Plasma Models

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Introduction

- **Defination**: a special class of gases made up of a large number of electrons and ionized atoms and molecules, in addition to neutral atoms and molecules as are present in a normal (non-ionized) gas.
- **Aim**: Studing the dynamics (Knowing the position and velocity at instant time t) of the plasma
- **Models**: Depending on the density of charged particles, a plasma behaves either as a fluid, with collective effects being dominant, or as a collection of individual particles.

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  II- Kinetic model.

  III- Fluid model.

- The plasma is a collection of charged particles. So in order to study various physical phenomena inside the plasma, we have to solve the equations of motion:

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \tag{1}$$

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{F}, \tag{2}$$

for each particle.

- Where the position vector $\mathbf{r}$ is given by

$$\mathbf{r} = x\mathbf{x} + y\mathbf{y} + z\mathbf{z}. \tag{3}$$

and the velcoity vector $\mathbf{v}$ is given by

$$\mathbf{v} = v_x\mathbf{x} + v_y\mathbf{y} + v_z\mathbf{z}. \tag{4}$$

- $\mathbf{F}$ is the combined influence forced, due to the externally applied forces and the internal forces generated by all the other plasma

Comments:

- If the plasma consists of **N** particles, we need to solve **6N** coupled nonlinear differential equation simultaneously.
- Hence, it will be an impossible task to solve this problem analytically and it will be waste of time and money computationally.
- A plasma is a system containing a very large number of interacting charged particles, so that for its analysis it is appropriate and convenient to use **a statistical approach** to describe the positions and velocities of plasma particles using **a probability distribution function**.
- Describing a plasma using a distribution function is known as **plasma kinetic theory**.

- **Phase space**: defined by the six coordinates x, y, z, v_x, v_y, and v_z. Thus, the position $\mathbf{r}$ and the velocity $\mathbf{v}$ of a particle at any given time can be represented as a point in this six-dimensional space.

$$\mathbf{d}V = d\mathbf{r}d\mathbf{v} = d^3rd^3v = dxdydzdv_xdv_ydv_z \tag{5}$$

- **Velocity distribution function** $f_s(t, \mathbf{r}, \mathbf{v})$:

$$f_s(t, \mathbf{r}, \mathbf{v})dV = f_s(t, \mathbf{r}, \mathbf{v})d^3rd^3v = dN \tag{6}$$

  the number of particles in a volume element dV in phase space at time t.
- $f_s(t, \mathbf{r}, \mathbf{v})$: the no of particles per unit volume in phase space at time t.
- $\int_{-\infty}^{\infty} f_s(t, \mathbf{r}, \mathbf{v})d\mathbf{v} = n(\mathbf{r}, t)$: the number particle density in real space only at time t.

Kinetic model #3

- The plasma kinetic equation:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{r}} + \frac{\mathbf{F}}{m_s}\cdot\nabla_{\mathbf{v}}\right] f_s(t,\mathbf{r},\mathbf{v}) = \left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{coll}} \tag{7}$$

- **Special cases**:

I- If $\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{coll}} = C(f_s)$: It is called '**Boltzmann**' equation, where $C(f_s)$ is the Coloumb collision operator.

II- If $\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{coll}} = FP(f_s)$: It is called '**Fokker-Plank**' equation, where $FP(f_s)$ is the FP collision operator.

III- If $\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{coll}} = 0$: It is called '**Vlasov**' equation. Thus the '**Vlasov**' equation (**??**) can be simply stated as

$$\frac{df_s}{dt} = 0, \tag{8}$$

Comments:

- The **measurable or macroscopic** (i.e., ensemble average) values of various plasma parameters (e.g., density, flux, current) can be can easily be derived from the moments of distribution function $f_s(t, \mathbf{r}, \mathbf{v})$.
- **For example:** The total number $N(t, \mathbf{r})d\mathbf{r}$ of velocity points in the entire velocity space, is given by

$$N(t, \mathbf{r}) = \int_{-\infty}^{\infty} f(t, \mathbf{r}, \mathbf{v}) d\mathbf{v} = \int\int\int_{-\infty}^{\infty} f(t, \mathbf{r}, \mathbf{v}) dv_x dv_y dv_z. \tag{9}$$

- Consider any property $g(\mathbf{r}, \mathbf{v}, t)$ of a particle. The value of this quantity averaged over all velocities (**weighted average**)is then given by

$$\bar{g}_{av}(t, \mathbf{r}) =< g(t, \mathbf{r}, \mathbf{v}) >= \int_{-\infty}^{\infty} g(t, \mathbf{r}, \mathbf{v}) \hat{f}(t, \mathbf{r}, \mathbf{v}) d\mathbf{v}. \tag{10}$$

Fluid model #1

$$\frac{\partial N_s}{\partial t} + \nabla \cdot [N_s \mathbf{u}_s] = 0,$$

$$m_s N_s \left[\frac{\partial}{\partial t} + \mathbf{u}_s \cdot \nabla\right]\mathbf{u}_s = q_s N_s \Big(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}\Big) - \nabla P_s + \nabla \cdot \mathbf{\Pi}_s + \mathbf{R}_{ij}$$

$$\frac{\partial \frac{3}{2} P_s}{\partial t} + \nabla \cdot (\frac{3}{2} P \mathbf{u}_s) = P_s \nabla \cdot \mathbf{u}_s + \nabla \cdot q_s + \mathbf{R}_{ij}$$

Gauss' Law $\quad \nabla \cdot \mathbf{E} = \frac{\rho_q}{\epsilon_0},$

Gauss' Law $\quad \nabla \cdot \mathbf{B} = 0,$

Faraday's Law $\quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Ampére's Law $\quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$

The charge density $\quad \rho_q = \sum q_s N_s,$

The current density $\quad \mathbf{J} = \sum q_s N_s \mathbf{u}_s$

Further reading

- Francis F. Chen: Introduction to Plasma Physics and Controlled Fusion, 3rd edn (Springer International Publishing Switzerland, 2016).
- Umran Inan, Marek Gołkowski: *Principles of Plasma Physics for Engineers and Scientists*, (Cambridge University Press, 2011).
- J. A. Bittencourt, *Fundamentals of Plasma Physics*, 3rd edn (New York: Springer-Verlag, 2004).
- N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics*, (San Francisco: San Francisco Press, 1986).

Thanks for your attention!