Dynamical Properties of Dust Acoustic waves in Space Plasma and Its Application in Image Encryption

For M.Sc. degree in Physics (Theoretical Physics)

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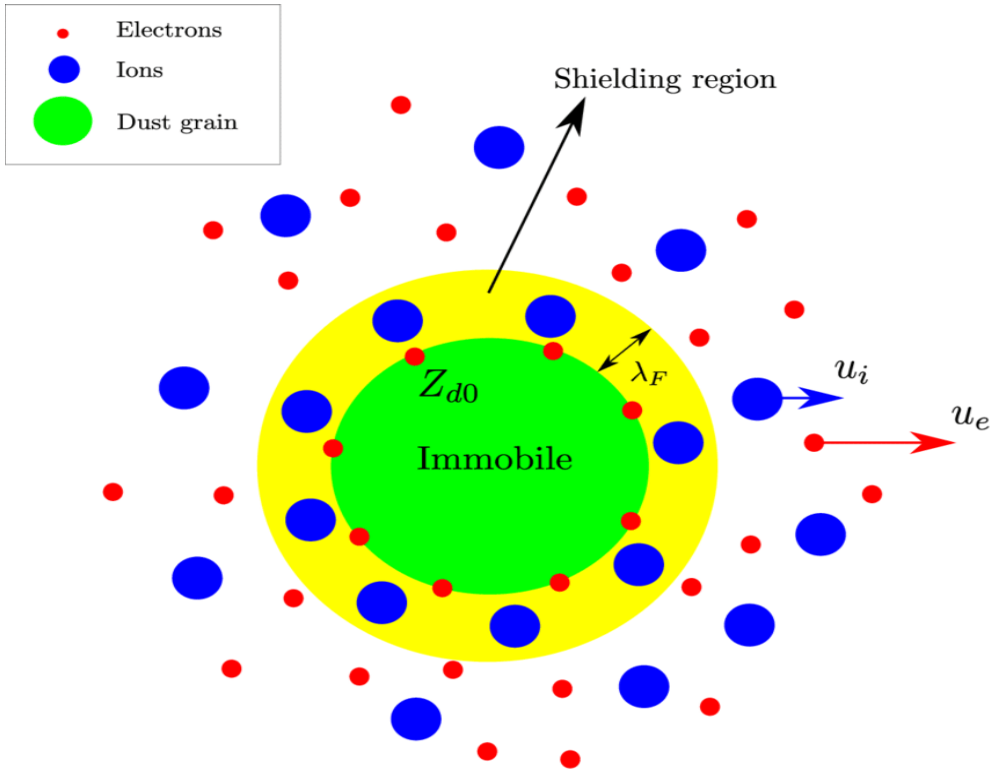
Outline

- Introduction
- Aim
- Physical Model
- Application
- Conclusion

Dusty (Complex) Plasma

A dusty plasma is an ionized gas containing dust particles, with sizes ranging from tens of nanometers to hundreds of microns.



a is an inter-grain distance

Criteria of dusty plasma

- *Dust in plasma* λ_d **<a**
- *Dusty Plasma* λ_d **>a**

Examples of Dusty Plasma

Orion Nebula

Comet Tails

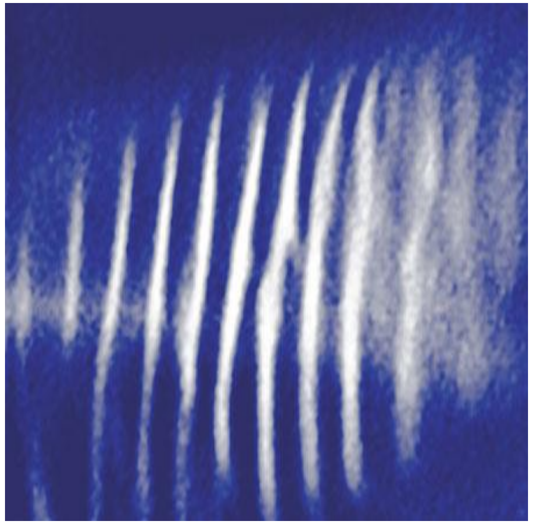
Planetary Rings

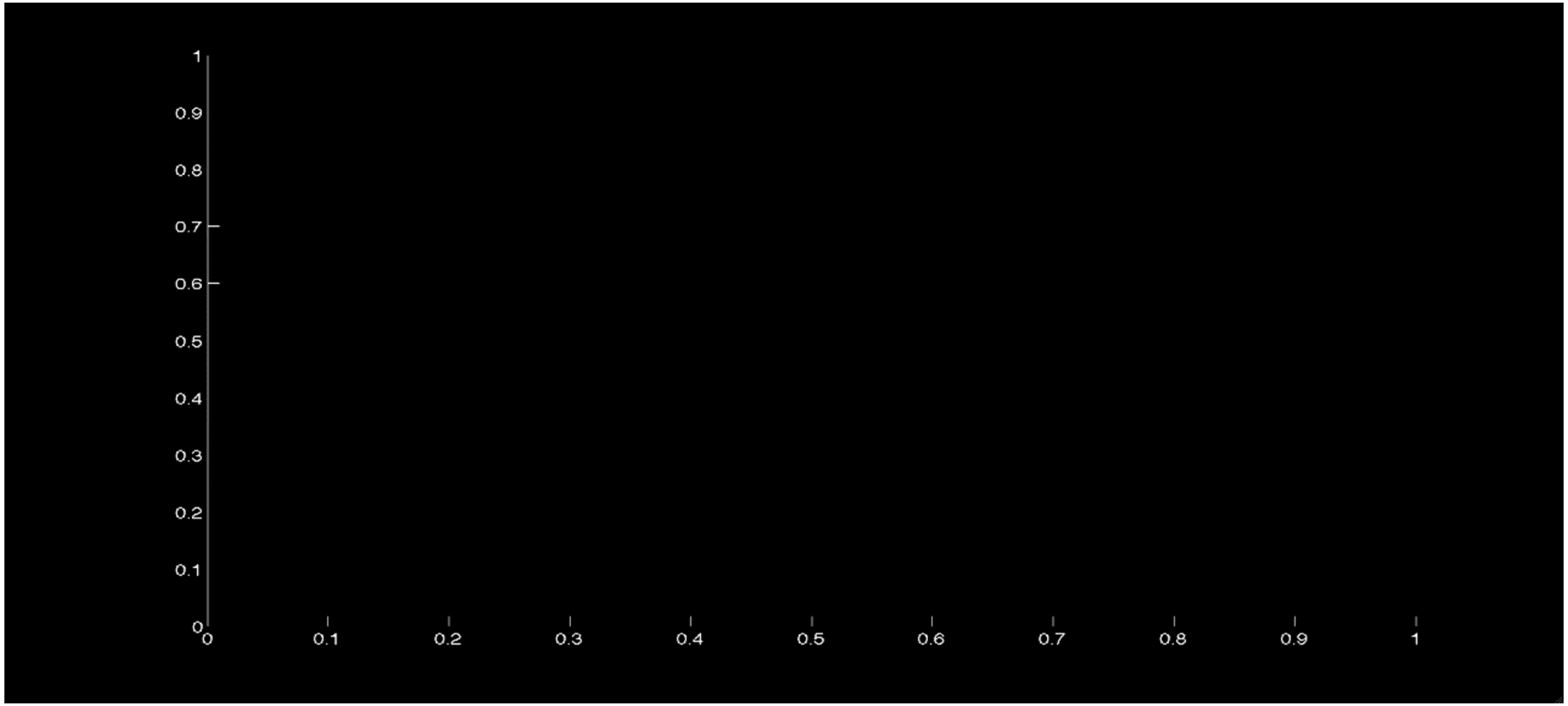
Thermonuclear Explosions

Rocket Exhausts

Dust acoustic waves

are extremely low-frequency EAWs in which the inertia is provided by the mass of the dust species and the restoring force is provided by the thermal pressures of ion species and electrons.



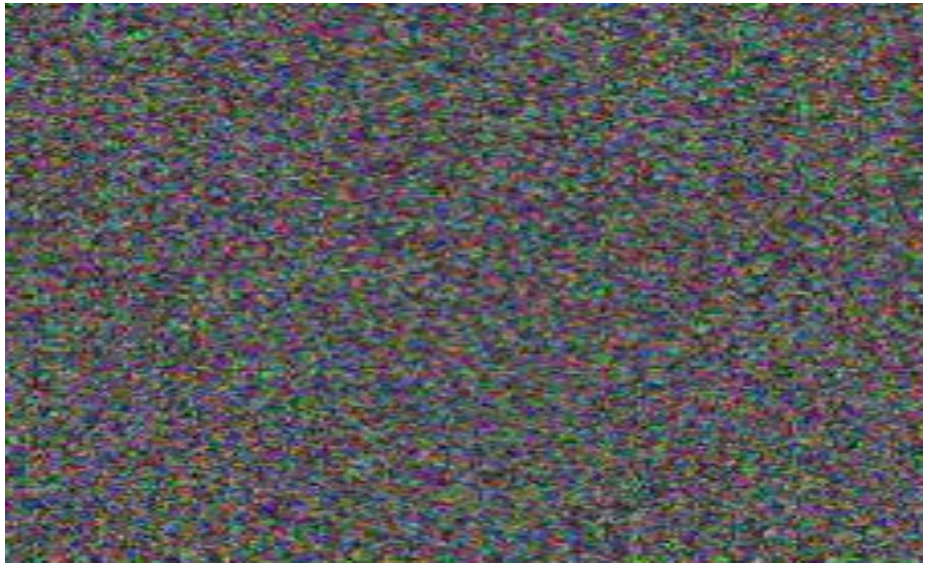


Encryption

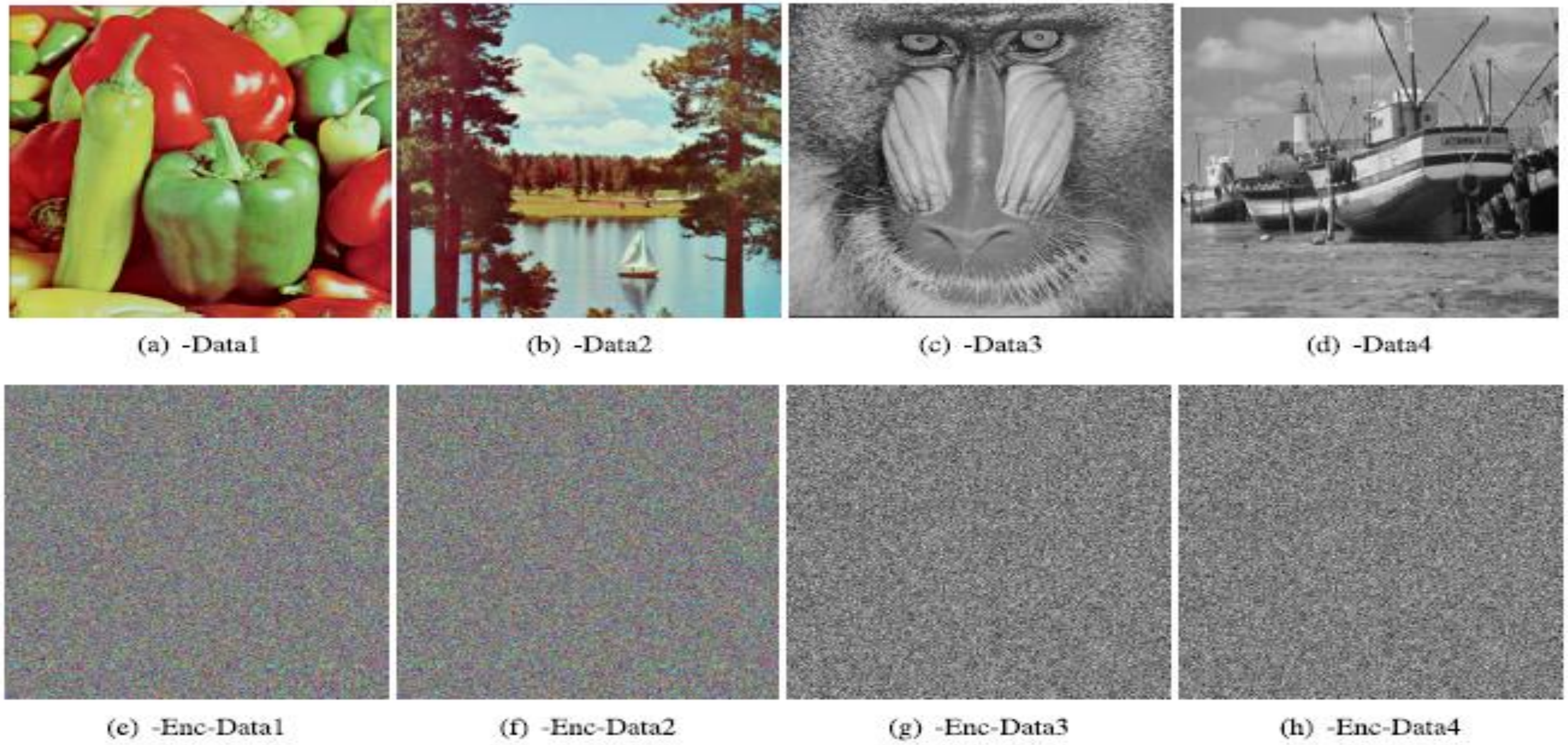
is the process of encoding information. This process converts the original representation of the information, known as plaintext, into an alternative form known as ciphertext.

Examples

```
f6c71eedc3d99bb183cb5b8d1568e606
4106dc419a06db149dd22bfb583ffeee
f6c71eedc3d99bb183cb5b8d1568e606
4106dc419a06db149dd22bfb583ffeee
8b6af01acb7464cb68c4a3548aaf95a6
469c7fcb75d5d9a1b418cb997b09a185
8b6af01acb7464cb68c4a3548aaf95a6
469c7fcb75d5d9a1b418cb997b09a185
```



Encrypted Image Examples



(a) -Data1 (b) -Data2 (c) -Data3 (d) -Data4

(e) -Enc-Data1 (f) -Enc-Data2 (g) -Enc-Data3 (h) -Enc-Data4

Aim of This Study

- Study of the nonlinear dust-acoustic (DA) wave in a space plasma
- Drive numerical periodic wave solution for DAW
- Study stability property of DAWs with change of initial conditions
- Using the chaotic feature in the perturbed DS to design efficient encryption algorithm

Plasma model

Governing equations (fluid model)

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i u_i) = 0$$

$$\frac{\partial u_i}{\partial t} + (u_i . \nabla) u_i = -\frac{e}{m_i} \nabla \phi + \frac{e B_0}{m_i c} u_i \times e_z + \eta_0 \nabla^2 u_i - \nu_0 u_i$$

$$\nabla^2 \phi = 4 \pi e \left[n_e - n_i + Z_d n_d \right]$$

In order to study the nonlinear dust ion-acoustic wave in x-z plane, the normalized equations in the component form can be written as

$$\frac{\partial n}{\partial t} + \frac{\partial (n u_x)}{\partial x} + \frac{\partial (n u_z)}{\partial z} = 0$$

$$\frac{\partial u_x}{\partial t} + \left(u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z} \right) u_x = -\frac{\partial \phi}{\partial x} + u_y + \eta_i \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u_x - \nu_{id} u_x$$

$$\frac{\partial u_y}{\partial t} + \left(u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z} \right) u_y = -u_x + \eta_i \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u_y - \nu_{id} u_y$$

$$\frac{\partial u_z}{\partial t} + \left(u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z} \right) u_z = -\frac{\partial \phi}{\partial z} + \eta_i \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u_z - \nu_{id} u_z$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\phi = \beta[n_e - \mu_1 n + \mu_2]$$

Nonlinear evaluation of DMKPB equation

Reductive perturbation technique (RPT) is employed to construct DMKPB equation for small-amplitude ion-acoustic two dimensional solitary wave in the magnetized dusty plasma

$$n = 1 + \epsilon n_1 + \epsilon^2 n_2 + + \epsilon^3 n_3 \ldots$$

$$u_x = \epsilon^2 u_{x1} + \epsilon^3 u_{x2} + \ldots$$

$$u_y = \epsilon^2 u_{y1} + \epsilon^4 u_{y2} + \ldots$$

$$u_z = \epsilon u_{z1} + \epsilon^2 u_{z2} + + \epsilon^3 u_{z3} \ldots$$

$$\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + + \epsilon^3 \phi_3 \ldots$$

$$\nu_{id} \approx \epsilon^3 \nu_{id0}$$

$$\eta_i \simeq \epsilon \eta_{i0}$$

$$\chi = \epsilon^2 x$$

$$\xi = \epsilon (z - V t)$$

$$\tau = \epsilon^3 t$$

Governing Equation

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1^2\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + D\phi_1 + E\frac{\partial^2 \phi_1}{\partial \xi^2}\right] + C\frac{\partial^2 \phi_1}{\partial \chi^2} = 0$$

where $A = \frac{15}{4V^3} - \frac{3RV}{2P}$, $B = \frac{V^3}{2\mu_1\beta}$, $C = \frac{V}{2}$, $D = \frac{1}{2}\nu_{id0}$, $E = -\frac{1}{2}\eta_{i0}$

The above equation is termed as damped modified KP Burgers (DMKPB) equation

TRAVELING PLANE WAVE ANALYSIS: PHASE-SPACE DYNAMICS

we perform the nonlinear analysis for traveling plane wave. Also, to incorporate the spatial dimensional effect, we consider oblique propagation of the plane wave. Thus, we search for traveling wave solutions of the nonlinear Equation that depends only on the independent variable

$$\theta = k_\xi \xi + k_\chi \chi - M\tau$$

$$\frac{\partial}{\partial \xi} = k_\xi \frac{d}{d\theta} \quad , \quad \frac{\partial}{\partial \chi} = k_\chi \frac{d}{d\theta} \quad , \quad \frac{\partial}{\partial \tau} = k_\tau \frac{d}{d\theta}$$

Nonlinear Dynamical System

$$\dddot{X} = -\alpha\ddot{X} - \frac{\left(AX - \rho B{k_\xi}^2\right)}{B{k_\xi}^2}\dot{X} - \gamma X$$

where $\phi_1 \equiv X$, $\frac{d\phi_1}{d\theta} \equiv \dot{X}$, $\alpha = \frac{E}{Bk_\xi}$, $\rho = \frac{C\left(1 - {k_\xi}^2\right) - Mk_\xi}{B{k_\xi}^4}$, $\gamma = \frac{D}{B{k_\xi}^3}$

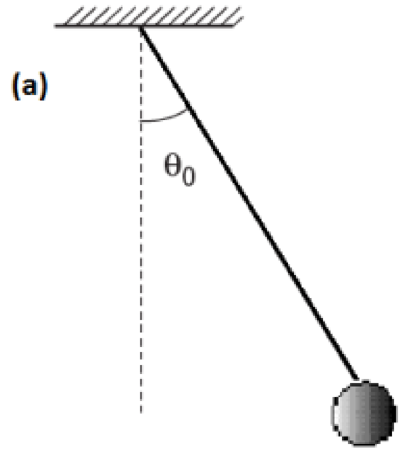
$$\dot{X} = Y$$

$$\dot{Y} = Z$$

$$\dot{Z} = -\alpha Z - \frac{\left(AX - \rho B{k_\xi}^2\right)}{B{k_\xi}^2}Y - \gamma X + f_0\cos(\omega t)$$

Fixed Point

$$S_0 = (0, 0, 0)$$



Types of Fixed Points

Stable

If the ball is displaced it will return to it's original position

Unstable

If the ball is displaced it will accelerate away from the equilibrium point

characteristic equation

$\dot{X} = Df(S_0)X$ Where X is a column vector $X = [X, Y, Z]^T$ and $Df(S_0)$ the Jacobian matrices at S_0

$$Df = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} & \frac{\partial f_x}{\partial z} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} & \frac{\partial f_y}{\partial z} \\ \frac{\partial f_z}{\partial x} & \frac{\partial f_z}{\partial y} & \frac{\partial f_z}{\partial z} \end{bmatrix}$$

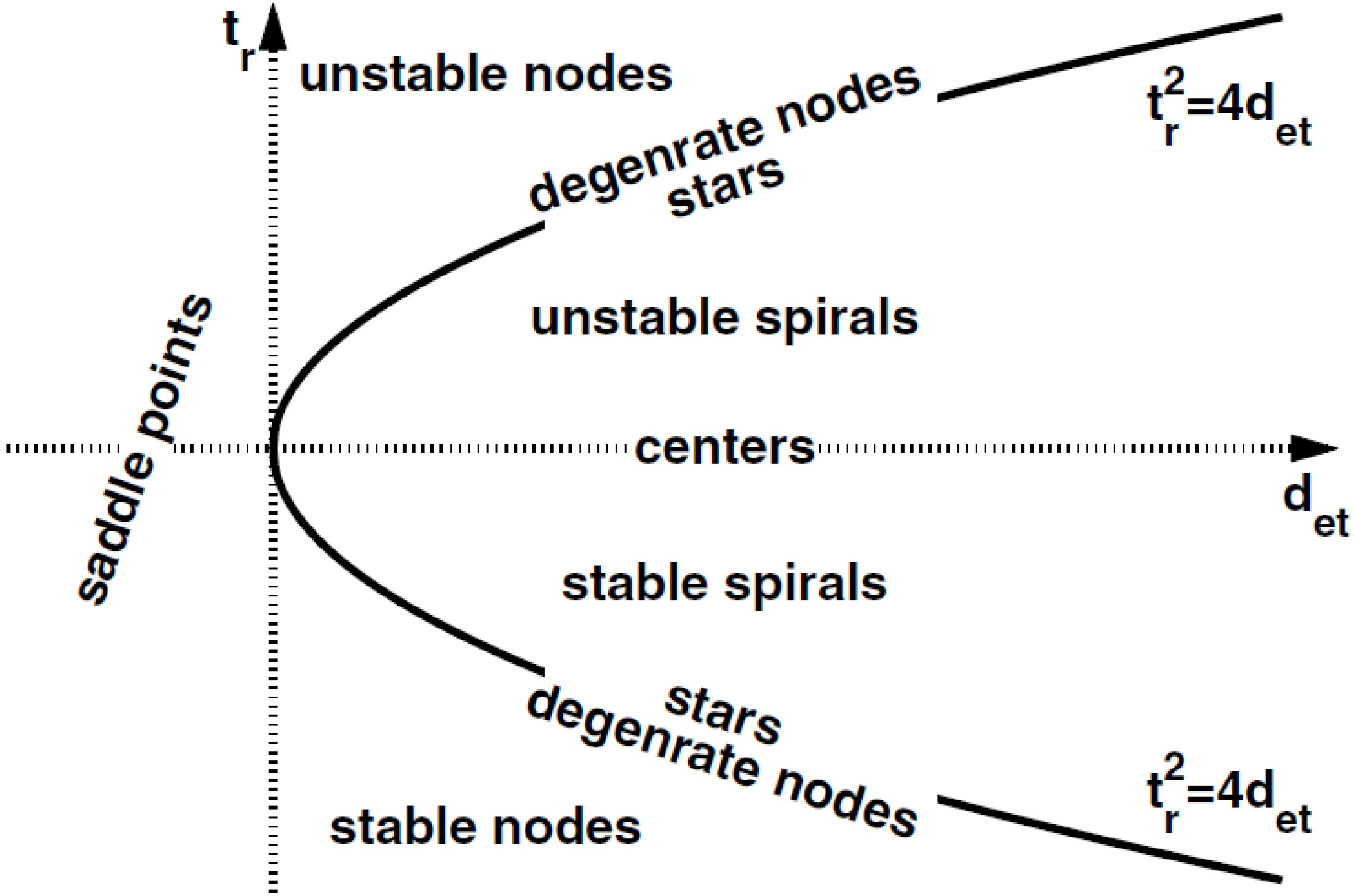
$$Df(S_0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\gamma & \rho & -\alpha \end{bmatrix}$$

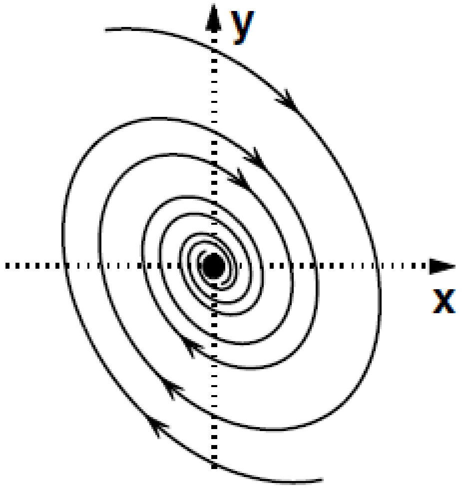
$$|Df(S_0) - \lambda I_3| = 0$$

$$|Df(S_0) - \lambda I_3| = \begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -\gamma & \rho & -(\alpha + \lambda) \end{bmatrix}$$

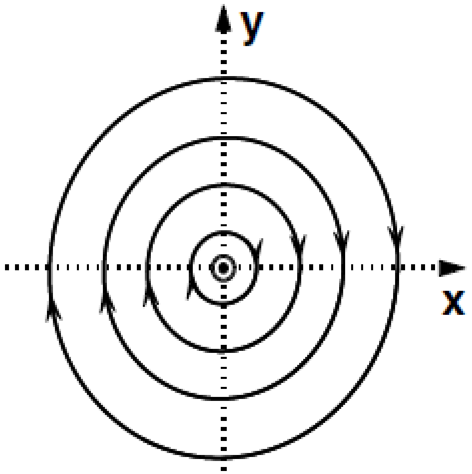
$\lambda^3 + \alpha\lambda^2 + \gamma = 0$ is characteristic equation and λ is eigenvalue

Classification diagram for two-dimensional linear systems in terms of the trace t_r and determinant d_{et} of the linear matrix

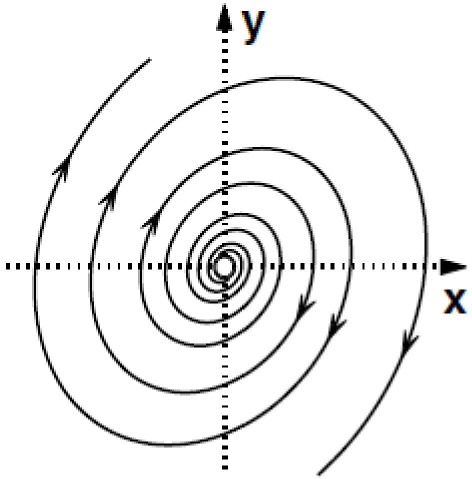




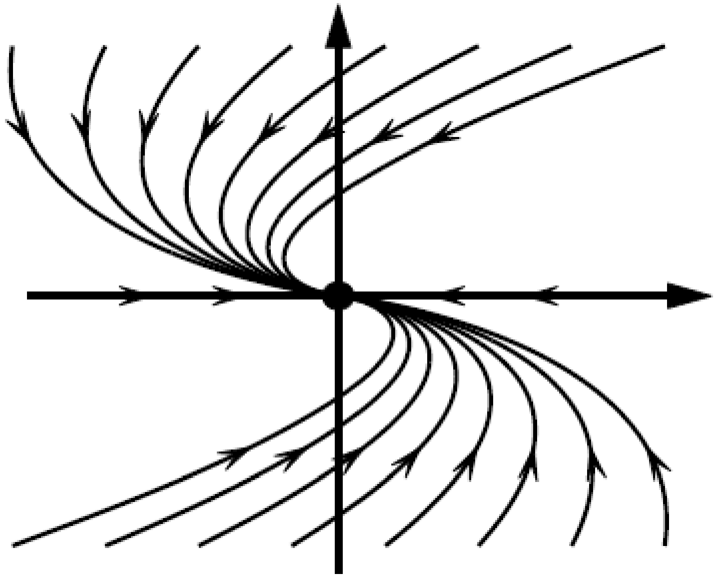
stable spiral



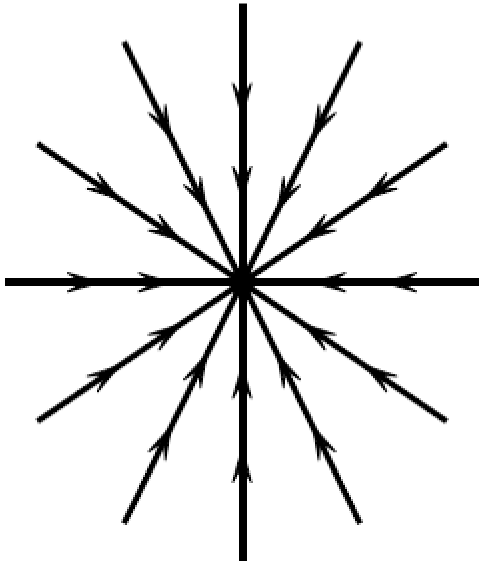
center



unstable spiral



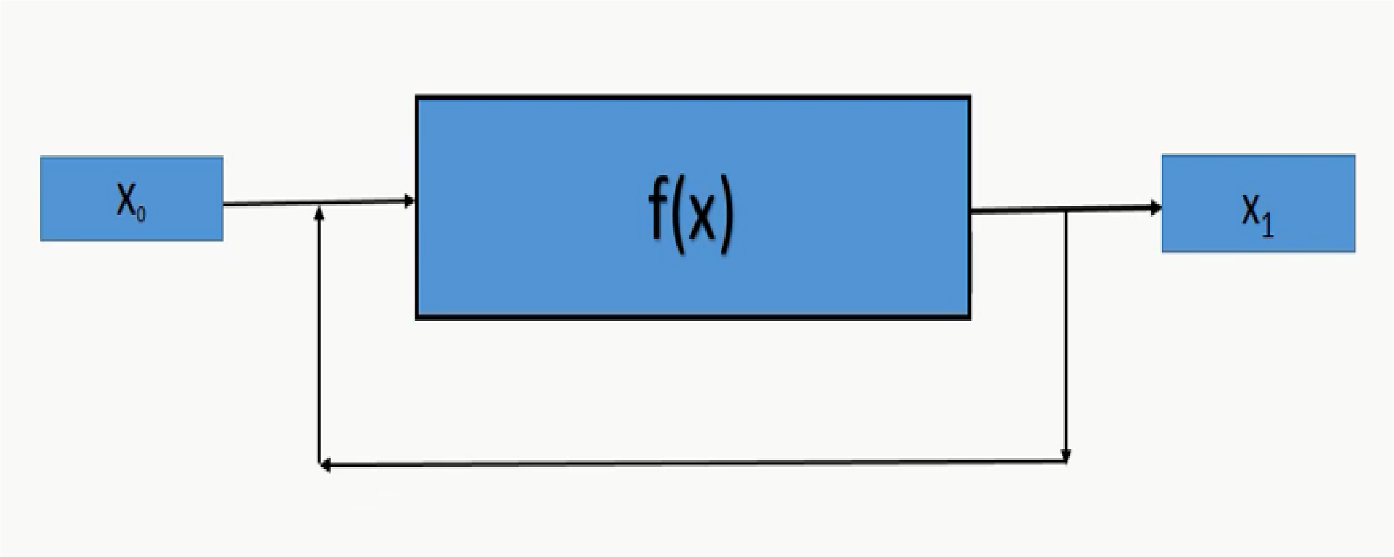
stable degenerate node



stable star node

Chaotic Behavior

- Feedback system
- Sensitive to initial conditions
- Unpredictable system



Logistic map

$$x_{n+1} = r x_n (1 - x_n)$$

Lorenz system

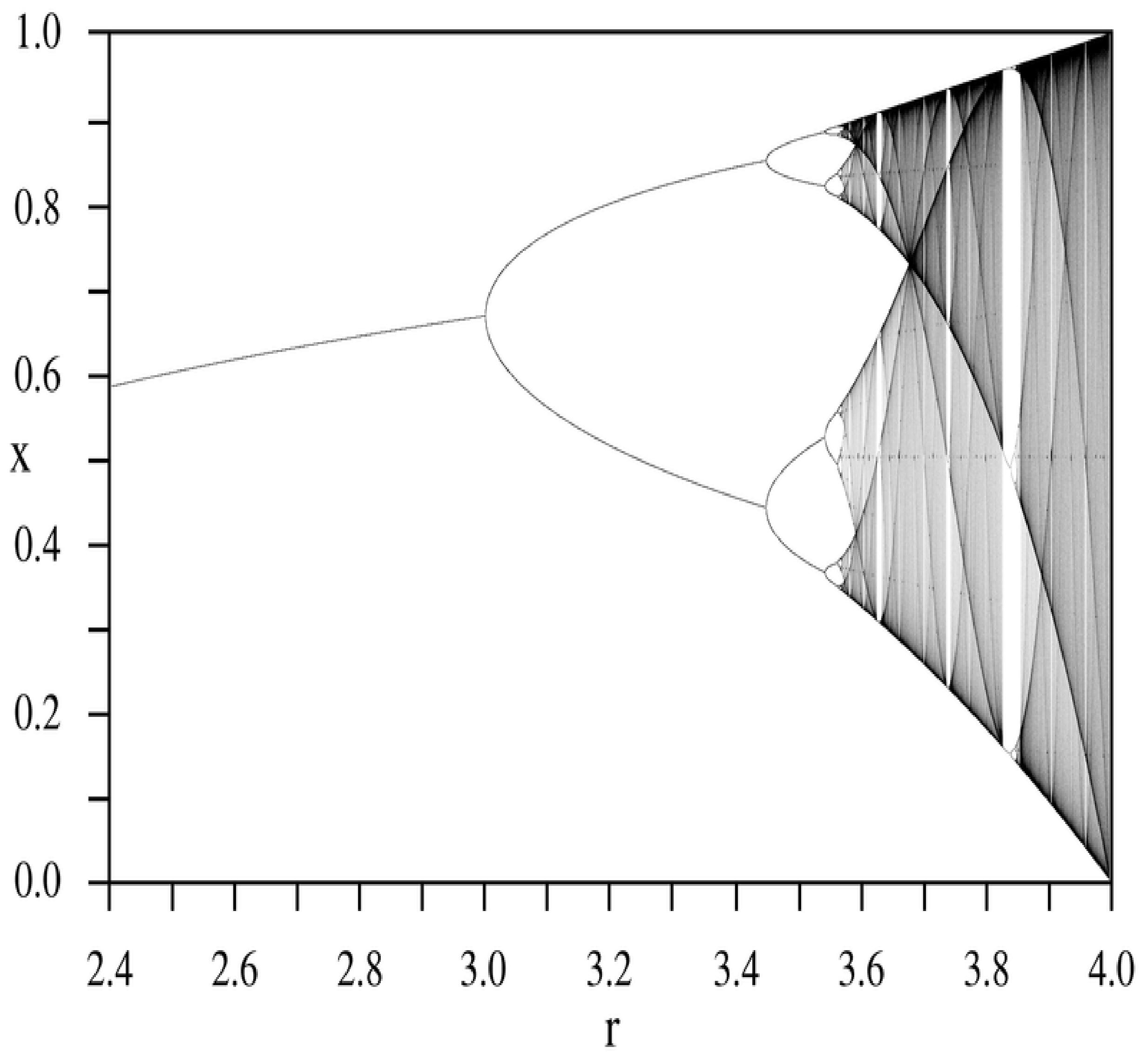
$$\frac{dx}{dt} = \sigma(y - x),$$

$$\frac{dy}{dt} = x(\rho - z) - y,$$

$$\frac{dz}{dt} = xy - \beta z.$$

Sensitivity to initial conditions

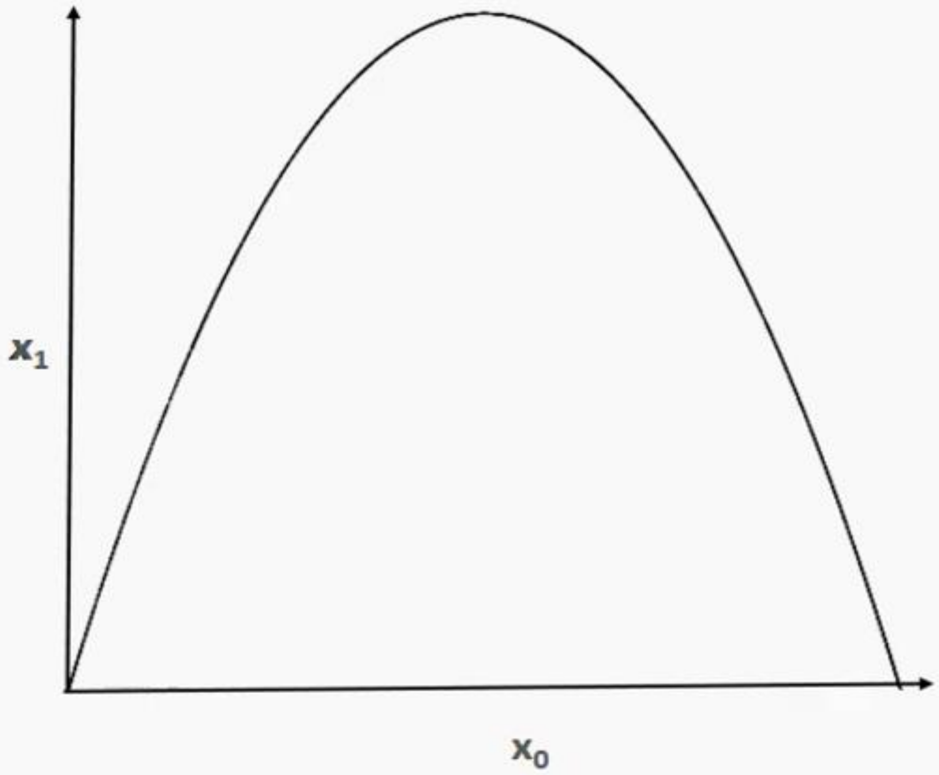
0.1	0.101
0.3465000000000003	0.3495761500000003
0.8717853375000001	0.875384761602033
0.4303363018570424	0.41998218091439093
0.9438158312700305	0.9378490222208136
0.20415590546925336	0.22440969990000684
0.6255336461654382	0.670092447989694
0.9018290191695807	0.8511139526769619
0.3408537415098691	0.4878681201069067
0.8649890033730808	0.9619333473374994
0.449614655554392	0.14097769307765673
0.9527260707006251	0.46624648505881117
0.17340055389569453	0.9581136958820925
0.5518312869467894	0.15450759033421121
0.9521570431197424	0.502944730223808



Nonlinear equation

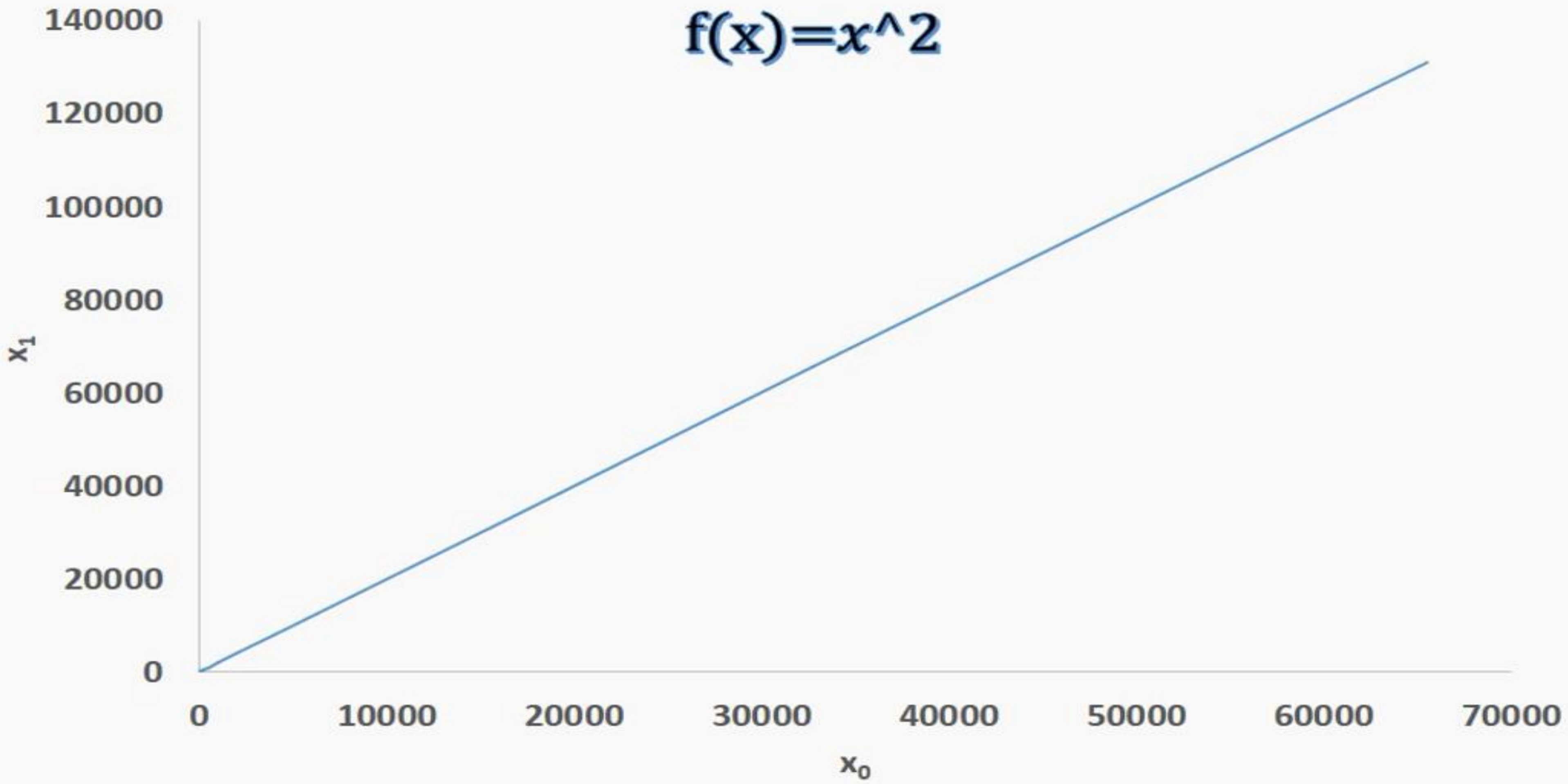
Logistic map $$x_{n+1} = r x_n (1 - x_n)$$

x_0	x_1
0.1	0.35910000000000003
0.35910000000000003	0.9182872881000002
0.9182872881000002	0.2993926210096504
0.2993926210096504	0.8369291511835427
0.8369291511835427	0.5445502008601223
0.5445502008601223	0.9895809656172578
0.9895809656172578	0.041138807640040874
0.041138807640040874	0.1573911605225264



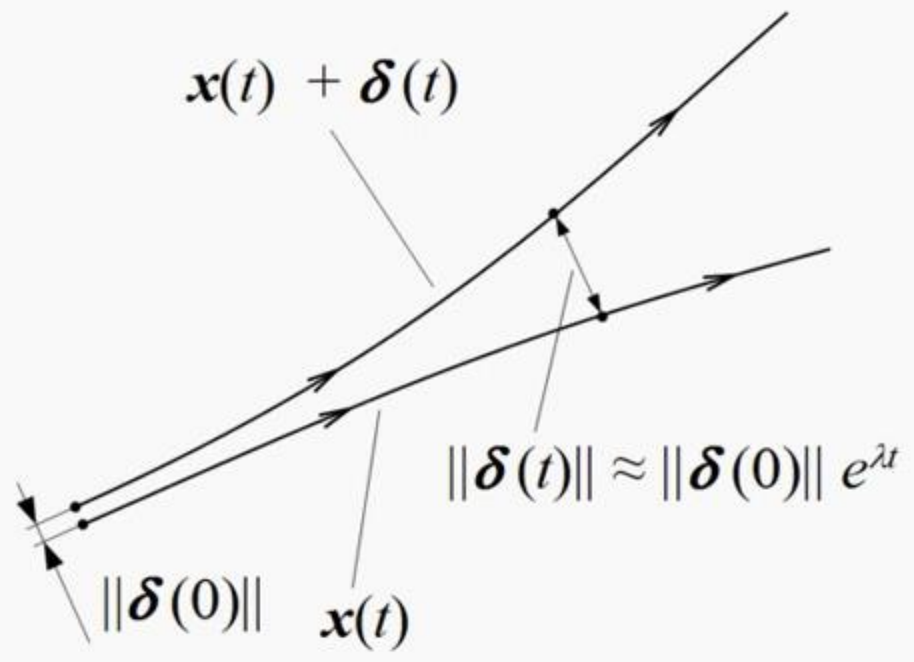
Linear equation

x_0	x_1
1	1
1	2
2	4
4	8
8	16
16	32
32	64
128	256
256	512
512	1024
1024	2048



Lyapunov Exponents

- Measure of sensitive dependence on initial conditions
- Measure rate of separation or convergence of infinitesimally close trajectories



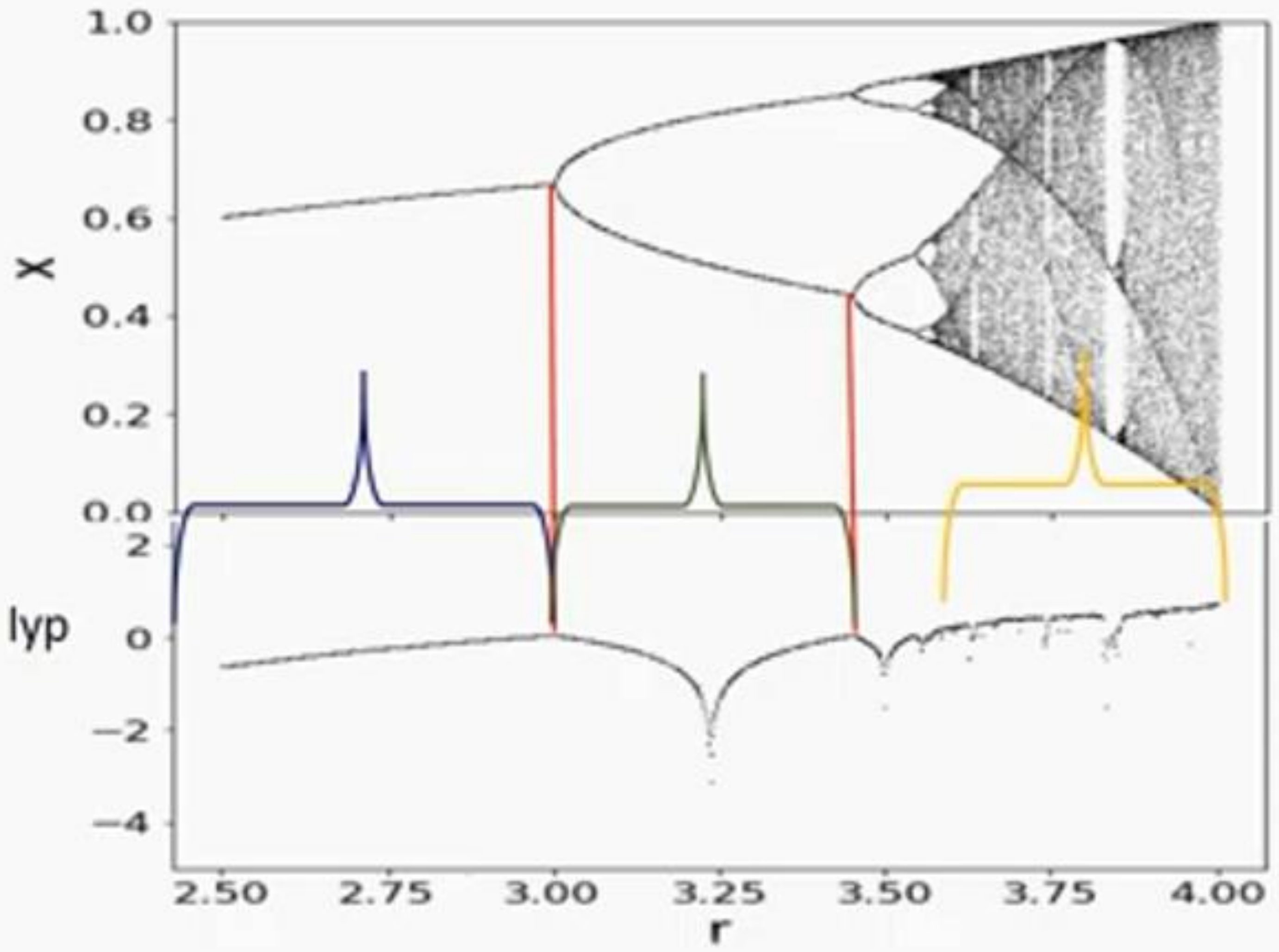
LYAPUNOV EXPONENT	INFERENCE
Positive	Dynamical system is chaotic
Negative	Dynamical system is periodic or refers to a fixed point
Zero	Bifurcation point

$$\lambda_i = \overline{\lim_{t\to\infty}} \frac{1}{t} \ln \frac{\|\delta \mathbf{x}(t)\|}{\|\delta \mathbf{x}_i(t_0)\|}.$$

$$\lambda_{F(x)} = \lim_{n\to\infty} \left\{ \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| F'(x_i) \right| \right\}$$

Bifurcation diagram

Lyapunov exponent



r = 2
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5
0.5

r = 3.3
0.47882410860516644
0.8235202193579928
0.4796044032996355
0.82362726479627 9
0.47937579848558104
0.8235963196292456
0.4794418923439434
0.8236053018916865
0.4794227083390485
0.8236026977240704
0.479428270284823
0.8236034529905348
0.4794266572015889
0.82360323396812 7
0.4794271249857199

r = 3.83
0.6196668543989299
0.9026538023601561
0.3365417761539989
0.8551677966887337
0.4743678526278855
0.9549836632707908
0.16465118738012915
0.5267826959391158
0.954752691969232
0.1654559584320021
0.5288474886826693
0.9543127597793735
0.1669876794353168
0.532763702368637 6
0.9533886474611906

r = 3.93
0.94344302075891 6
0.20969806924701 6
0.6512984207742825
0.8925375363338481
0.3769431304959847
0.92298803699073 6
0.27934880381150456
0.7911602850089033
0.6493369555437893
0.8948550016760254
0.36977184367010413
0.9158496652847409
0.3028813896184961
0.8297969160241709
0.5550495971311176

r=2 f5f5f5f5f5f5f5f5f5f5f5f5f5f5f5f5

r=3.3 df3ae03ae03ae03ae03ae03ae03ae0

r=3.83 d89515adbbda410bda512bda616bc

r=3.93 b2d18d7f799d181a8b8172962f402d

Numerical Method (RK4)

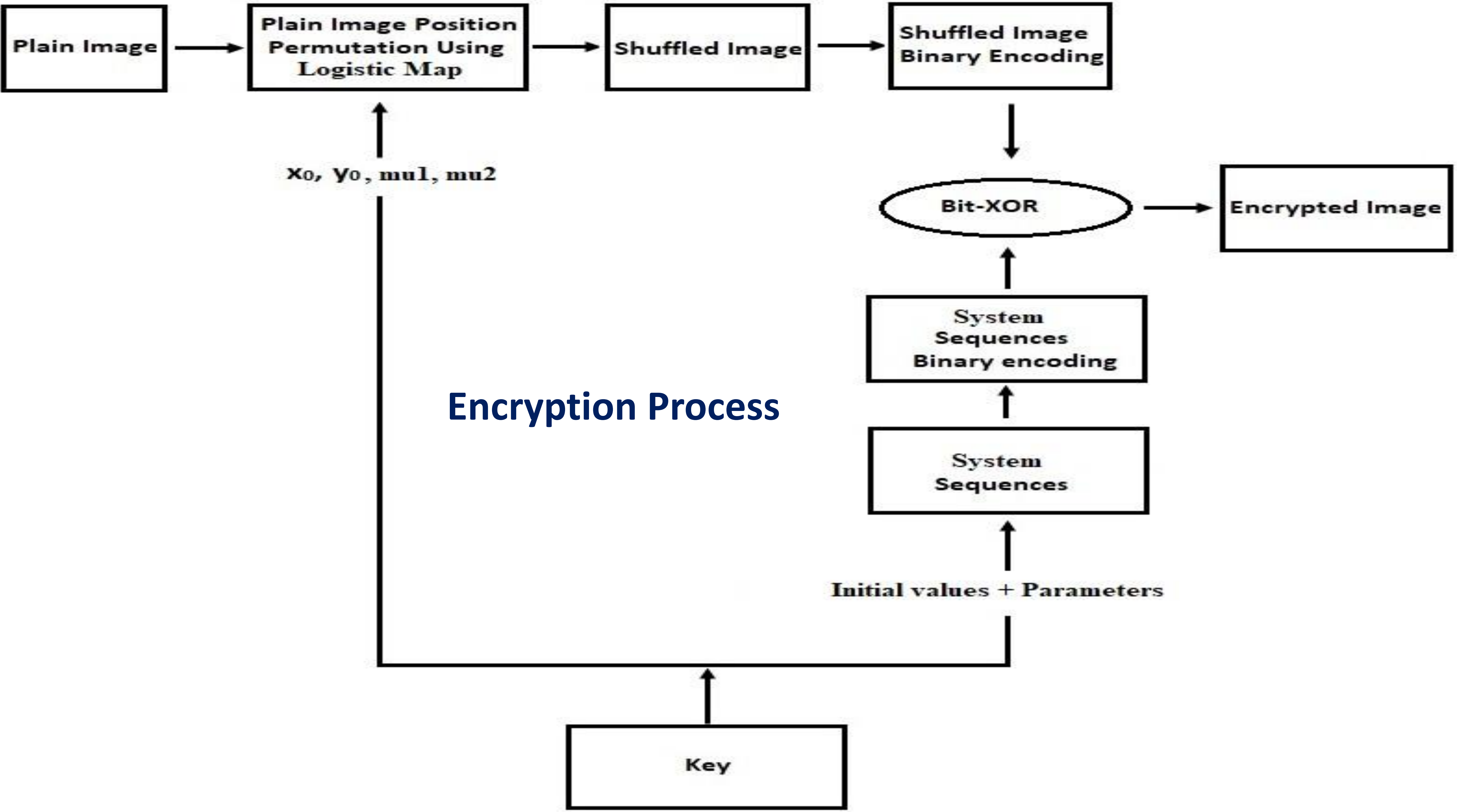
$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$K_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$K_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + k_1/6 + k_2/3 + k_3/3 + k_4/6 + O(h^5)$$



Encryption Process

Conclusion

- Studying plasma Fluid model for DAWs
- Using traveling plane wave analysis to drive dynamical system
- Appling stability analysis to the fixed points and Classify it
- Solving dynamical system numerically and determine chaotic behaviour
- Using chaotic values to design efficient encryption algorithm

Thank You