal electrons**

$$n_e = n_e^{(0)} \left[exp\left(\frac{e\phi}{T_{fe}}\right) - G\left(\frac{e\phi}{T_{fe}}\right) \right]$$

$$n_e = n_e^{(0)} \left[1 + \left(\frac{e\phi}{T_{ef}}\right) - \frac{4}{3} b \left(\frac{e\phi}{T_{ef}}\right)^{3/2} + \frac{1}{2}\left(\frac{e\phi}{T_{ef}}\right)^2 + \cdots \right]$$



- Some electrons interact with the wave → is not free electrons
- Characterized by free-to-trapped electron temperature ratio
- Free electrons = 0 → flat-topped
- Lab. & Bow shock measurements

Plasma approximations

- **Pressure gradient force**

$$m_e N_e \left[\frac{\partial \mathbf{u}_e}{\partial t} + (\mathbf{u}_e \cdot \nabla)\mathbf{u}_e \right] = -\nabla p_e + q_e N_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$$

$$p_e = k_B T_e N_e \qquad (p_d + A n_d^2)(1 - B\, n_d) = n_d \kappa_d T_d$$

Which expression of pressure can be used:

- Equation of state of ideal gas OR real gas?
- What is the final decision ---> Obs. or Exp.
- Usually in dusty plasma

Plasma approximations

- **Cold plasma waves**

Approximation: Ignore thermal motion of particles

$$\text{`Cold Plasma'} \qquad \nabla p = 0 \quad (e.g.\ T \simeq 0)$$

$$m_e N_e \left[\frac{\partial \mathbf{u}_e}{\partial t} + (\mathbf{u}_e \cdot \nabla)\mathbf{u}_e \right] = -\nabla p_e + q_e N_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$$

$$\text{Generally requires wave phase velocity} \gg v_{\text{thermal}}$$

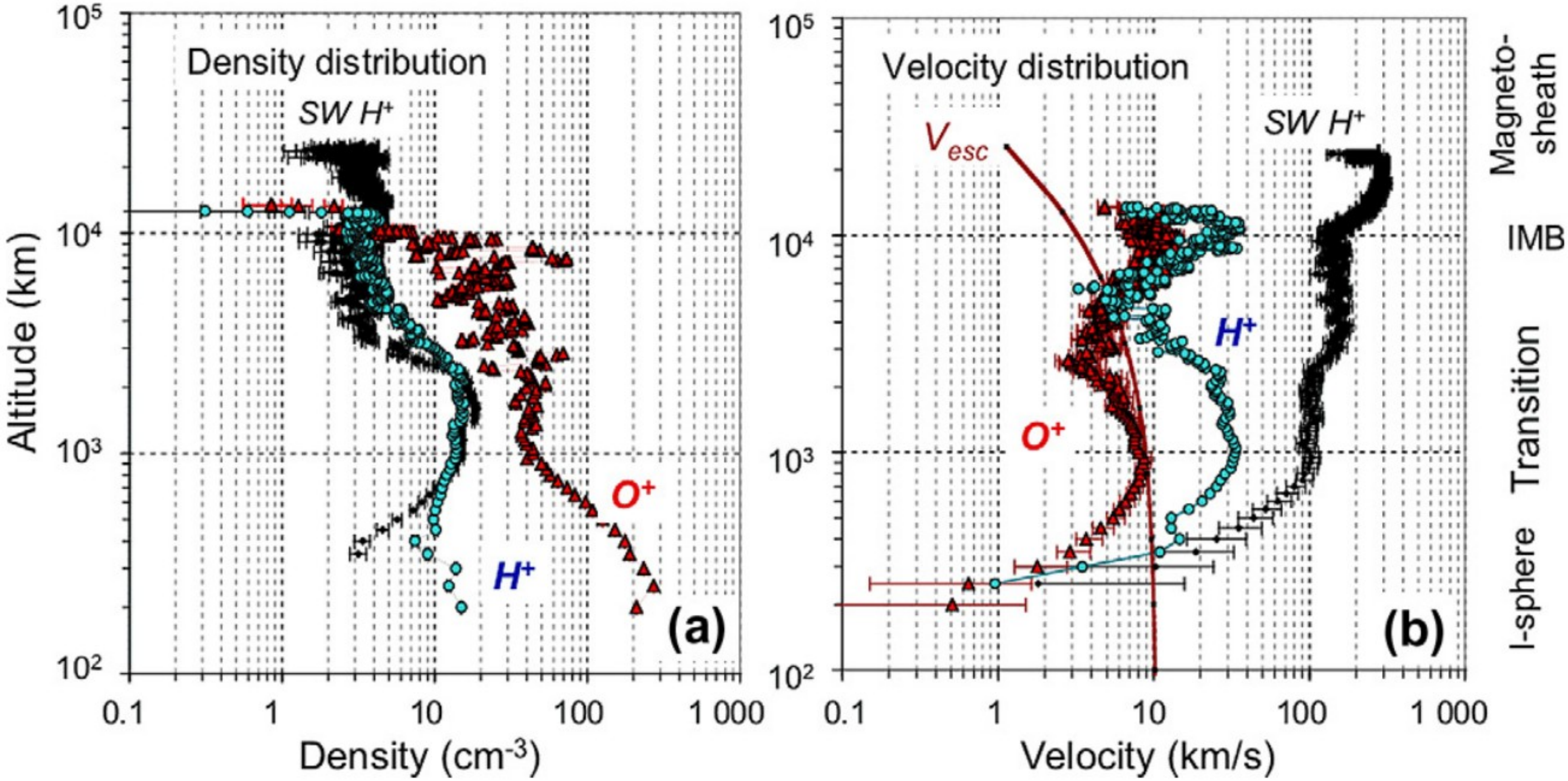
Plasma approximations

- **Collisional force**

- In the absence of collisions, ordinary sound waves would not occur.
- Kinds of collisions in plasma ---> ν_{ee} ν_{ei} ν_{ii}
- When the collision can be ignored?
- $\nu_{ee} > \nu_{ei} > \nu_{ii}$
- The plasma can be considered collisionless if the electron–electron collsion frequency is much smaller the characteristic frequency of interest $\omega_{p_{O,H}} \gg \nu_{ee}$

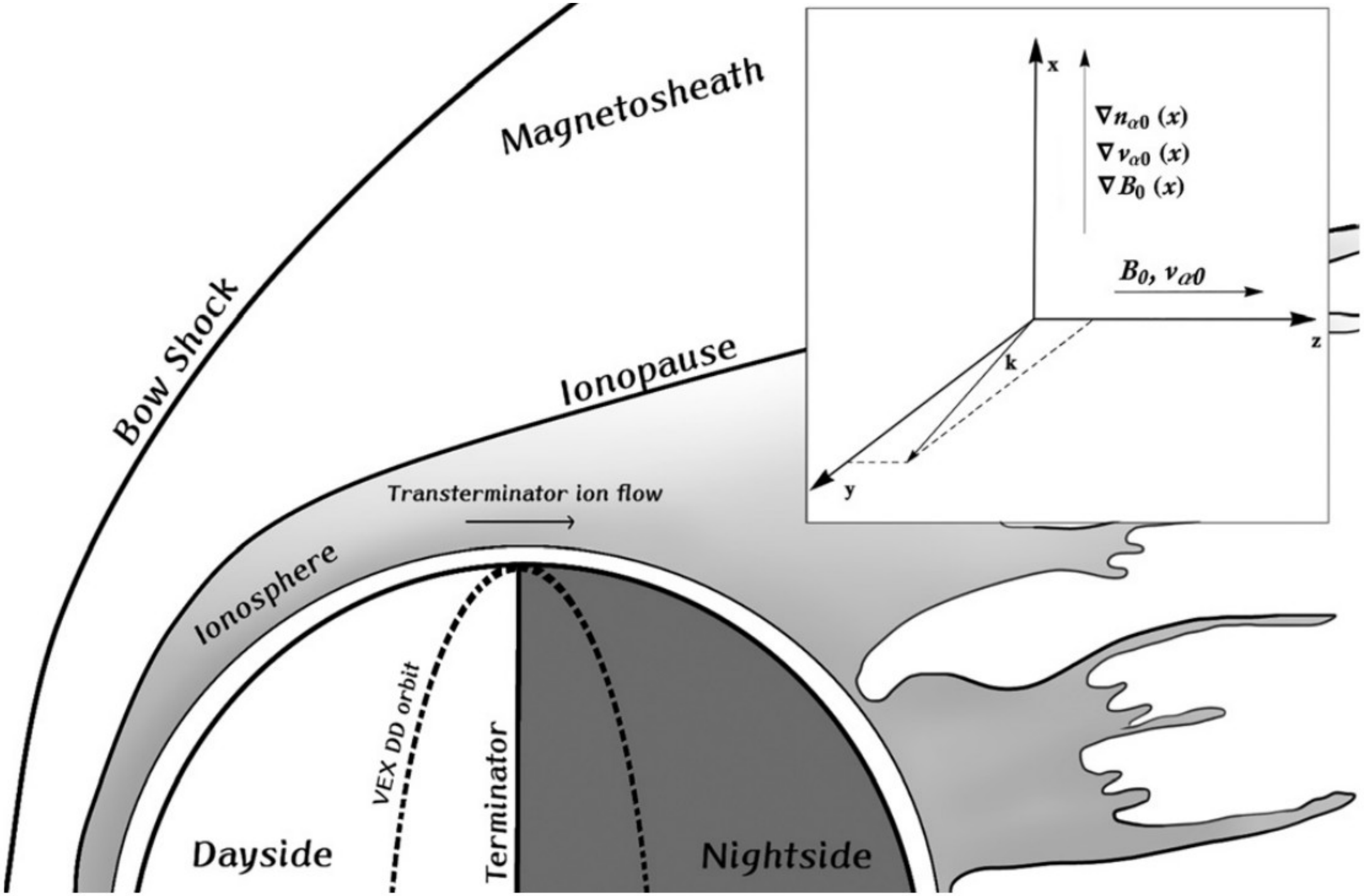
Plasma approximations

- **Homogeneous & Inhomogeneous plasma (Space approximation)**



Plasma approximations

- **Homogeneous & Inhomogeneous plasma (Space approximation)**



Plasma approximations

- Homogeneous & Inhomogeneous plasma (Space approximation)

- Inhomogenity scale length (L)

$$\left(\frac{\nabla n}{n}\right)^{-1}, \left(\frac{\nabla v}{v}\right)^{-1}, \left(\frac{\nabla B}{B}\right)^{-1}, \left(\frac{\nabla T}{T}\right)^{-1}$$

- Inhomogenity scale length > Larmor radius (weak)
- Inhomogenity scale length < Larmor radius (steep)
- Inhomogenity scale length ---> infinity (homogeneous)

Plasma approximations

- **Concept of β**

$$\therefore\ P + \frac{B^2}{2\mu_0} = \text{const.}$$

P → thermal pressure

$\frac{B^2}{2\mu_0}$ → magnetic field pressure

Plasma approximations

- **Concept of β**

$$\beta = \frac{\text{Particle Pressure}}{\text{Mag. field Pressure}}$$

$$\beta = \frac{\sum n k_B T}{B^2 / 2\mu_0}$$

- high → space
- low → fusion (< 0.1)

Plasma approximations

Hydromagnetic Waves
(Alfvén waves)

Shear Modes $(B_{z1} = 0)$
In case of parallel propagation:
$$\omega^2 = k_z^2 v_A^2$$

Kinetic Alfvén wave
Requires oblique propagation and small-but-finite β
$(m_e/m_i << \beta_e << 1)$
$$\omega^2 = k_z^2 v_A^2 (1 + k_x^2 \rho_i^2)$$

Inertial Alfvén wave
Requires oblique propagation and ultra-low β: $(\beta_e << m_e/m_i << 1)$
$$\omega^2 = \frac{k_z^2 v_A^2}{1 + k_x^2 \lambda_e^2}$$

Compressional Modes $(B_{z1} \neq 0)$

Compressional Alfvén wave
(finite B_{z1})
In case of perpendicular propagation:
$$\omega^2 = k_z^2 v_A^2$$

Magnetosonic wave
(finite B_{z1} and finite KT)
In case of perpendicular propagation:
$$\omega = k(v_A^2 + C_s^2)^{1/2}$$

Finally ....!!!!!!

Whom want to
model a plasma