• Observation of electrostatic waves in plasma

Presented by

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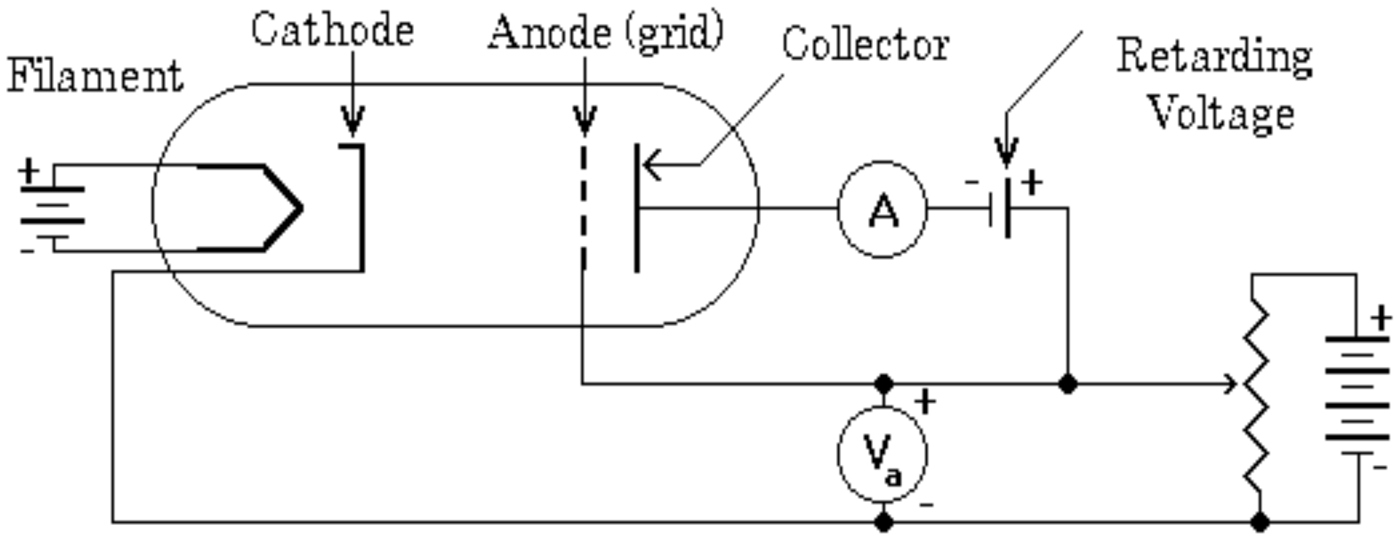
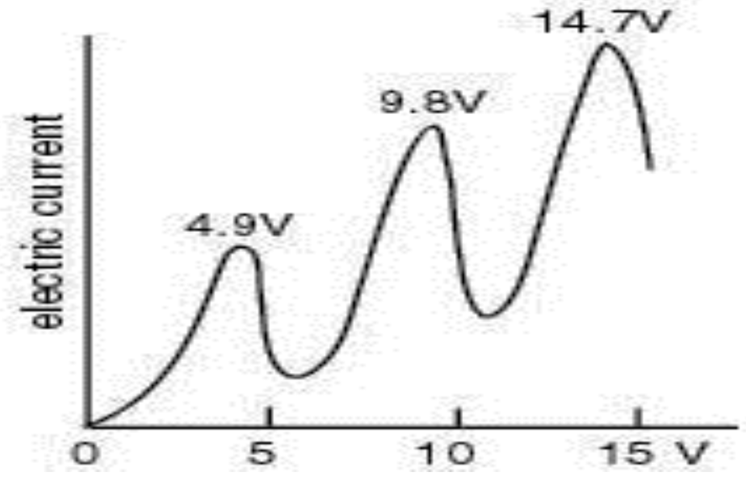
•Outlines

- Introduction
- The concept of plasma oscillation
- The concept of plasma wave
- The dispersion relation

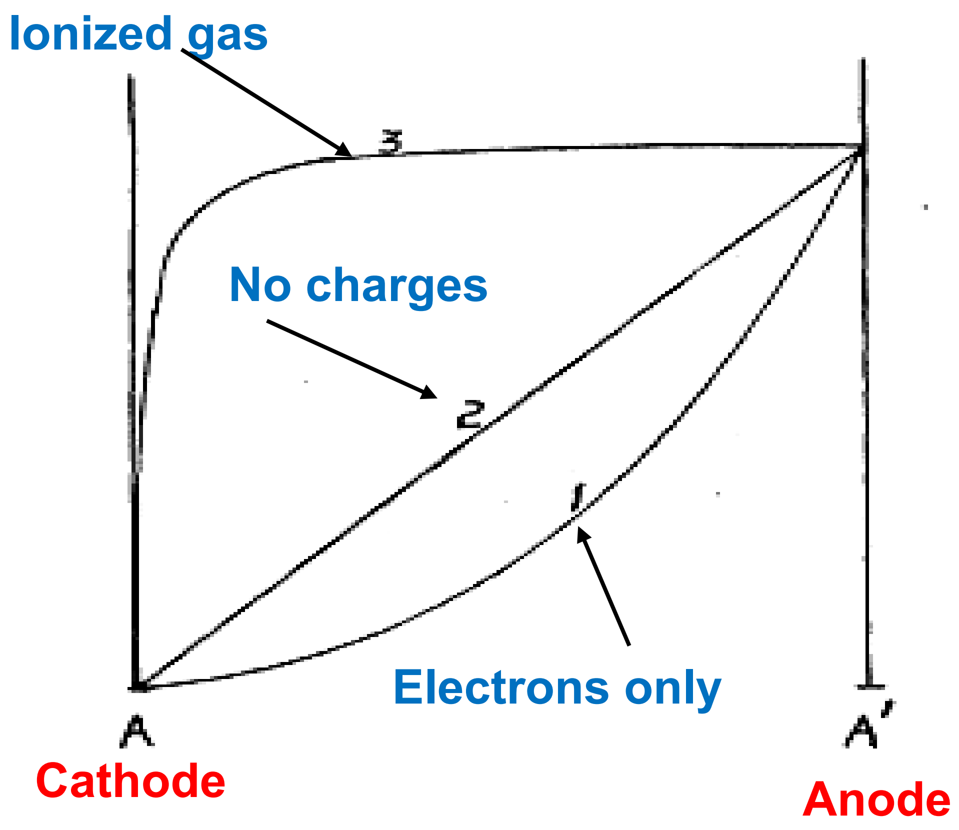
In 1914 James Franck and Gustav Hertz (nephew of Heinrich) performed an experiment on a vacuum tube with a small amount of mercury enclosed to verfy Bohr assumption 1913.

The Franck-Hertz Experiment

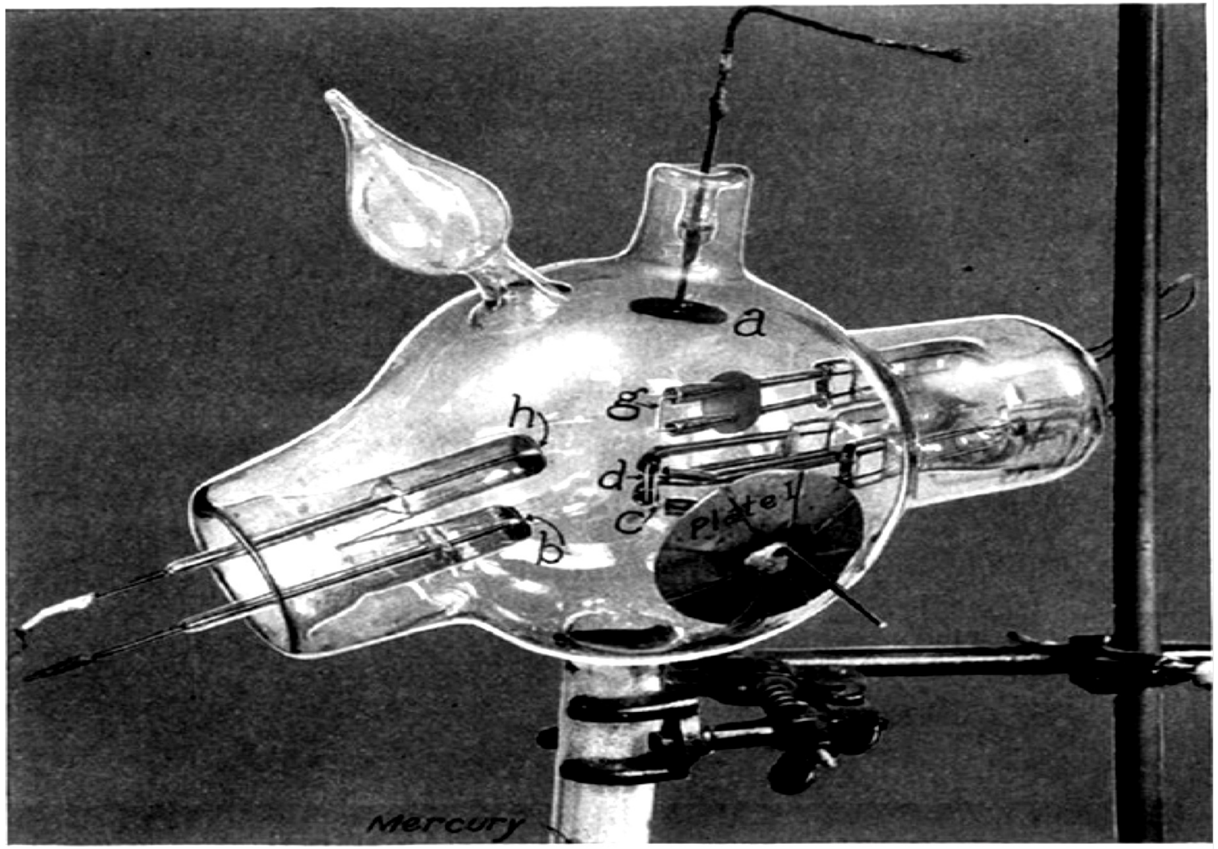


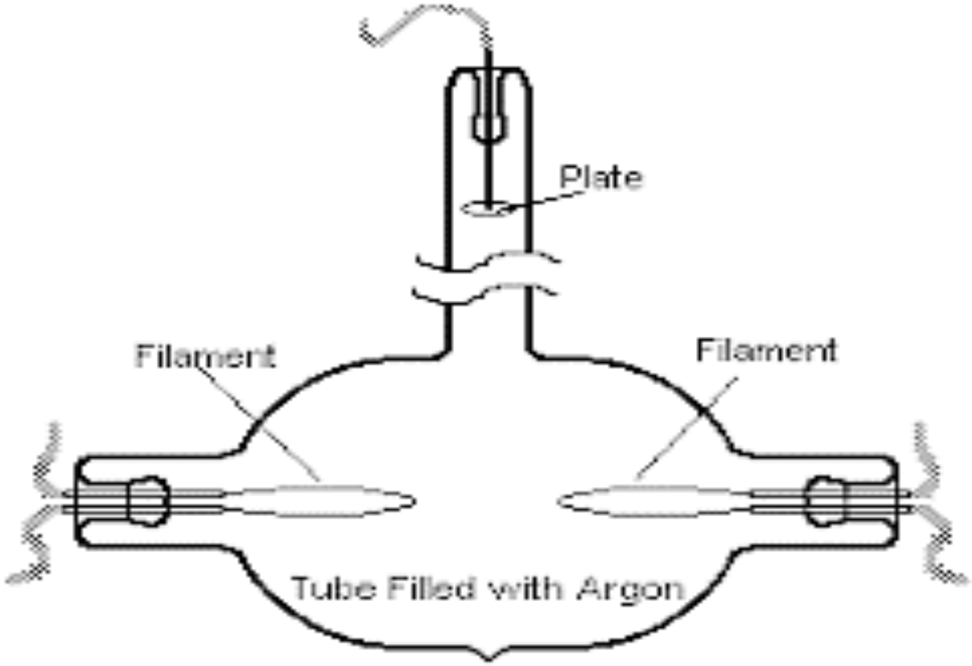


In 1926, Dittmer predicted the possibility of plasma oscillation and suggested that very small disturbances in the balance between the concentration of positive ions and of electrons could thus produce wide variations.

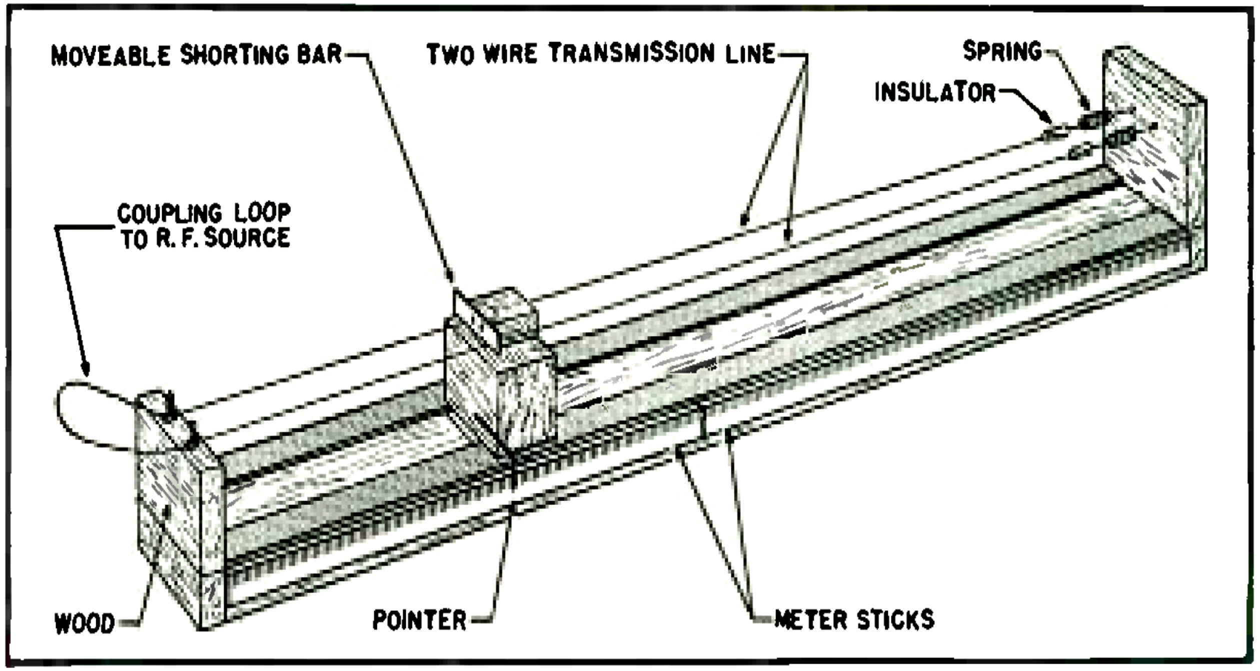


In 1929, Tonks and Langmuir observed the plasma oscillation

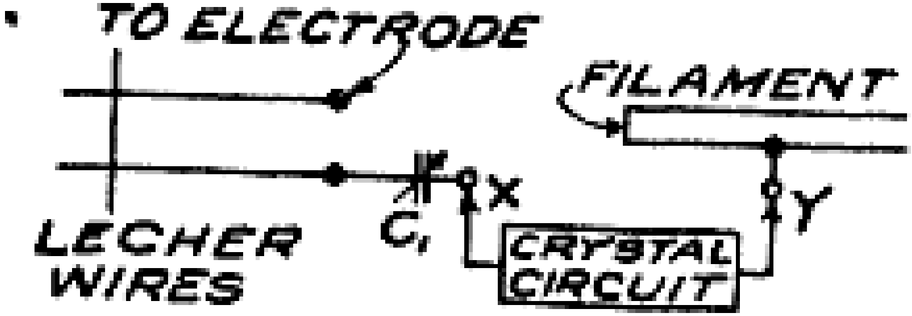




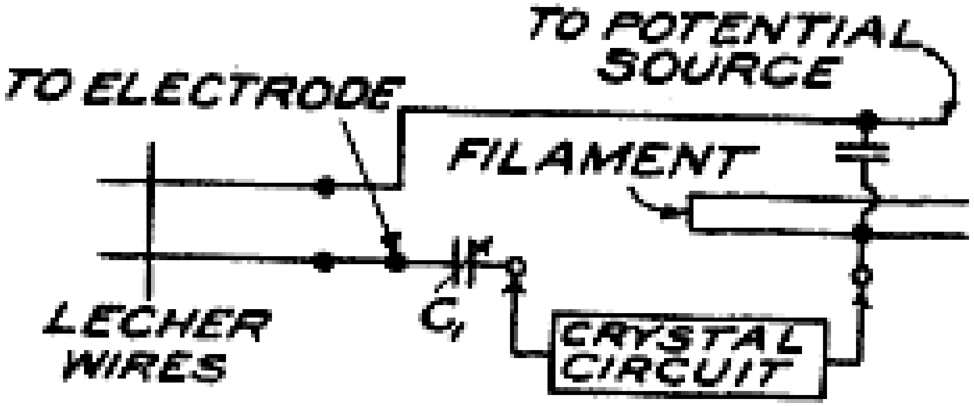
The higher frequencies of the oscillations were determined from their wave-length as measured on a pair of Lecher wires.



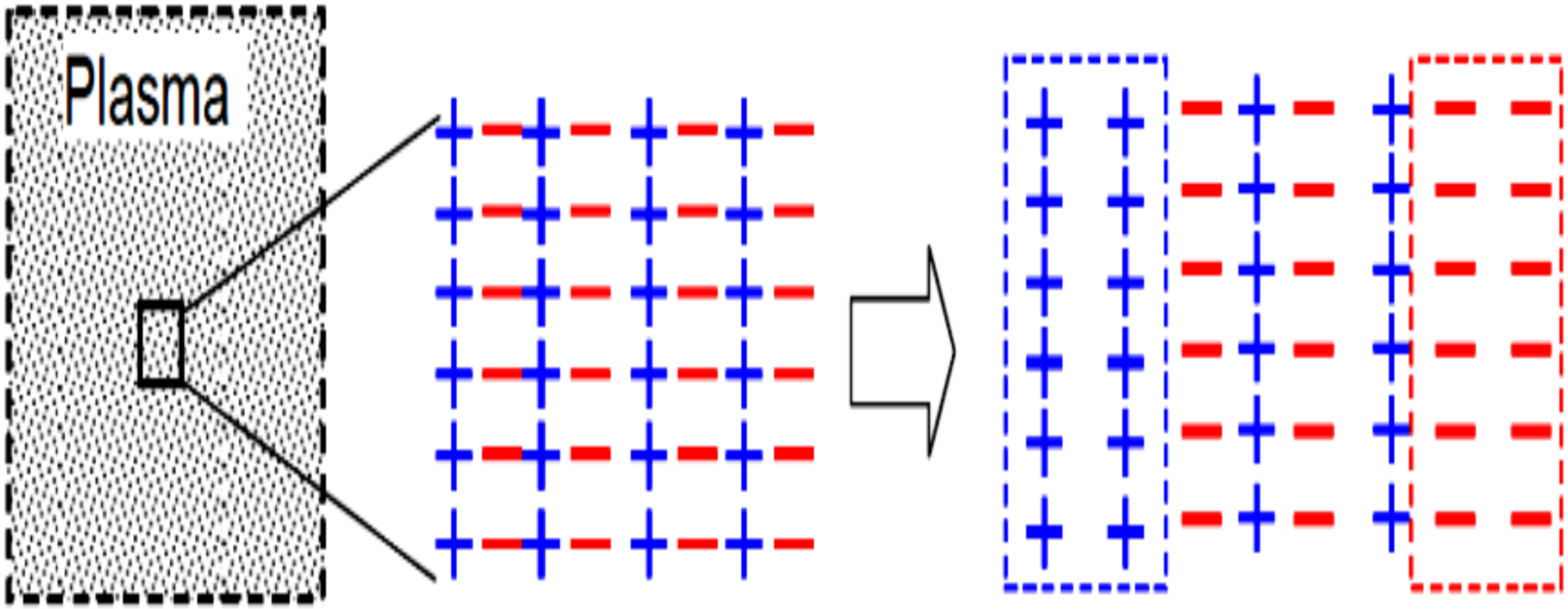
RADIO NEWS

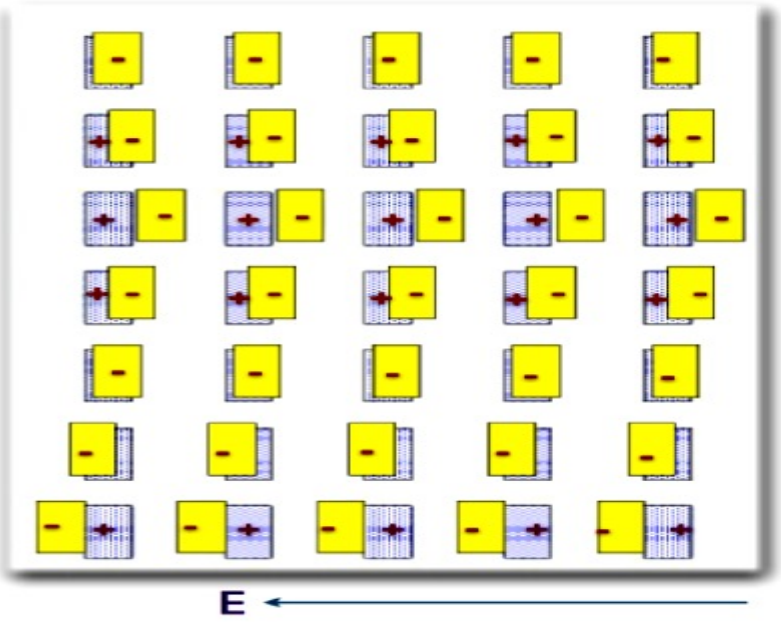


CIRCUIT A



CIRCUIT B


Plasma


E

The formula of plasma frequency

$$E = \frac{Q}{\varepsilon_o A},$$

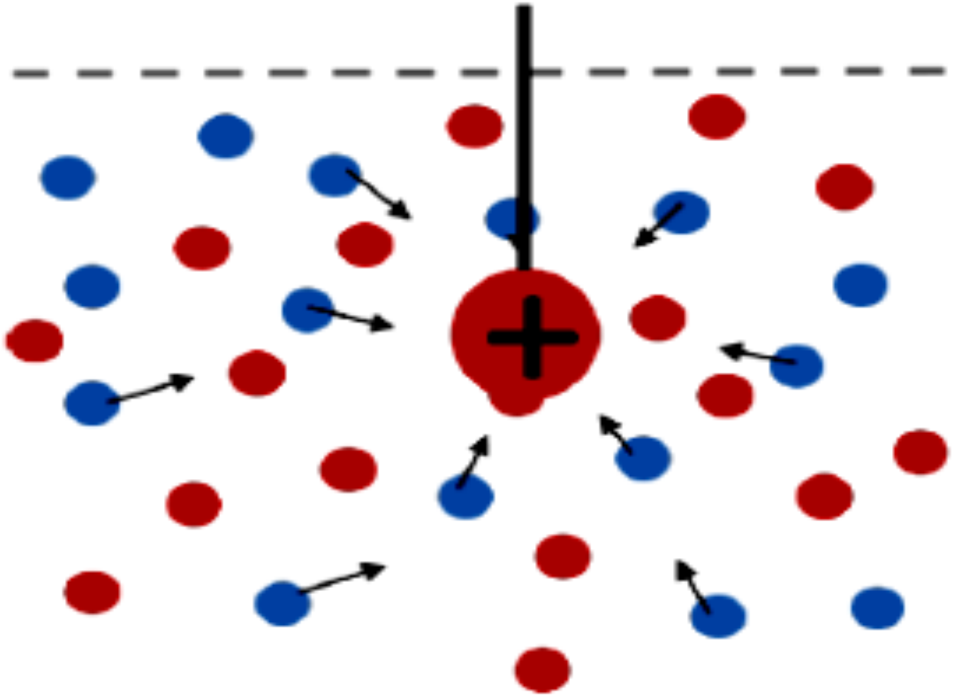
$$Q = \rho V = (en_e)(Ax),$$

$$E = \frac{en_e Ax}{\varepsilon_o A} = \frac{en_e x}{\varepsilon_o}.$$

$$m_e \frac{d^2 x}{dt^2} = -e \cdot \frac{en_e}{\varepsilon_o} x = -\frac{e^2 n_e}{\varepsilon_o} x,$$

$$\omega_{pe} = 2\pi f_{pe} = \sqrt{\frac{e^2 n_e}{\varepsilon_o m_e}}.$$

This quantity can be obtained via another route by asking how quickly it would take the plasma to adjust to the insertion of the foreign charge. For a plasma of temperature , the response time to recover quasi-neutrality is just the ratio of the Debye length to the thermal velocity ; that is



Fluid Theory

Continuity Equation

$$\frac{\partial n_j}{\partial t} + \nabla . (n_j \mathbf{v}_j) = 0 \quad \textbf{(3)}$$

Momentum Equation

$$m_j n_j \left[\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j . \nabla) \mathbf{v}_j \right] = n_j q_j (\mathbf{E} + \mathbf{v}_j \wedge \mathbf{B}) - \nabla p_j \quad \textbf{(4)}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$\varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \partial \mathbf{E} / \partial \mathbf{x} = e(n_i - n_e)$$

$$\nabla n_0 = \mathbf{v}_0 = \mathbf{E}_0 = 0$$

$$\frac{\partial n_0}{\partial t} = \frac{\partial \mathbf{v}_0}{\partial t} = \frac{\partial \mathbf{E}_0}{\partial t} = 0$$

$$m\left[\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \cdot \overset{0}{\nearrow} \nabla) \mathbf{v}_1\right] = -e\mathbf{E}_1$$

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1 + n_1^{\nearrow 0} \mathbf{v}_1) = 0$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 + \mathbf{v}_1 \cdot \nabla^{\nearrow 0} n_0 = 0$$

we note that $n_{i0} = n_{e0}$ in equilibrium and that $n_{i1} = 0$

$$\varepsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1$$

The oscillating quantities are assumed to behave sinusoidally:

$$\mathbf{v}_1 = v_1 e^{i(kx-\omega t)} \hat{\mathbf{x}}$$
$$n_1 = n_1 e^{i(kx-\omega t)}$$
$$\mathbf{E} = E_1 e^{i(kx-\omega t)} \hat{\mathbf{x}}$$

$$-im\omega v_1 = -eE_1$$

$$-i\omega n_1 = -n_0 ikv_1$$

$$ik\varepsilon_0 E_1 = -en_1$$

$$-im\omega v_1 = -e\frac{-e}{ik\varepsilon_0}\frac{-n_0 ikv_1}{-i\omega} = -i\frac{n_0 e^2}{\varepsilon_0 \omega}v_1$$

$$\omega^2 = n_0 e^2/m\varepsilon_0$$

The *plasma frequency* is therefore

$$\boxed{\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0 m}\right)^{1/2} \quad \text{rad/ sec}}$$

Numerically, one can use the approximate formula

$$\omega_p/2\pi = f_p \approx 9\sqrt{n} \ \ (n \text{ in } m^{-3})$$

Plasma oscillation could be propagated: (i) in a finite medium because of fringing fields. (ii) By taking into account the thermal motion.

Plasma frequency including ion motion?

- In 1949, Bohm and Gross were the first to derive the dispersion relation of electron plasma wave.

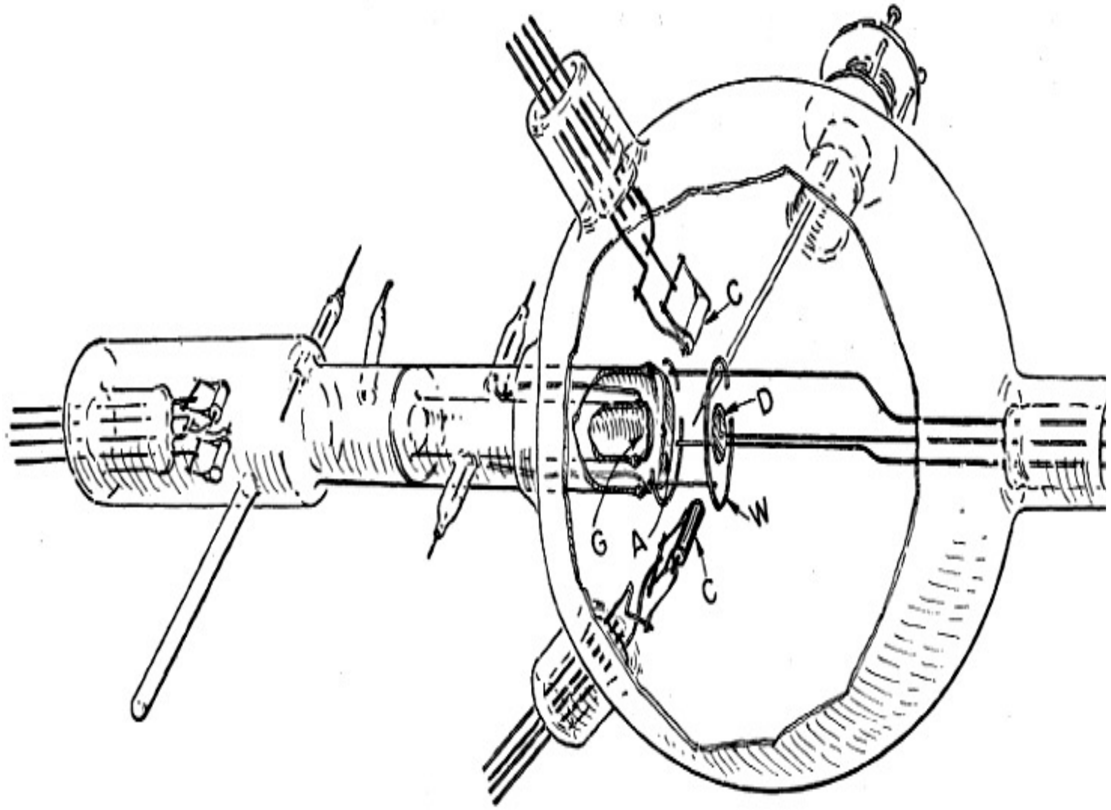
$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2$$

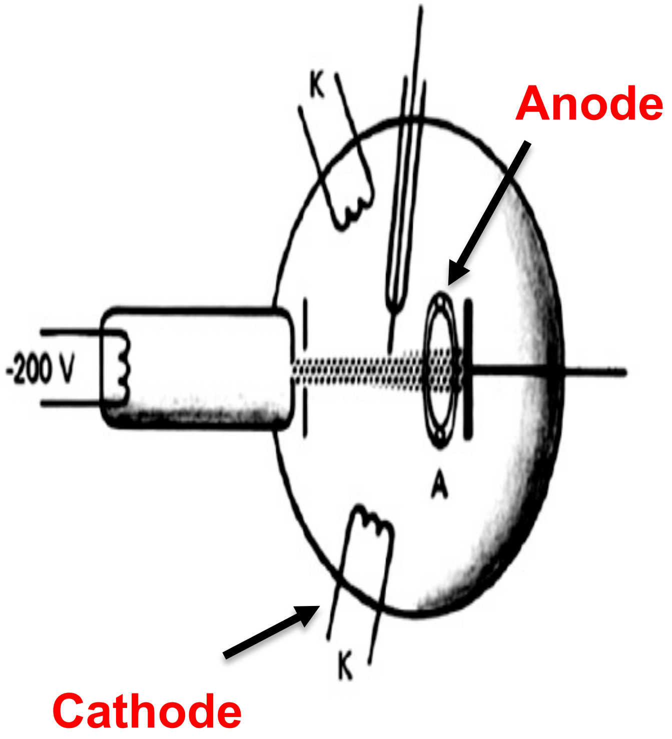
- Moreover, they discussed the possibility of electron plasma waves theoretically and reported that whenever a plasma contains beams of particles of well-defined velocity, or groups of particles for above mean thermal speeds, the system can become unstable, and small oscillations grow until they are limited by the appearance of non-linear effects.

- Collisions tend to damp the wave, and thus provide a stabilizing effect

Bohm

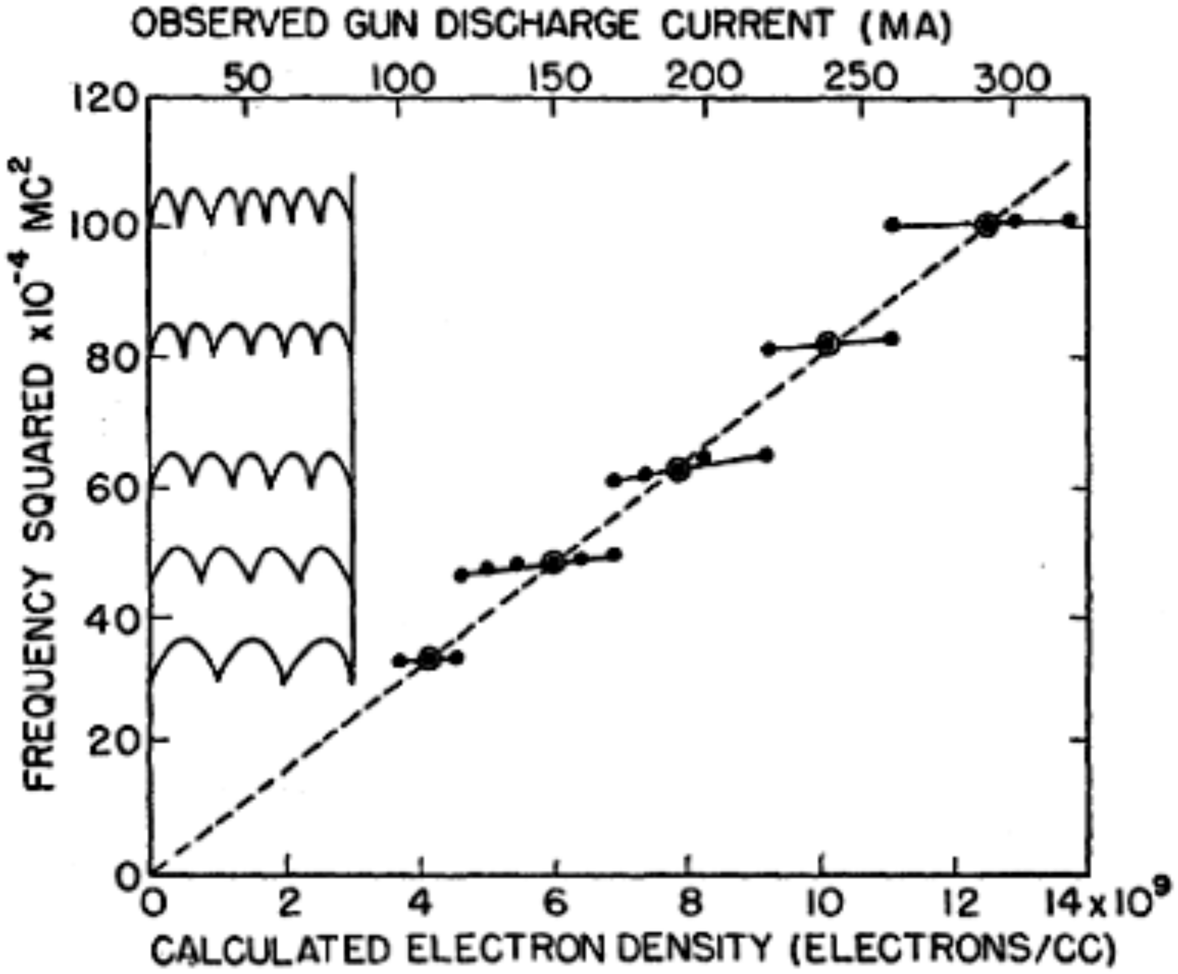
In 1954, Looney and Brown constructed an experiment to test the Bohm Gross theory.



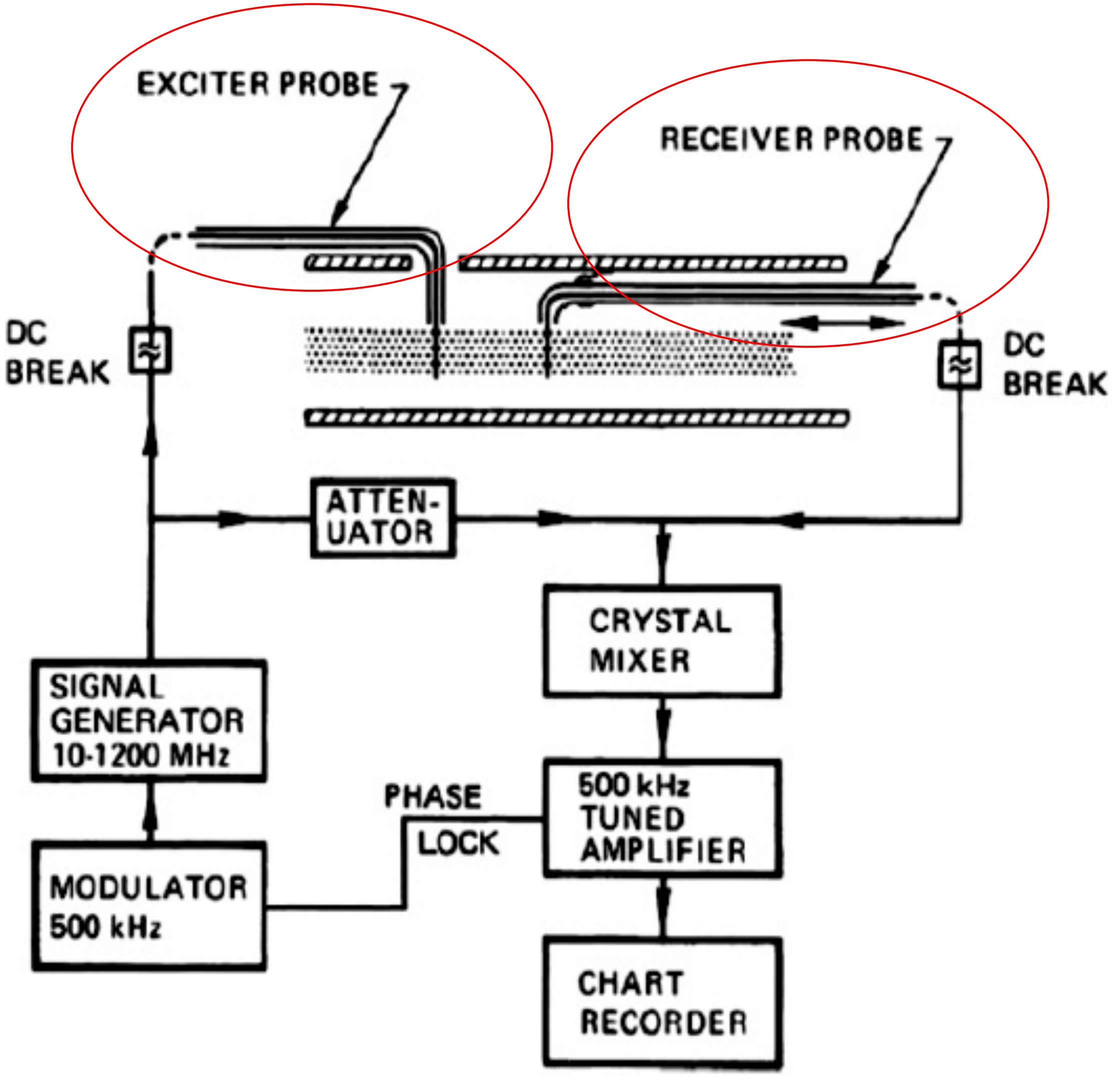


The *plasma frequency* is therefore

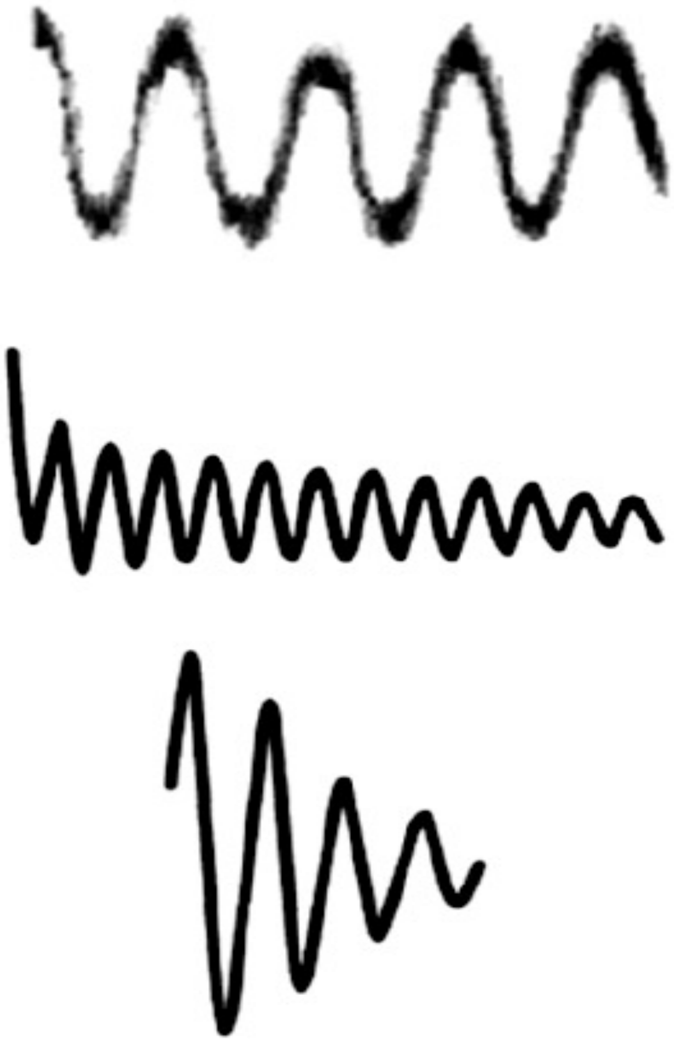
$$\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0 m}\right)^{1/2} \quad \text{rad/ sec}$$



In 1968, Barrett, Jones, and Franklin constructed a successful experiment to verify the Bohm-Gross dispersion relation



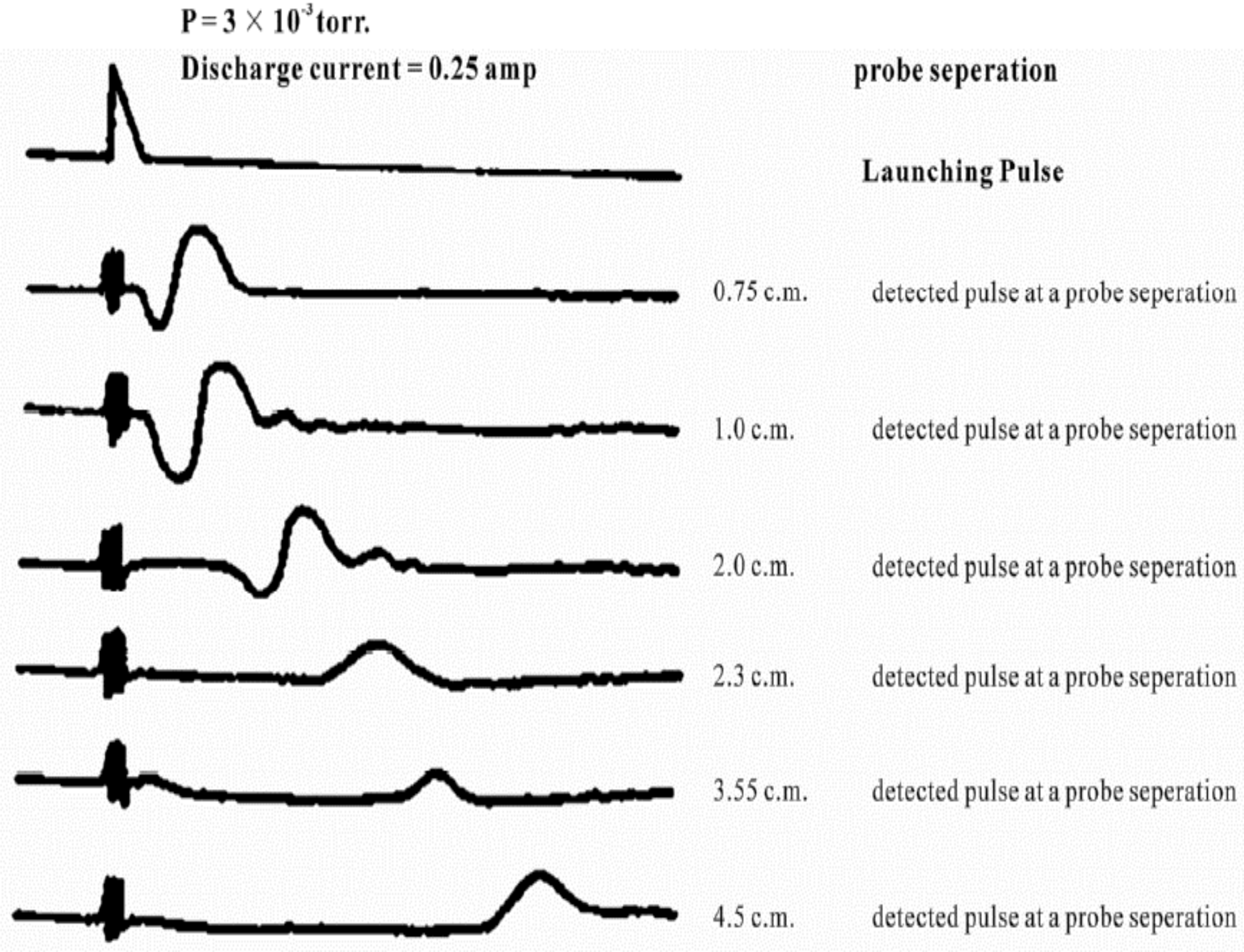
Waveform recordings at different electron density

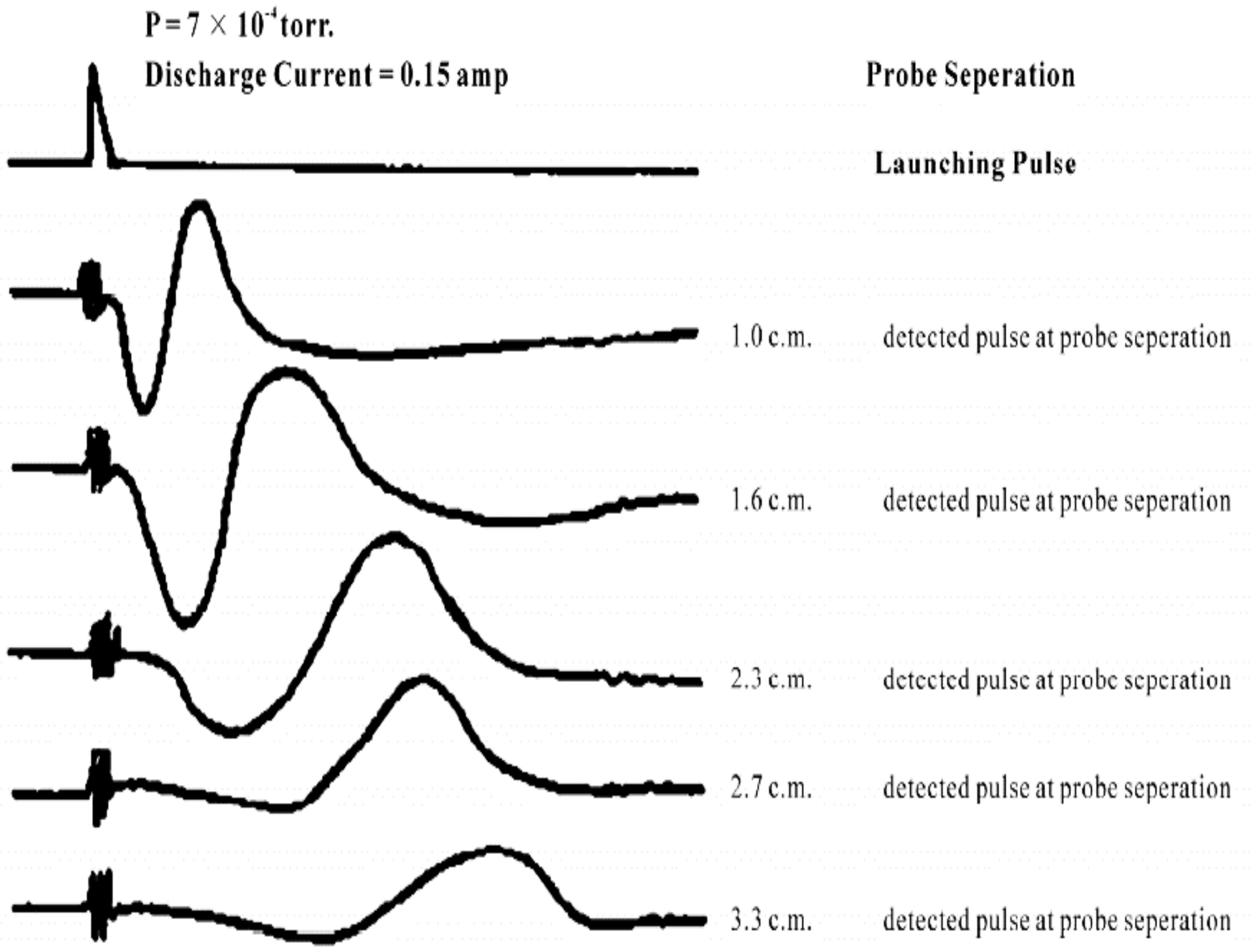


$n_o = 2\times10^{10}\ cm^{-3}$
$f = 950\ MHz$
$\lambda = 3.5\ cm$

$n_o = 4\times10^{8}\ cm^{-3}$
$f = 170\ MHz$
$\lambda = 1.3\ cm$

$n_o = 1\times10^{7}\ cm^{-3}$
$f = 20\ MHz$
$\lambda = 5.1\ cm$


P = 3 × 10⁻³ torr.
Discharge current = 0.25 amp
probe seperation
Launching Pulse
0.75 c.m.
detected pulse at a probe seperation
1.0 c.m.
detected pulse at a probe seperation
2.0 c.m.
detected pulse at a probe seperation
2.3 c.m.
detected pulse at a probe seperation
3.55 c.m.
detected pulse at a probe seperation
4.5 c.m.
detected pulse at a probe seperation


P = 7 × 10^-4 torr.
Discharge Current = 0.15 amp
Probe Seperation
Launching Pulse
1.0 c.m. detected pulse at probe seperation
1.6 c.m. detected pulse at probe seperation
2.3 c.m. detected pulse at probe seperation
2.7 c.m. detected pulse at probe seperation
3.3 c.m. detected pulse at probe seperation

Electron plasma wave

$$mn_e\left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla)\mathbf{v}_e\right] = -en_e\mathbf{E} - \nabla p_e \qquad (4.12)$$

$$\nabla p_e = 3KT_e \nabla n_e = 3KT_e \nabla(n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x}\hat{\mathbf{x}}$$

and the linearized equation of motion is

$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x} \qquad (4.28)$$

The oscillating quantities are assumed to behave sinusoidally:

$$\begin{aligned} \mathbf{v}_1 &= v_1 e^{i(kx-\omega t)}\hat{\mathbf{x}} \\ n_1 &= n_1 e^{i(kx-\omega t)} \\ \mathbf{E} &= E_1 e^{i(kx-\omega t)}\hat{\mathbf{x}} \end{aligned} \qquad (4.20)$$

Note that in linearizing we have neglected the terms $n_1 \, \partial v_1 / \partial t$ and $n_1 E_1$ as well as the $(\mathbf{v}_1 \cdot \nabla)\mathbf{v}_1$ term. With Eq. (4.20), Eq. (4.28) becomes

$$-im\omega n_0 v_1 = -en_0 E_1 - 3KT_e ikn_1 \tag{4.29}$$

E_1 and n_1 are still given by Eqs. (4.23) and (4.22), and we have

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1 + \overset{0}{n_1 \mathbf{v}_1}) = 0$$
$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 + \overset{0}{\mathbf{v}_1 \cdot \nabla n_0} = 0 \tag{4.18}$$

$$\varepsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1 \tag{4.19}$$

$$-i\omega n_1 = -n_0 ikv_1 \tag{4.22}$$

$$ik\varepsilon_0 E_1 = -en_1 \tag{4.23}$$

$$im\omega n_0 v_1 = \left[e n_0 \left(\frac{-e}{ik\varepsilon_0} \right) + 3KT_e ik \right] \frac{n_0 ik}{i\omega} v_1$$

$$\omega^2 v_1 = \left(\frac{n_0 e^2}{\varepsilon_0 m} + \frac{3KT_e}{m} k^2 \right) v_1$$

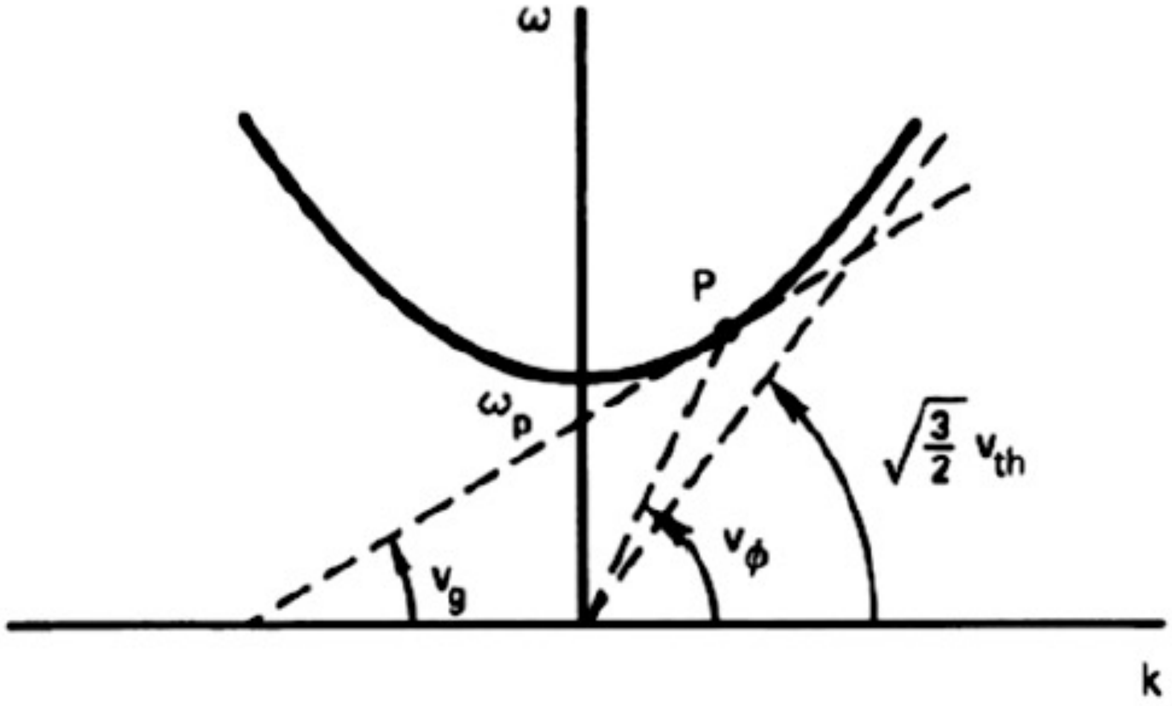
$$\boxed{\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{\mathrm{th}}^2} \tag{4.30}$$

where $v_{\mathrm{th}}^2 \equiv 2KT_e/m$. The frequency now depends on k, and the group velocity is finite:

$$2\omega\, d\omega = \frac{3}{2} v_{\mathrm{th}}^2 2k\, dk$$

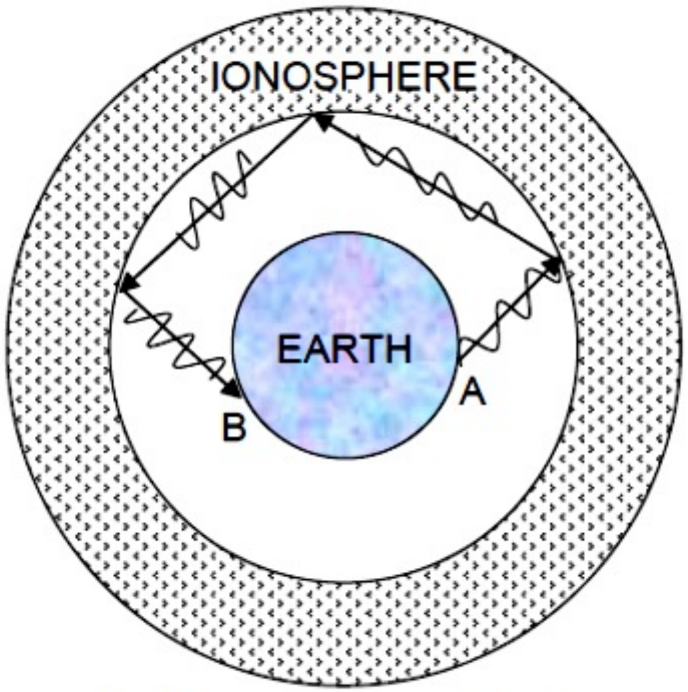
$$v_g = \frac{d\omega}{dk} = \frac{3}{2} \frac{k}{\omega} v_{\mathrm{th}}^2 = \frac{3}{2} \frac{v_{\mathrm{th}}^2}{v_\phi} \tag{4.31}$$

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2$$



The ionosphere.—**The upper layer of the Earth's atmosphere, above an altitude of roughly 90 km) that is ionized by solar UV radiation.**

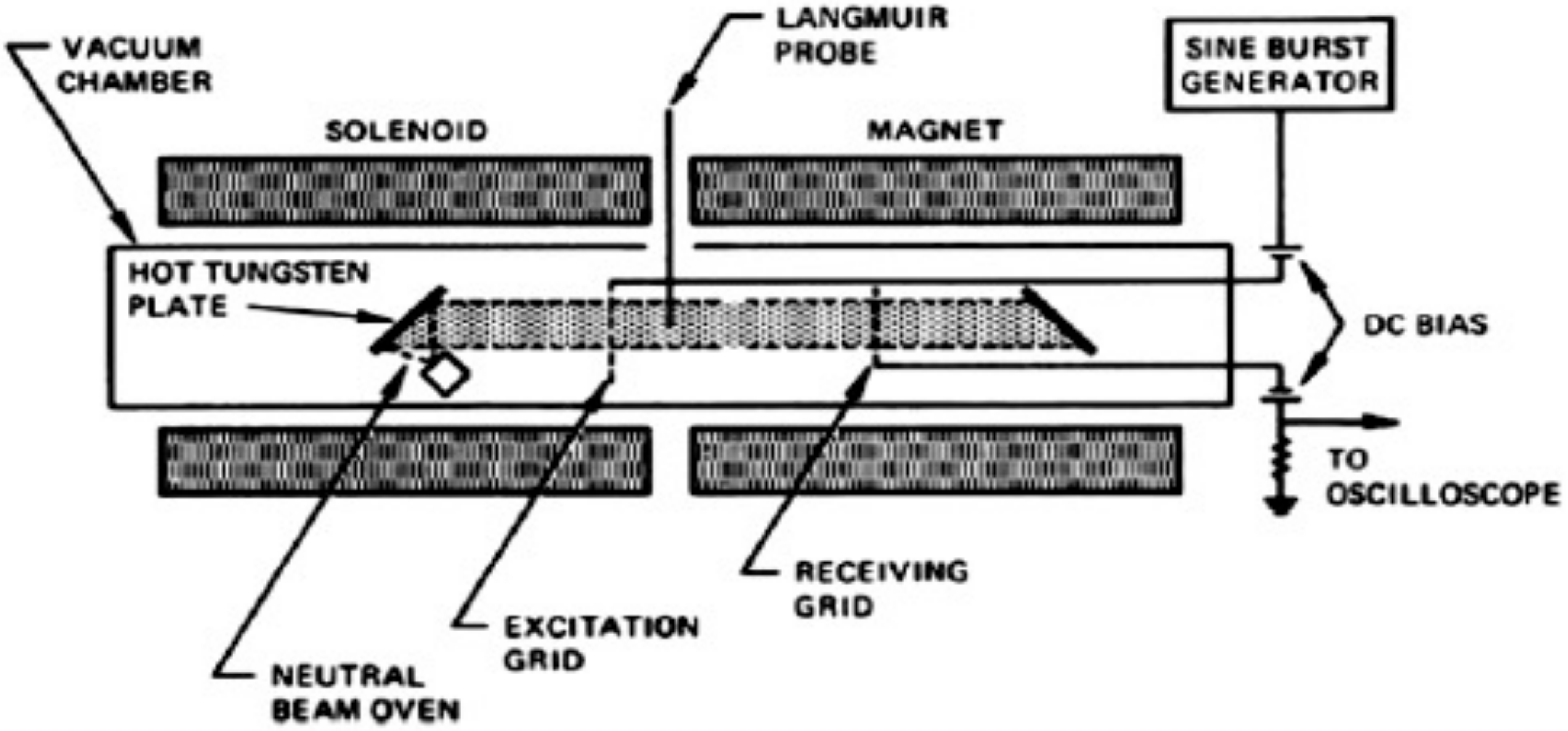
To communicate with satellites in low earth orbit, frequencies above f_{pe} must be used. Otherwise, it will be reflected by the ionosphere.



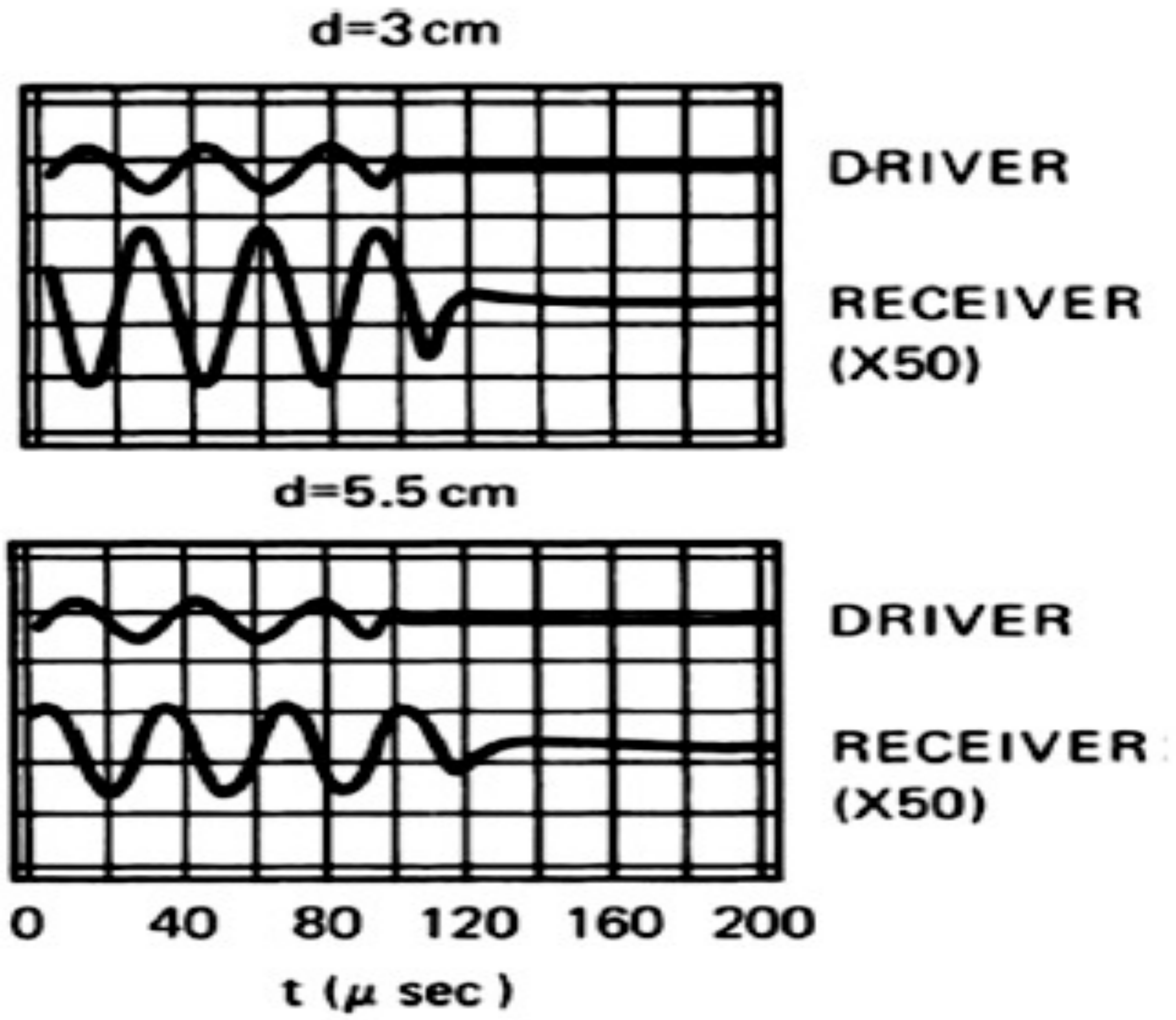
Multiple reflections of radio waves from the ionosphere, allowing for communication between points A and B.

Ion Acoustic Waves

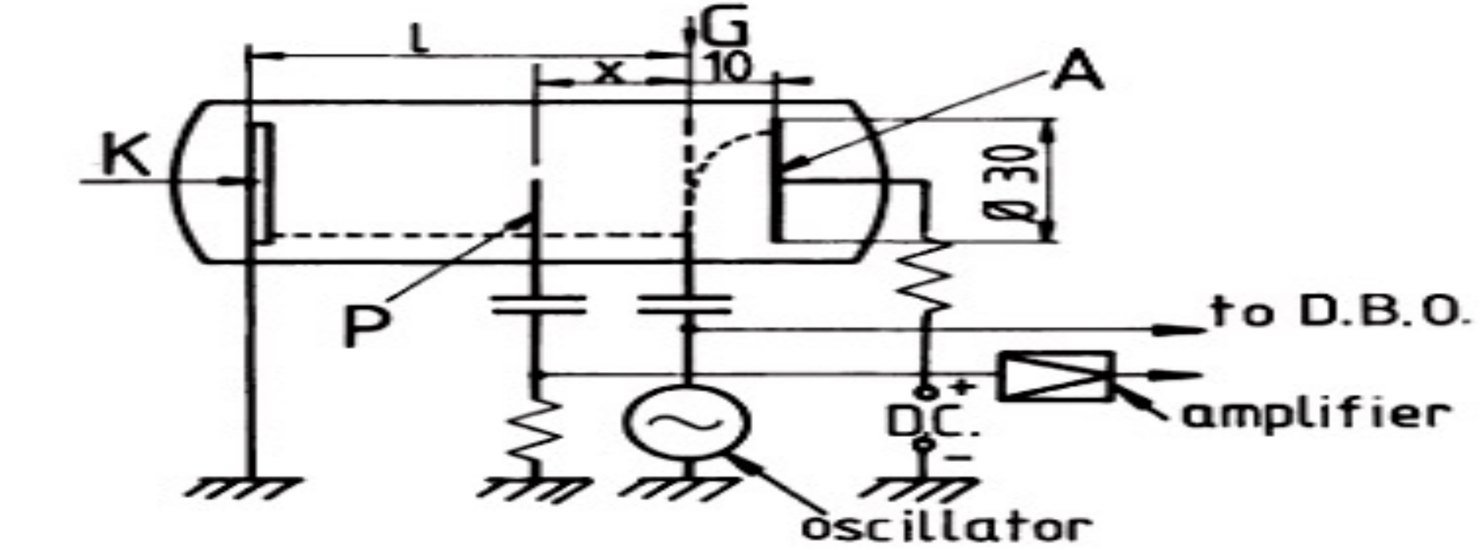
(1960)First observation of ion waves by Wong, Motley, and D'Angelo. Waves were launched and detected by grids inserted into the plasma.



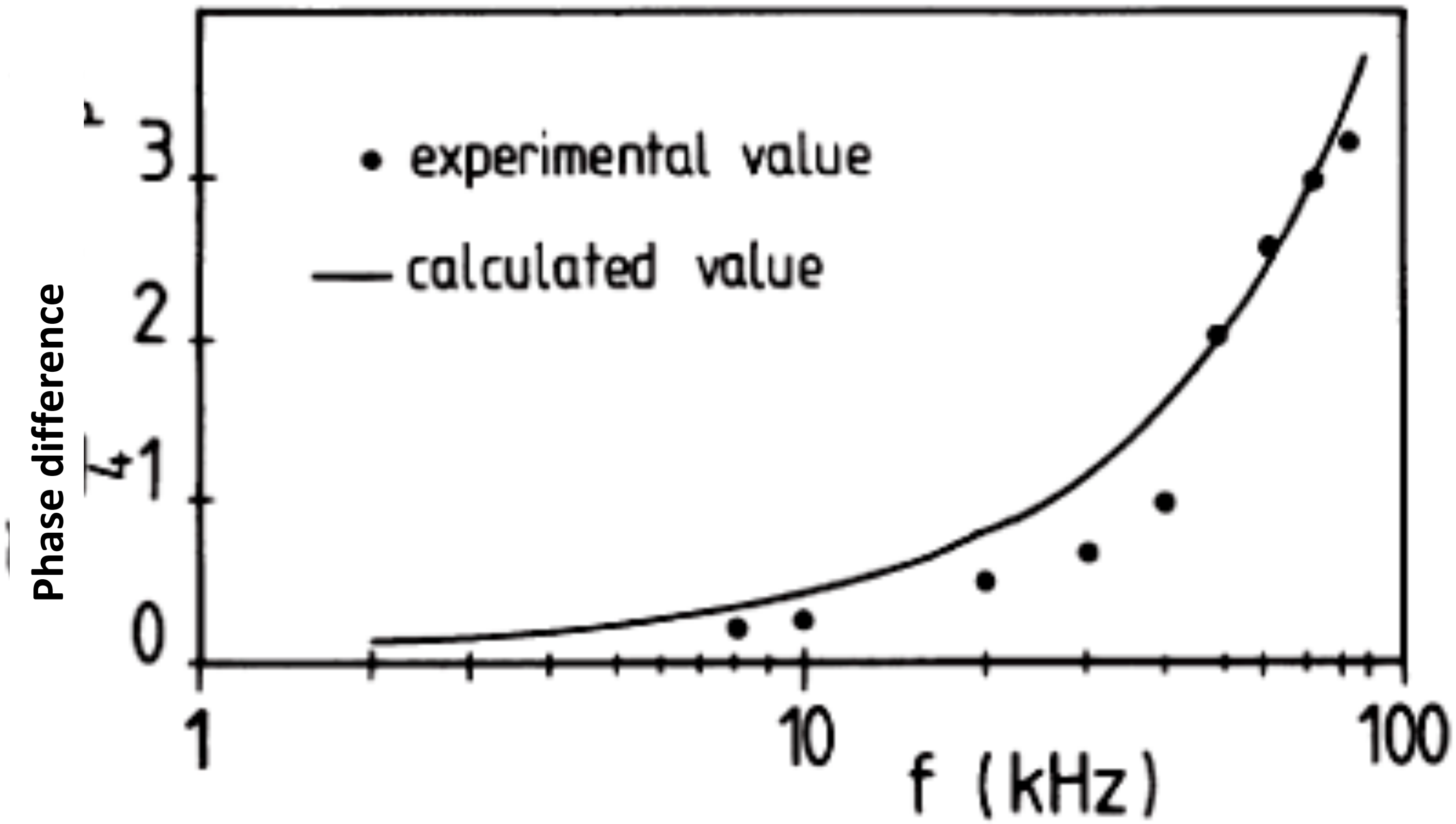
Plasma oscilloscope traces of the transmitted and received signals. From the phase shift, one can find the phase velocity (same as group velocity in this case).



This figure shows the experimental facility used by Hatta and Sato (1962). The argon plasma between the hot cathode K and the grid G was formed from a mixture of electrons emitted from the hot cathode and ions which drifted from the plasma-generating region between A and G. A sinusoidally time-varying voltage was placed on the floating grid G to produce a continuous modulation of the ion density in the vicinity of G. This *ion density variation* then propagated away from the grid as an *ion-acoustic wave* toward K. The floating Langmuir probe P was used to measure both the amplitude and phase of the wave at the position of the probe.



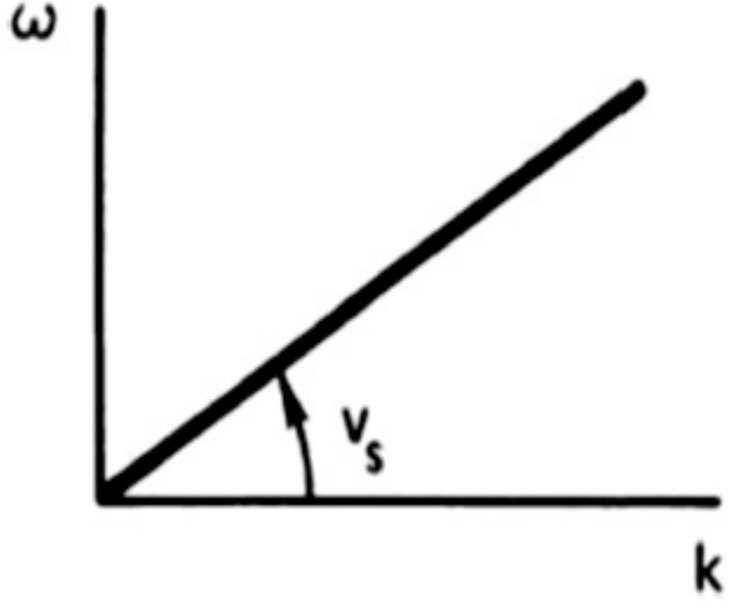
A anode K hot cathode
G mesh grid P probe
------ potential distribution



Phase difference versus frequency (f). (After Hatta and Sato, 1962.)

Ion acoustic waves

$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M}\right)^{1/2} \equiv v_s$$



- Plasma oscillations are basically constant-frequency waves, with a correction due to thermal motions.
- Ion waves are basically constant-velocity waves and exist only when there are thermal motions.
- For ion waves, the group velocity is equal to the phase velocity.
- In electron plasma oscillations, ions remains essentially fixed.
- In ion acoustic waves, electrons is far from fixed