Turbulence on Earth, in Galaxies and Beyond

Amr El-Zant

Centre for Theoretical Physics

The British University in Egypt

What is Turbulence?

- Causes mentioned are called ‘driving’.

Driving → large scale → Drives ‘chaos’

++ ‘**Dissipation**’ scale

→ Much smaller

→ Where things are ordered again

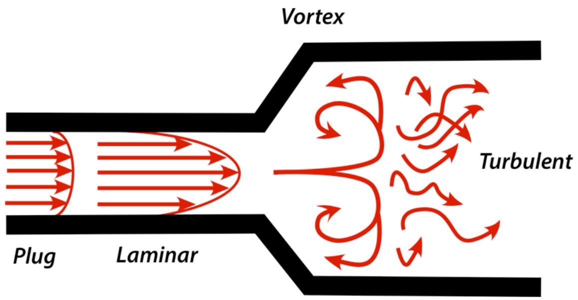
From CBC

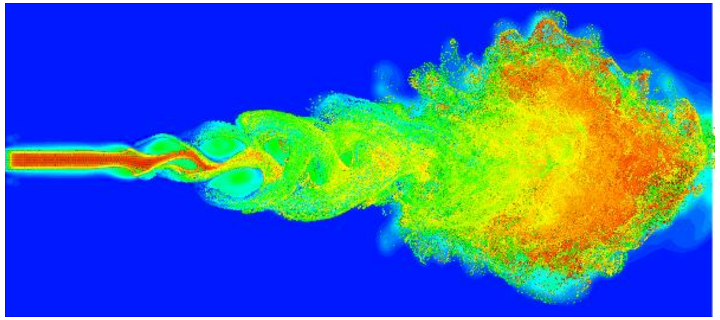
Turbulence

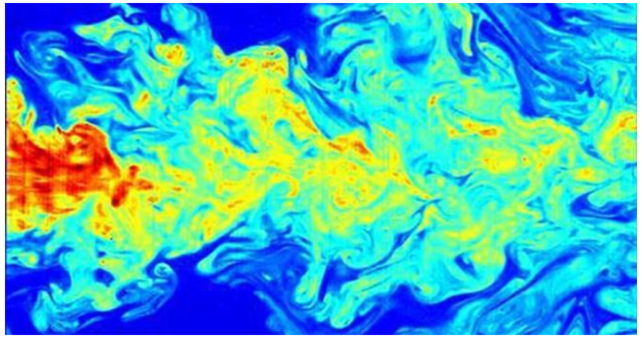
- Extremely hard to describe in detail, but easy to understand!
- And is very familiar… and (quasi) universal

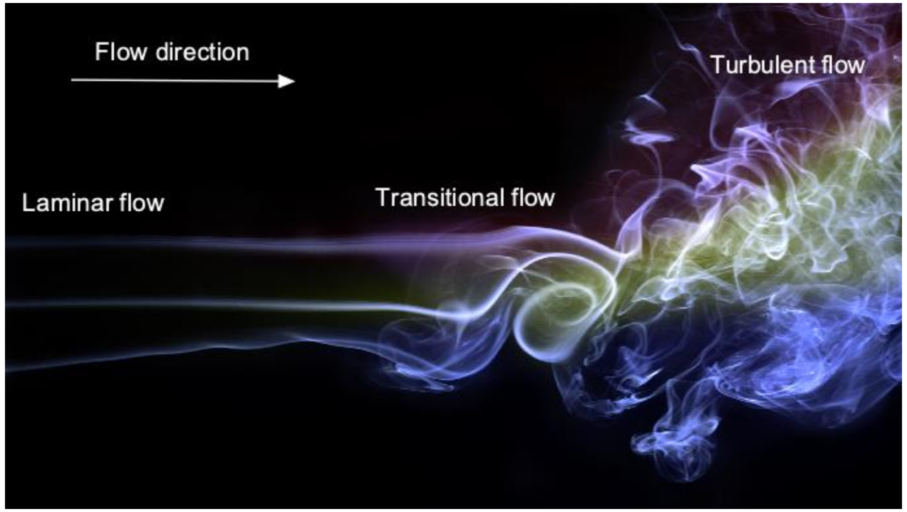
Examples

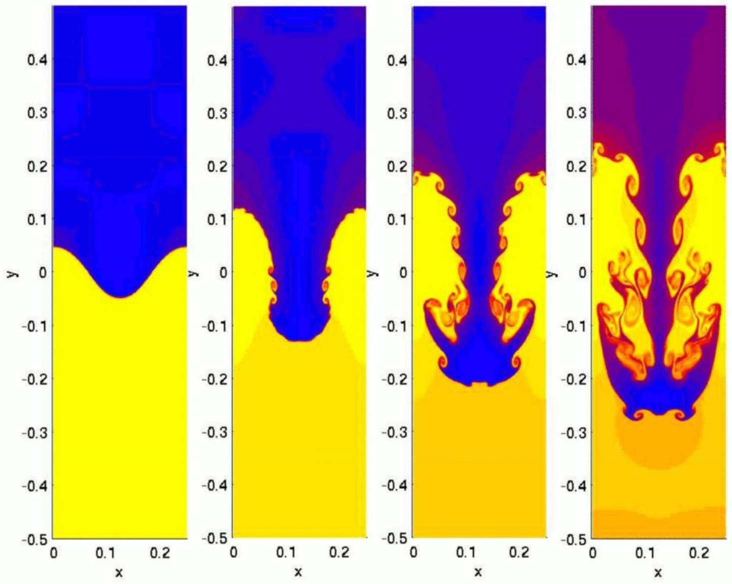
Examples

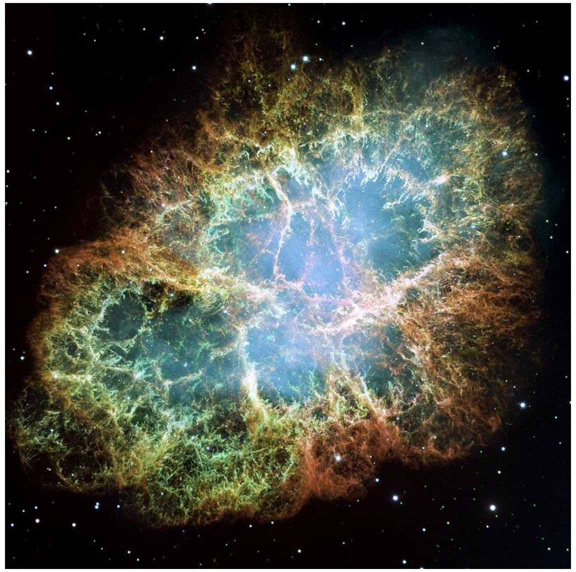

Vortex
Turbulent
Plug
Laminar






Flow direction
Turbulent flow
Laminar flow
Transitional flow


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
y
0
0.1
0.2
x



To start: *Inviscid* Fluids

- **Mass Conservation** (continuity eq.):

$$\partial_t \rho = -\boldsymbol{\nabla}_r \cdot (\rho \boldsymbol{u})$$

→ Mass flux from small unit vol

In fixed 'Eulerian' coordinates where vely ***u* = *u* (*x*)**

- **Momentum Conservation** (Euler eq.):

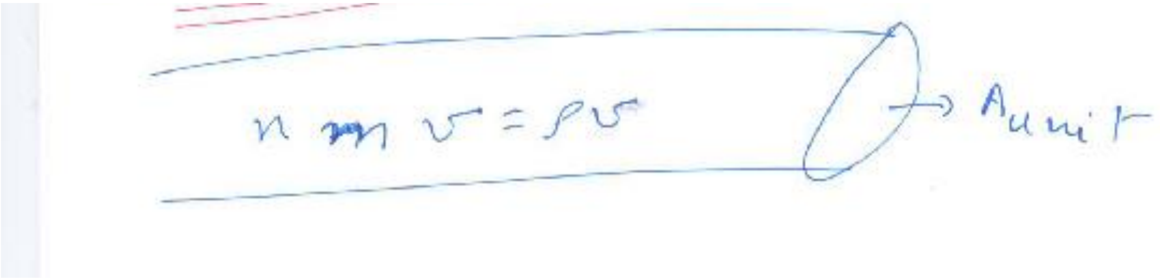
$$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}_r)\, \boldsymbol{u} = -\frac{\boldsymbol{\nabla}_r P}{\rho} - \boldsymbol{\nabla}_r \Phi$$

- ***F = m a = du/dt !*** → ***appears simple BUT highly non-linear***

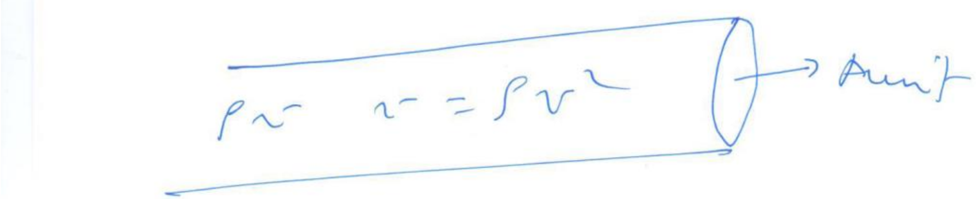
- $\frac{du}{dt} \to \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} \to$ $\quad D_t \boldsymbol{u} \equiv (\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u}$

Pressure: Diagonal (trace) of Stress Tensor

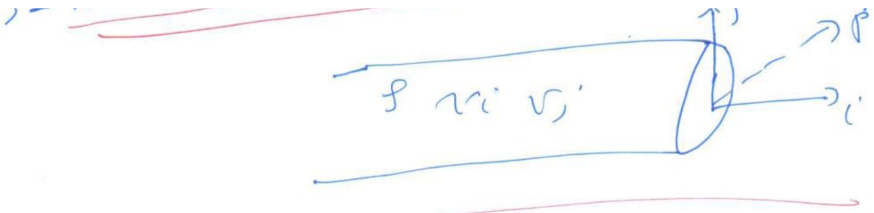
Mass Flux



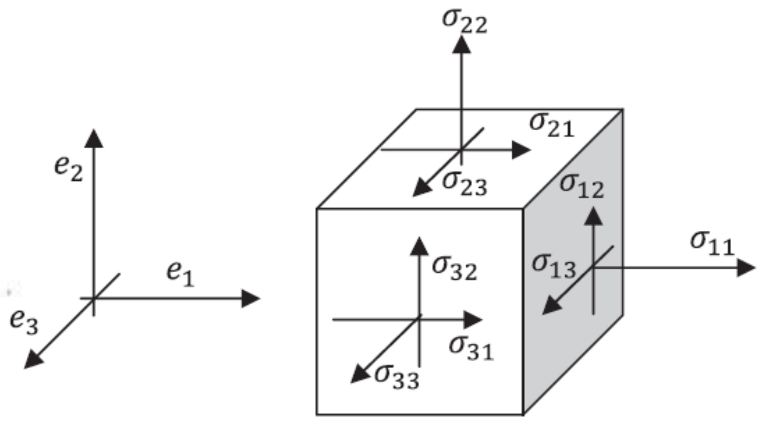
(Isotropic) Momentum Flux



In General



Statistically (dir*) $\rho \langle v_i v_j \rangle$



Pic from U. Alberta engineering course

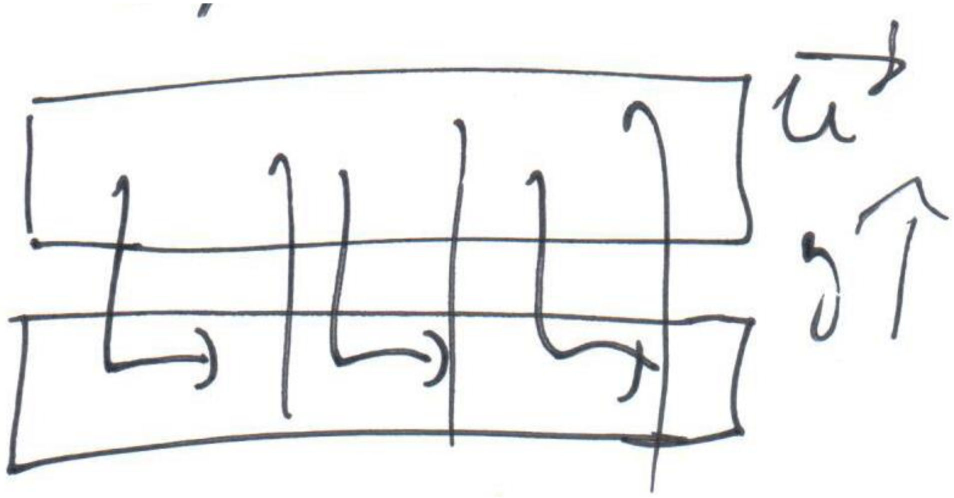
Components of - σ_{iJ}

Viscosity Includes Shearing Forces → Dissipation

- Gradients of $\rho \left\langle v_i^2 \right\rangle$ → Isotropic Pressure Forces
- Gradients of $\rho \left\langle v_i v_j \right\rangle_{i \neq j}$ → Frictional Forces Possible (Note averages over *atomic* particles important)

- Example: Incompressible Newtonian Fluid

→Viscous force arises from shear stress:



→ $$\sim \rho\, u\, \lambda \, \frac{du}{dy} = \eta \frac{du}{dy}$$

(momentum from vely difference *within reach* λ between 2 layers)

(Estimate λ: particle passing through balls of crossectional area σ will likely encounter one ball when N (ball per unit Vol) $*$ Obstructing volume $= n\, \lambda\, \sigma \sim 1$ → $\lambda \sim 1/n\sigma$)

Navier Stokes Eq. and Reynolds Number

In general $\frac{d\boldsymbol{u}}{dt} = \frac{1}{\rho}\,\nabla.\boldsymbol{\sigma} + \boldsymbol{F}_{ext}$

$Incompressible\ (\nabla.\boldsymbol{u} = 0)$ $\rightarrow$

$\rightarrow$ Simplifies to

$$\rho\frac{d\boldsymbol{u}}{dt} = \eta\,\nabla^2\boldsymbol{u} - \nabla P + \mathrm{F}_{\mathrm{ext}}$$

++ Neglect pressure and external forces and define $\boldsymbol{\nu} = \frac{\boldsymbol{\eta}}{\boldsymbol{\rho}}$

$\rightarrow \frac{\partial u}{\partial t} = \nu\,\nabla^2\boldsymbol{u} - (\boldsymbol{u}\,.\nabla)\,\mathbf{u}$ → **Restoring .VS. Inertial Force**

→Turbulence when Reynolds Number $\frac{(\mathbf{u}\cdot\nabla)\mathbf{u}}{\nu\,\nabla^2\mathbf{u}} \sim \frac{u^2/L}{\nu u/L^2} = \frac{uL}{\nu}$ **>> 1**

Basic Interpretation

System **forced**, such that: **time to restore to rest via viscosity long compared to crossing time:**

$$t_L = \frac{L}{u} = u \,/\, (u^2 \,/L) \;\sim\; \frac{\mathrm{u}}{\mathrm{a}_{\mathrm{adv}}} \ll \frac{\mathrm{u}}{a_{\mathrm{visc}}}$$

++ **Excited degrees of freedom coupled**

→ **complex behaviour**

Kolmogorov's Basic Insight: Universality

- There are two scales to the problem

1- The scale at which we drive *L*

2- The scale at which energy dissipates *d* << *L*

→Between these two the system is **scale free** → Described by **power law**.

With the **central characteristic** $(a\,x)^n = a^n x^n$

→**Basic character** of complicated system **simple**: By dimensional analysis!

Powerful Consequences

- In intermediate, '**inertial range**' energy transferred in **cascade *from L to d***
- With little loss or time dependence

→Specific energy E_l at intermediate scale *l* conserved ++ energy flow

→**rate** $\boldsymbol{\varepsilon = \frac{E_l}{t_l} \sim \frac{u_l^2}{\frac{l}{u_l}} = \frac{u_l^3}{l}}$ **constant across eddies *l*** →

→ $\boldsymbol{u_l \sim (\varepsilon l)^{\frac{1}{3}}}$

How is energy distributed among scales?

- **Fourier**: A powerful method of **analyzing** fields on different scales

If a field ***F*** is periodic in L you can expand

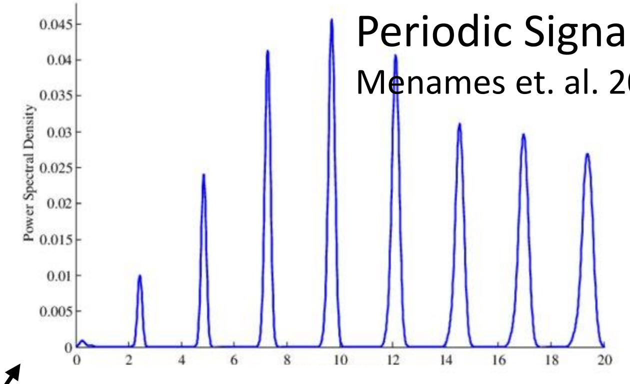
$$F(\mathbf{x}) = \sum F_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$k_i = n_i \frac{2\pi}{L} \quad k = \frac{2\pi}{\lambda}, \lambda \sim l$$

Cardiogram →

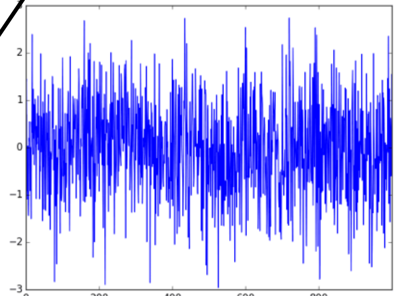
Periodic Signal

Menames et. al. 2002



Let L … → number modes increases arbitrarily → continuity →

$$F(\boldsymbol{x}) = \left(\frac{L}{2\pi}\right)^3 \int F(\boldsymbol{k}) e^{-i\,\boldsymbol{k}.\boldsymbol{x}}\ d\,\boldsymbol{k}$$



White noise -

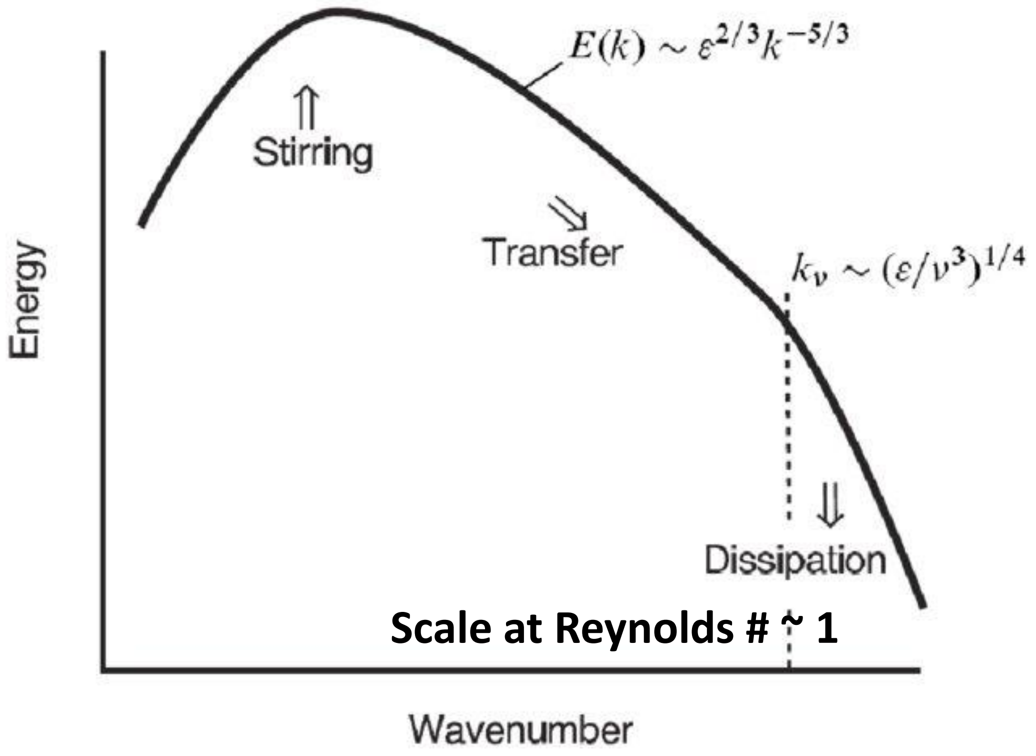
Frequency →

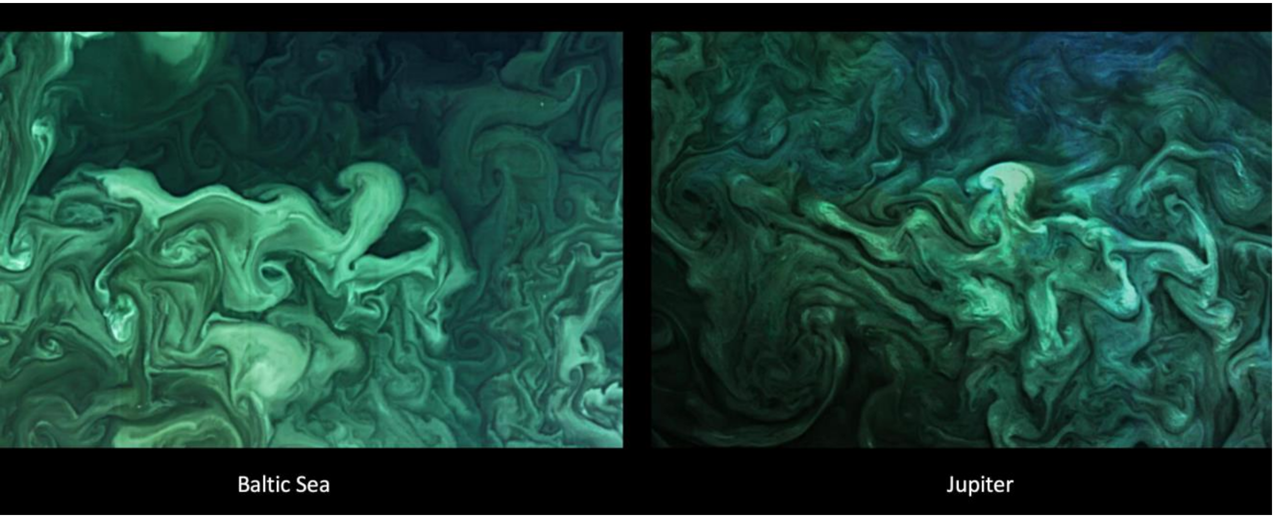
There are different conventions and L does not appear in **power spectra** $\boldsymbol{F^2}$

Universal Energy Spectrum

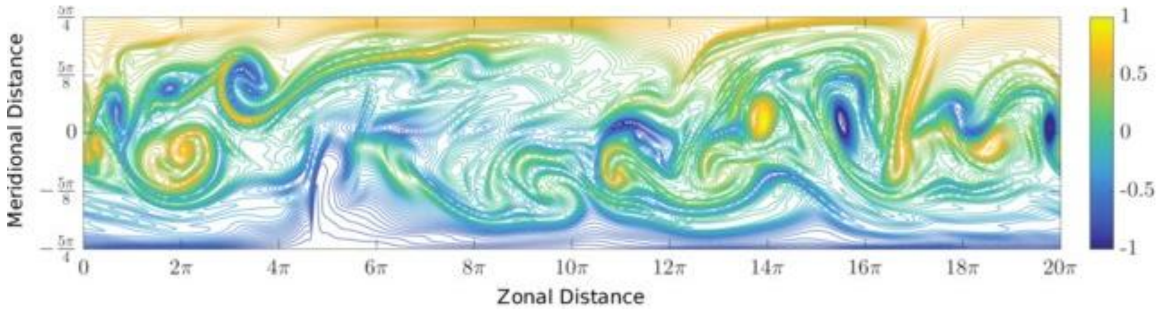
Energy per unit $k \sim \frac{1}{l} \sim u_l^2 l \; + + \; u_l \sim (\varepsilon l)^{\frac{1}{3}}$

$$E(k) \sim \varepsilon^{\frac{2}{3}} \, l^{\frac{5}{3}} \sim \varepsilon^{\frac{2}{3}} \, k^{-\frac{5}{3}}$$





(NASA)



Mesoscale Ocean turbulence
After Tomos David Ecole Normale, Lyon

From Solar System to Galaxies and Beyond

Many similarities ++ Main difference: **Often supersonic → Compressible**

Elementary estimate of effect: Consider ideal fluid with adiabatic sound speed in 1-D →

$$\frac{dP}{dx} = -\rho \frac{du}{dt} \quad \text{and} \quad \frac{dP}{d\rho} = c_s^2$$

$$\longrightarrow \quad M^2 \frac{du}{u} = -\frac{d\rho}{\rho}$$

Small ***Mach Number*** $\boldsymbol{M = u/c_s}$ → **Little density perturbations**

Large *M* → Highly Compressible

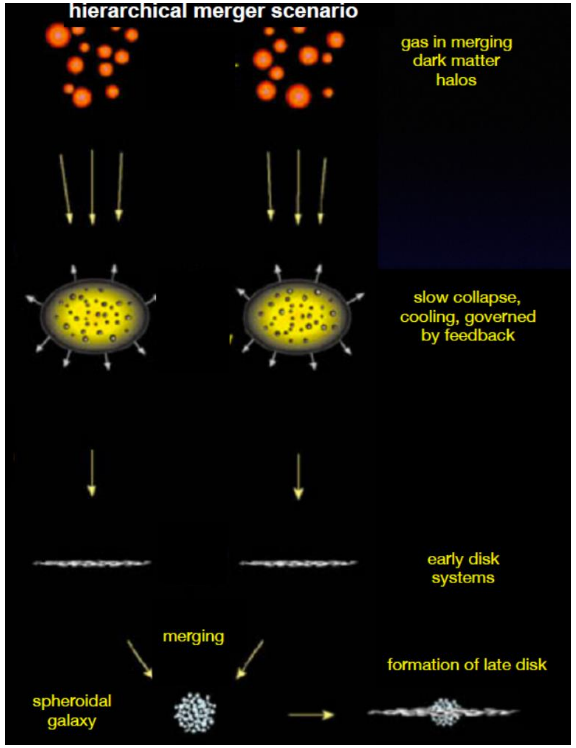
Actual astrophysical situation quite complicated!

Galaxy M 101 driven gas in red

Gould Belt ~ kpc across
NASA Spitzer telescope

Driving Turbulence in Galaxies

- Gravitational Energy, incl. infall (accretion) can be important



After Franck van den Bosch (lecture slides)

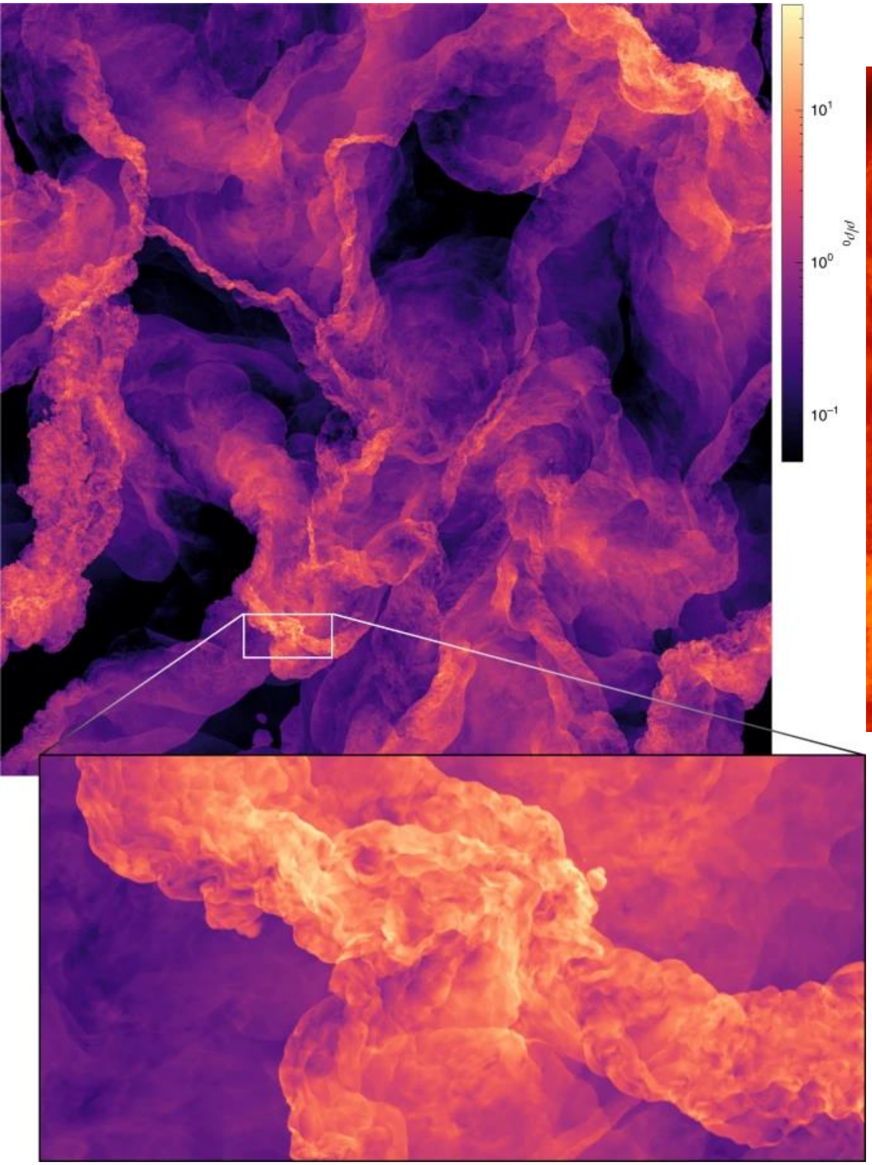
- Stellar feedback and AGN can also be important

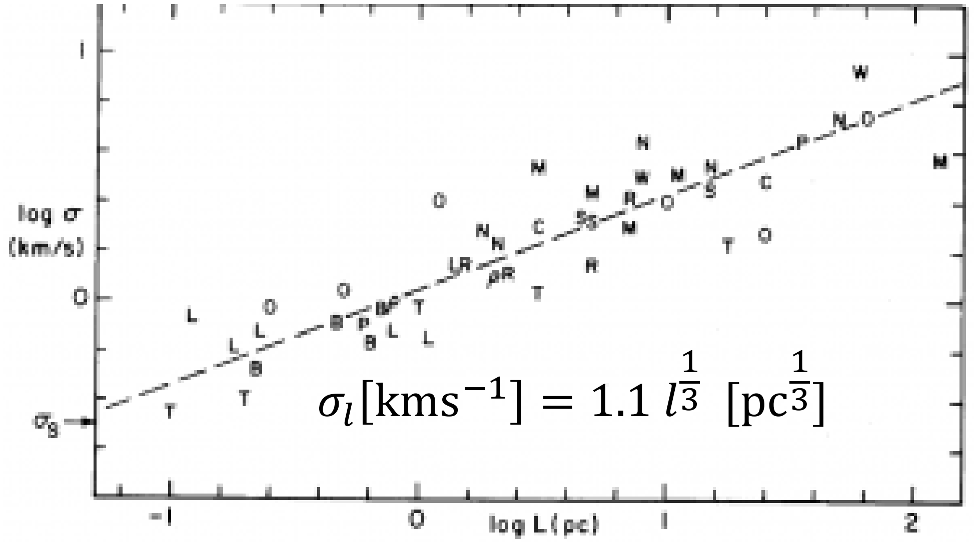
Single SN (Carb Nebula)

Starburst galaxy with outflow

Galaxy scale AGN

Simulations and Observations of Turbulent Clouds



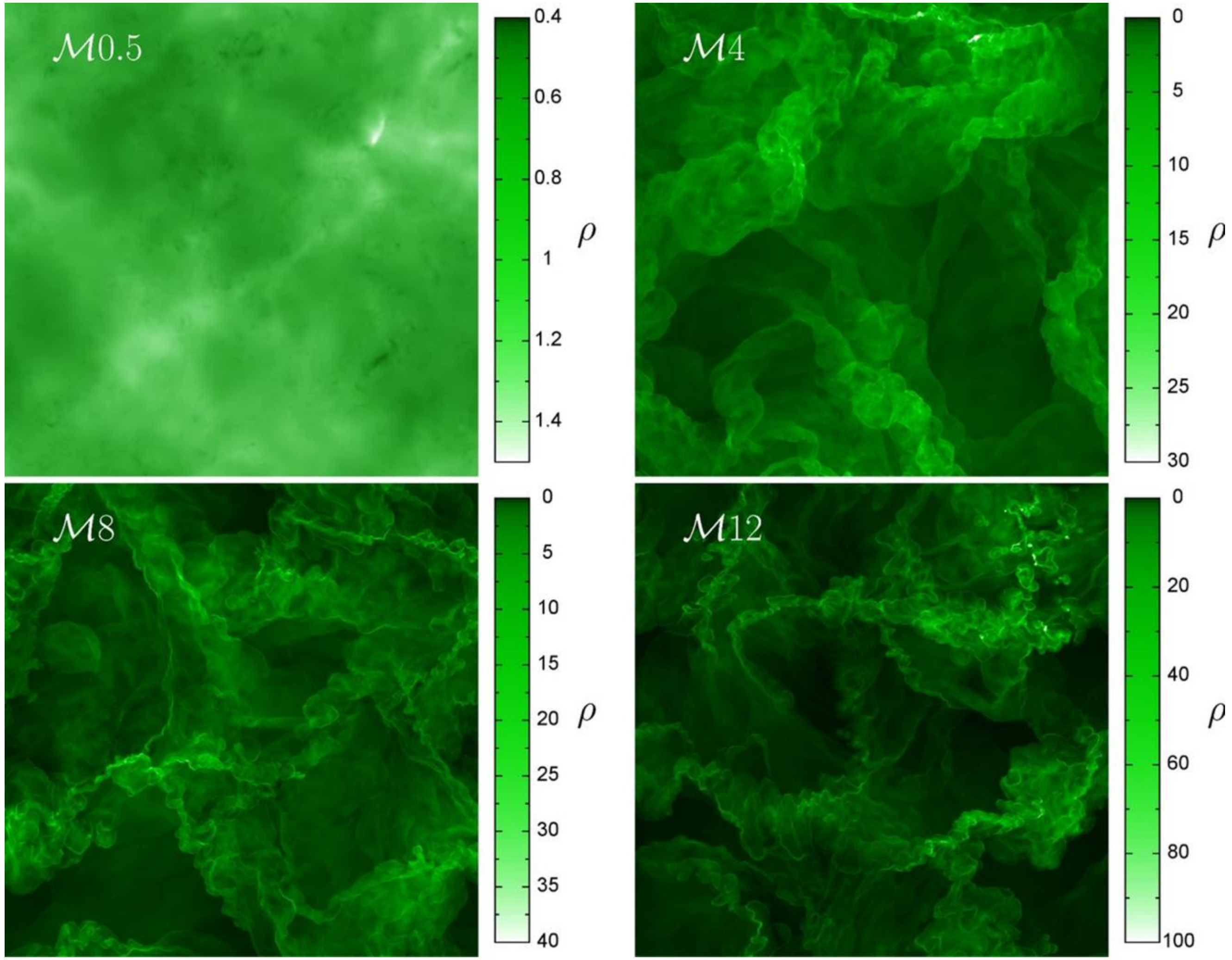


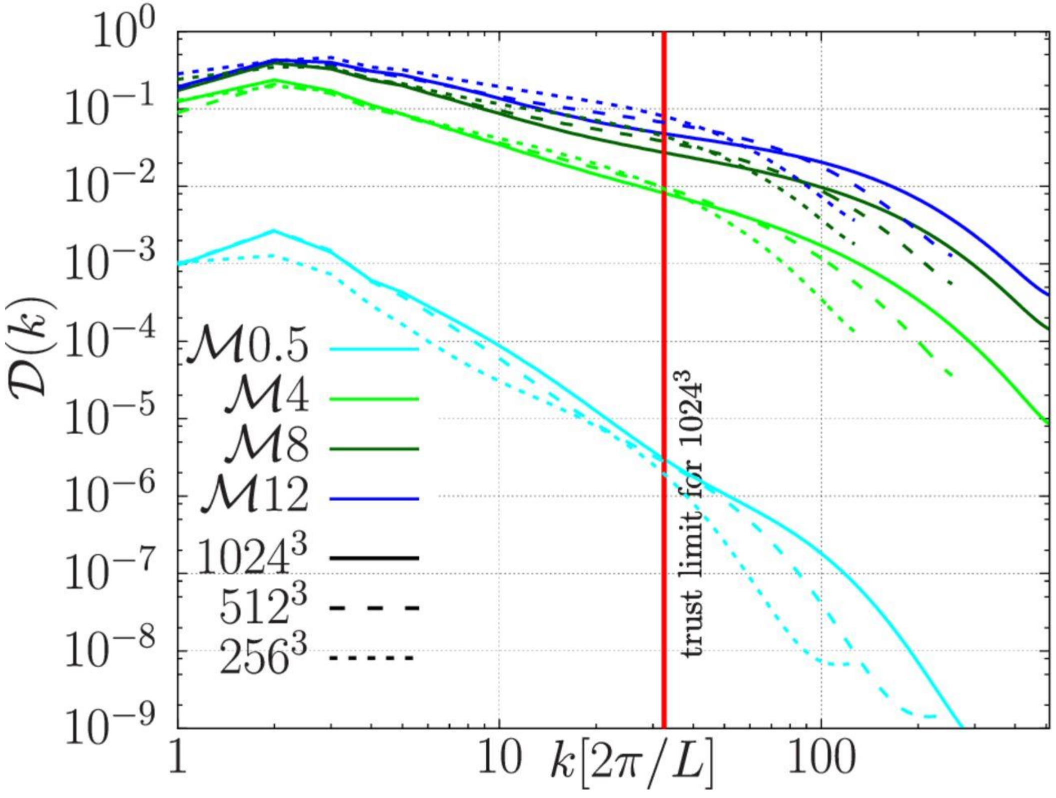
Larson (1981) relation. Kolmogorov Law → $\sigma_l \sim u_l \sim l^{\frac{1}{3}}$

Mammoth simulation, Federrath et. al. (2021).

Confirms Larson's relation with (index ~ 0.5 for M>>1,
Which is in line with recent observations.

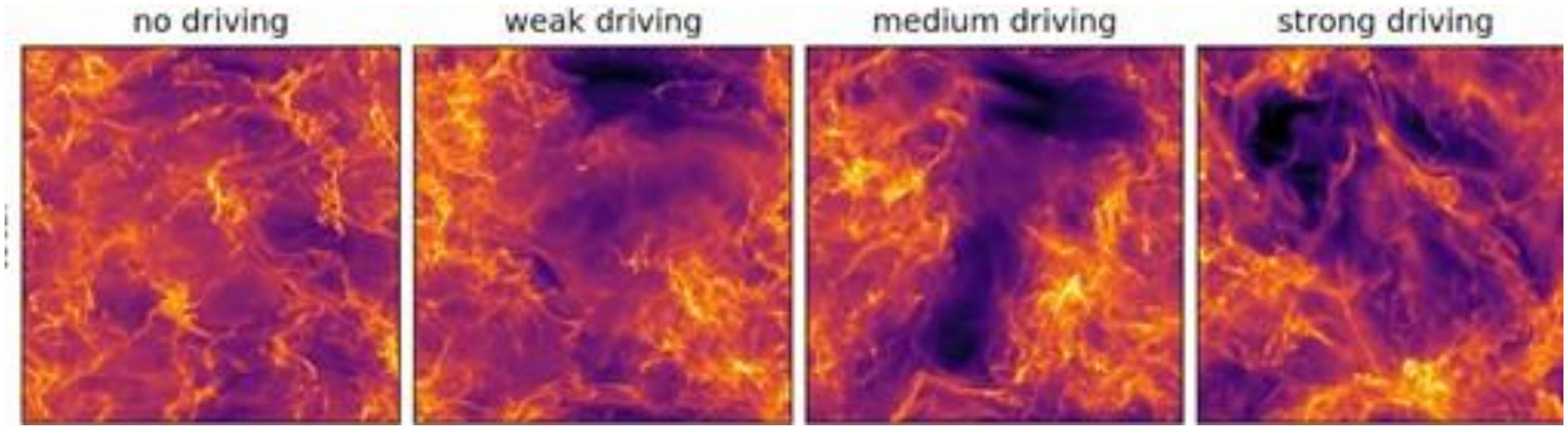
Density Power Spectra: The Effect of Mach



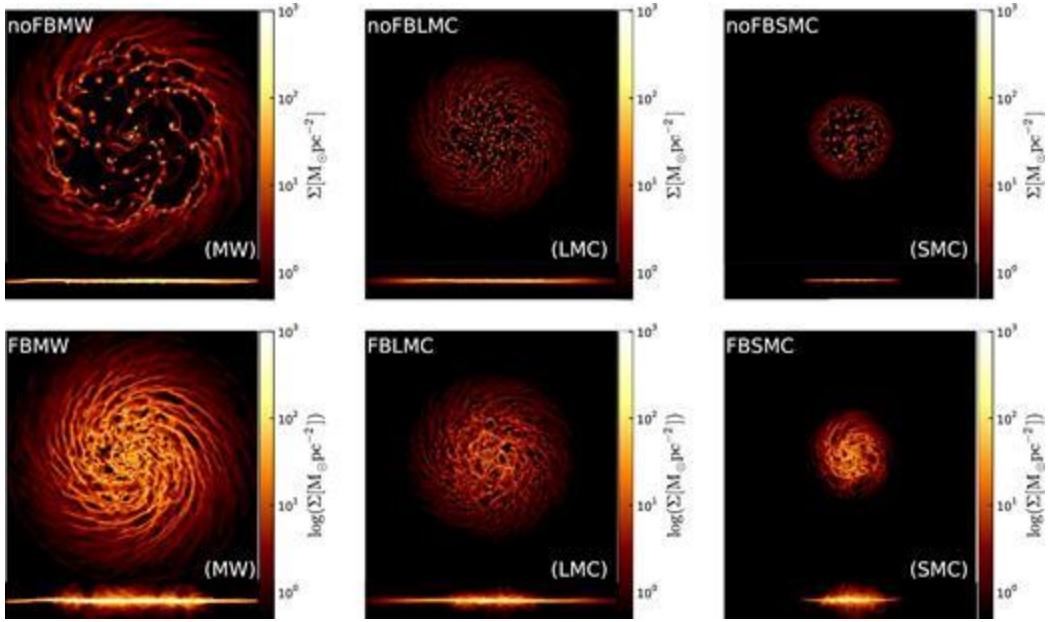


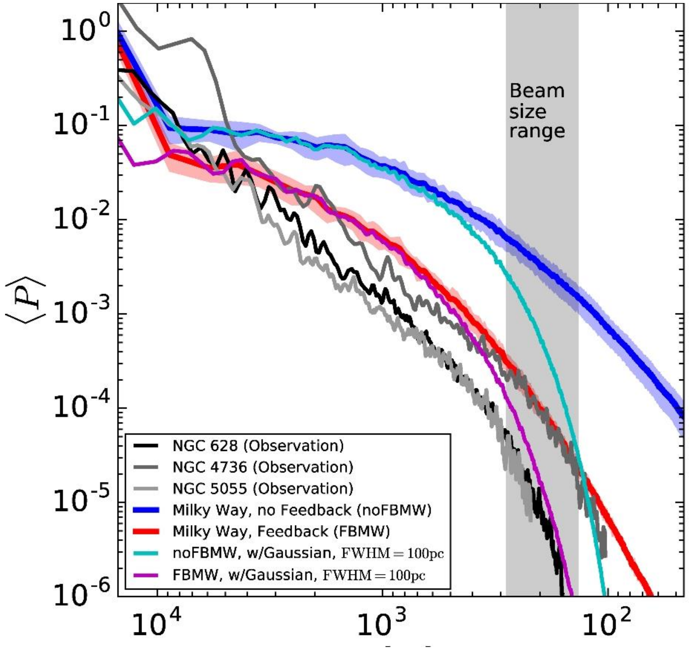
After Konstandin et. al. (2016)

The Signatures of (non-gravitational) Driving



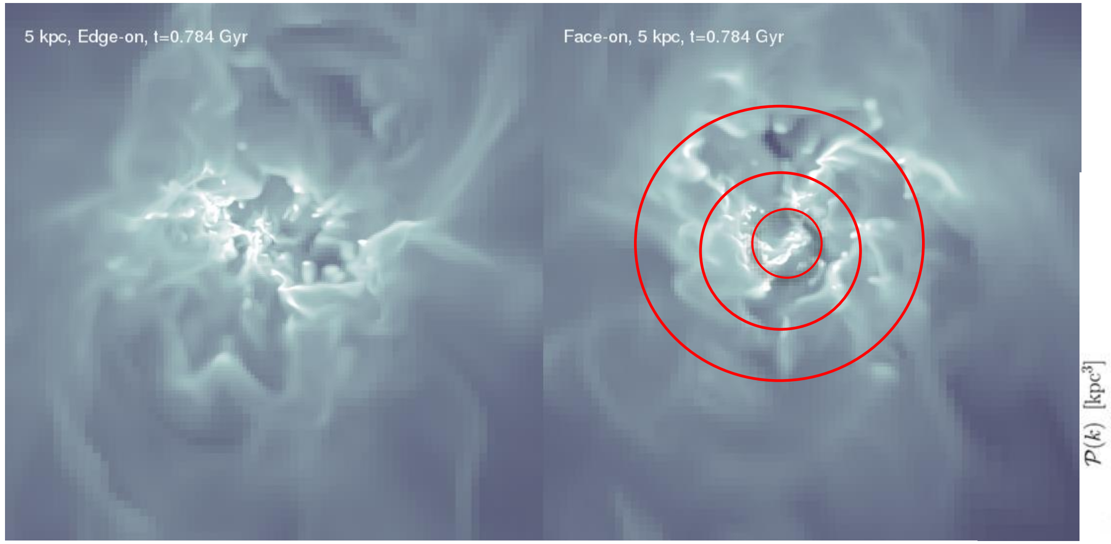
Colman et. al. (2022)

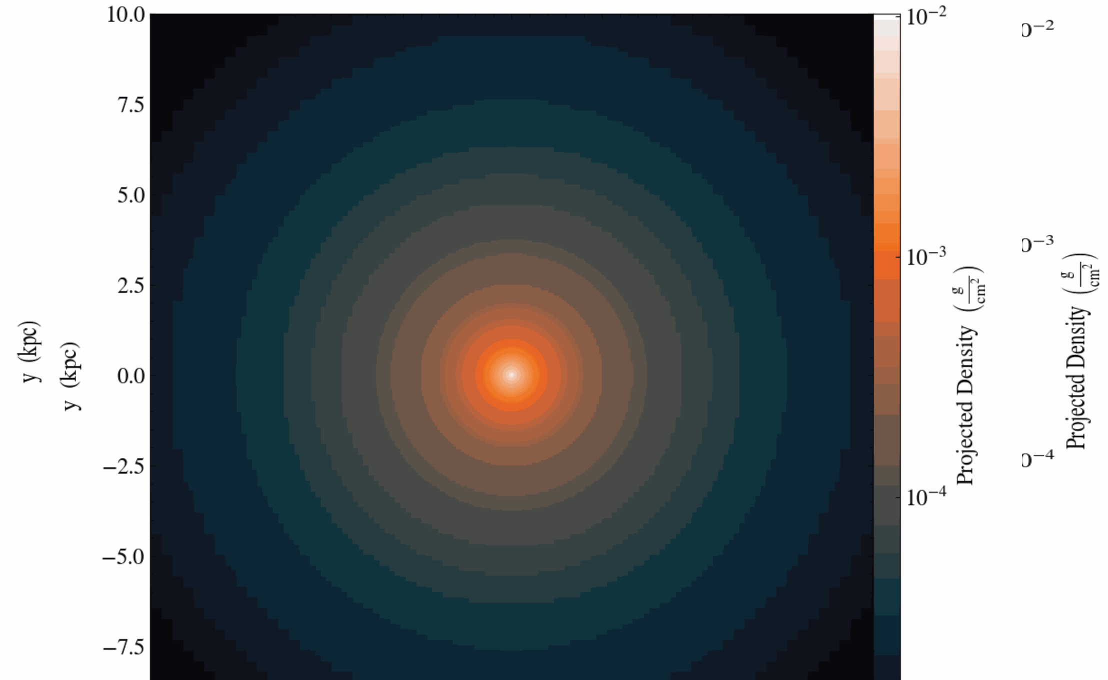


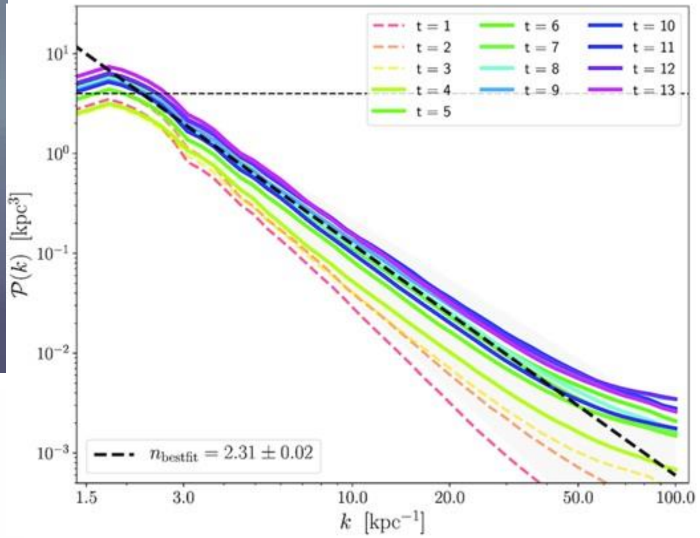


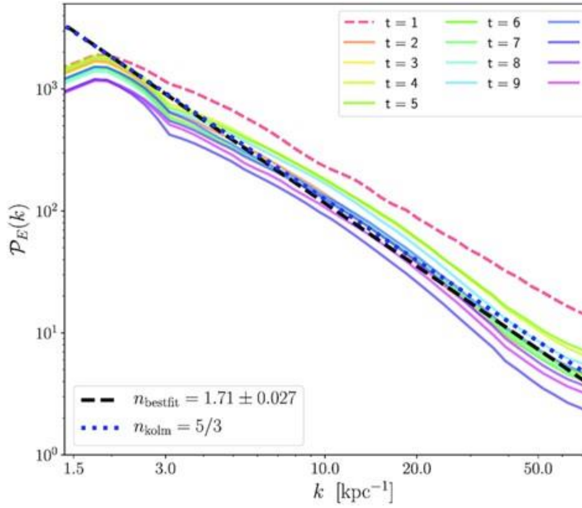
Grisdale et. al. (2017)

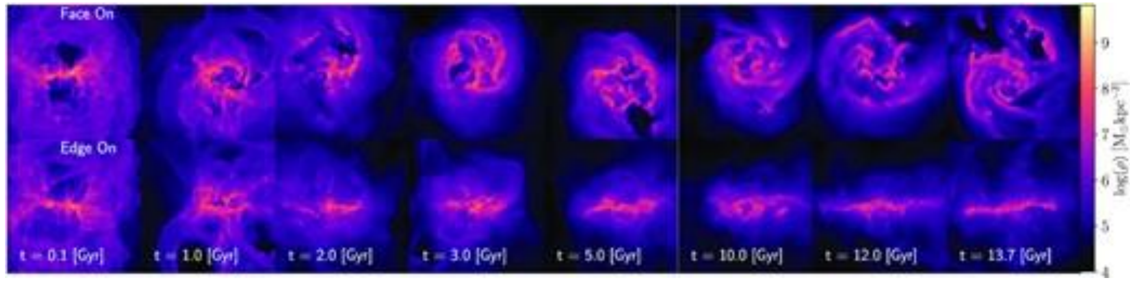
The Macroscale: **Strongly driven gas** and dark matter haloes



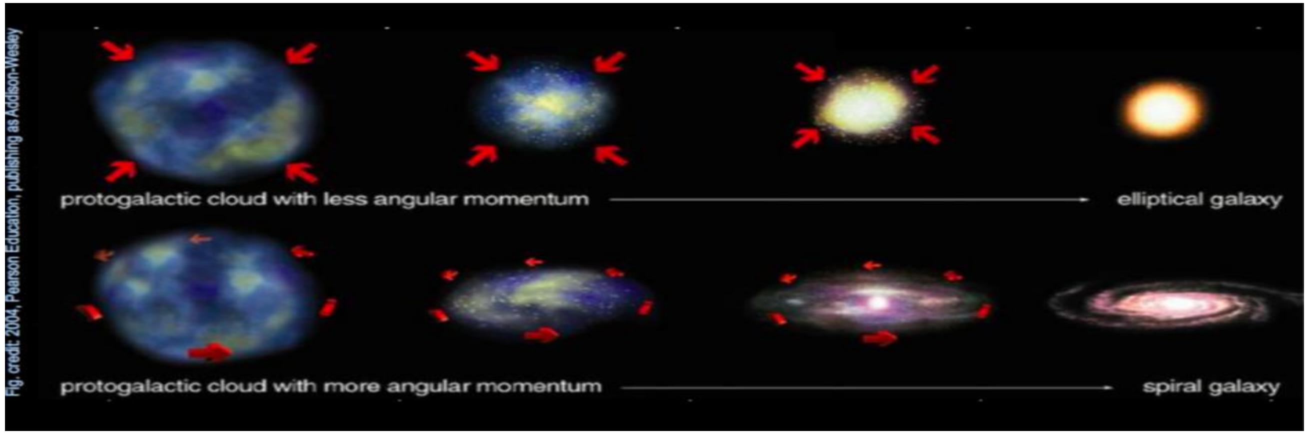






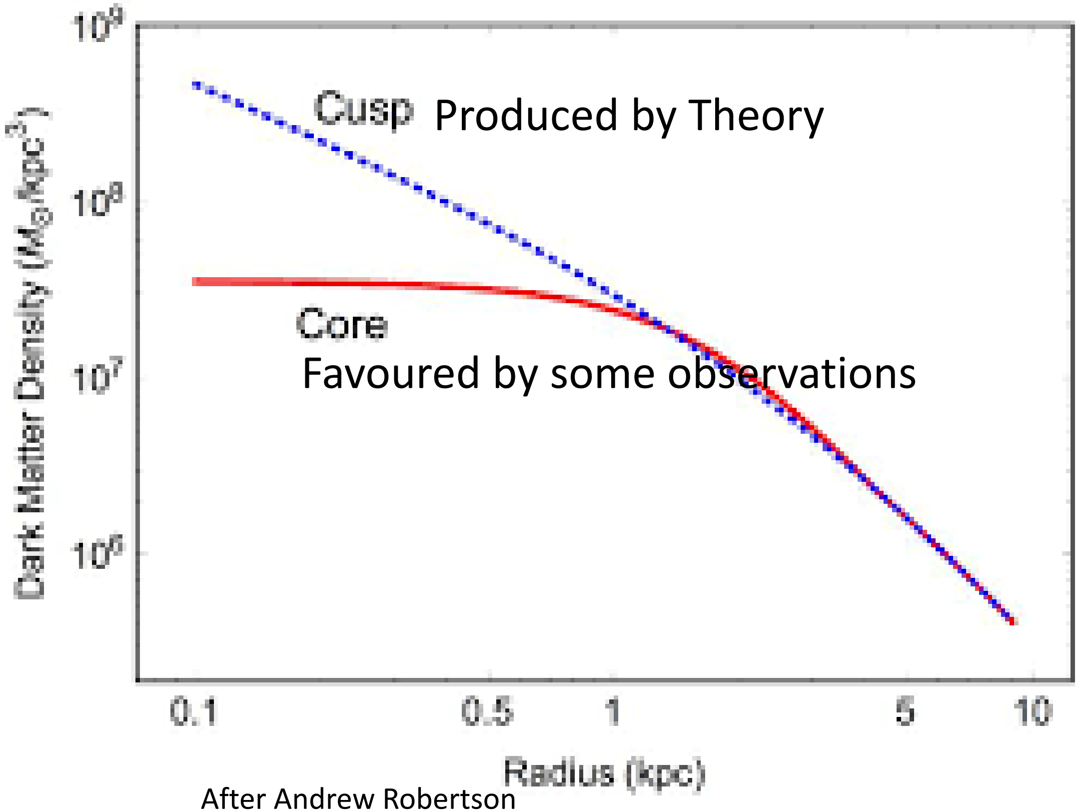


Hashim et. al. (2023)

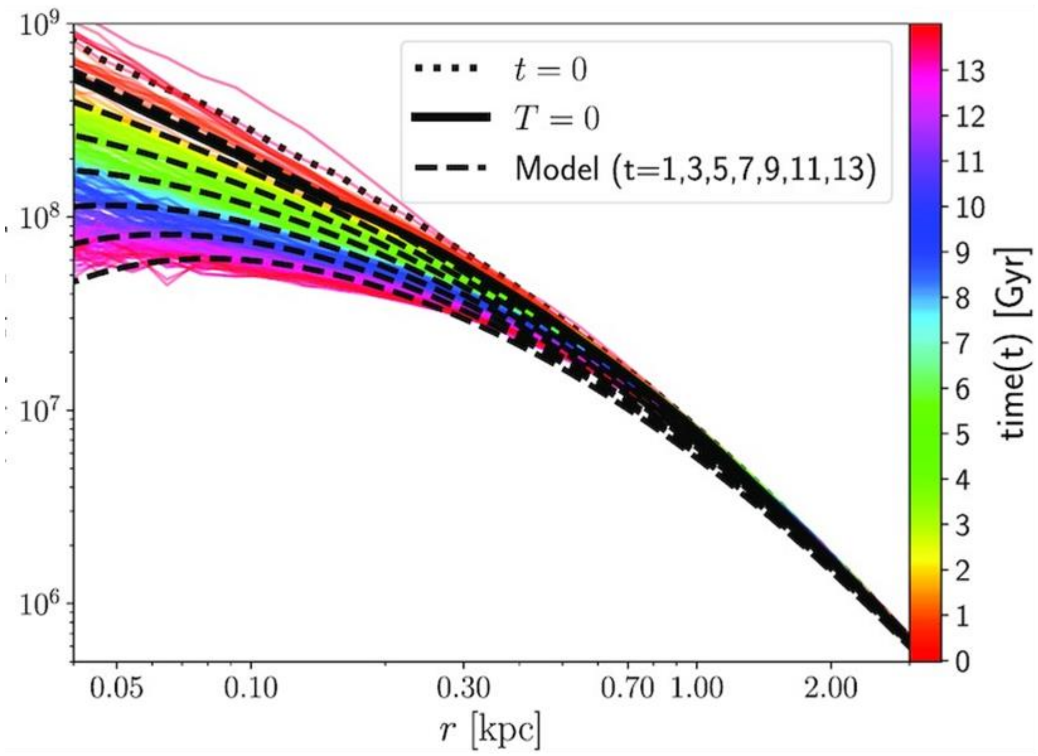


Cusp → core transformation via
Violent feedback → Turbulence
As galaxy settles into halo

Other models (El-Zant et. al 2001) involve clumpiness with no feedback.



After Andrew Robertson



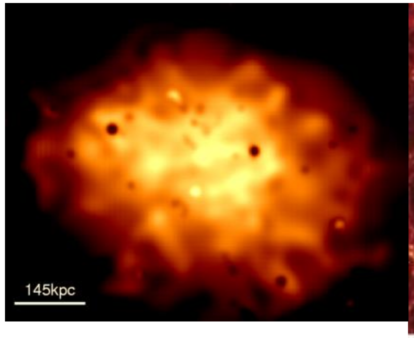
In Conclusion

- Turbulence everywhere → convection in room → clusters and beyond

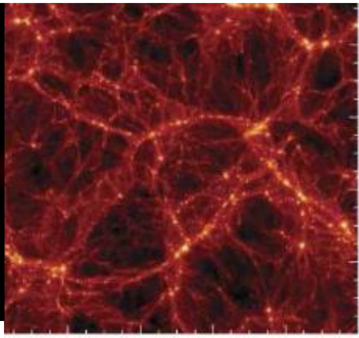
- Turbulence difficult in details:

Highly nonlinear ++ Nearly scale free → not amenable to perturbation

→one of the ‘big questions’ in physics and mathematics



Gas pressure I n Coma cluster
Schuecker et. al. (2004) r



Large scale gas density
Pfommer et. al 2006)

Astrophysical turbulence particularly complex (compressibility, mag-fields etc)

But all the more interesting for understanding our universe

→Upcoming observations at high redshift to shed light on clumsiness, gravitational instabilities and driving in nascent disks, feedback, interaction between luminous and dark components etc....

→ stay tuned.