Introduction to Plasma Physics

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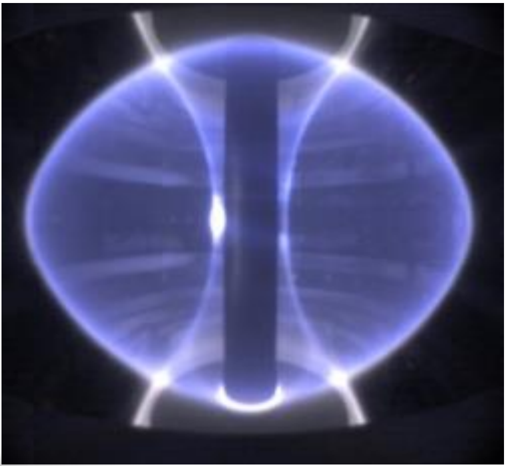
What is plasma?

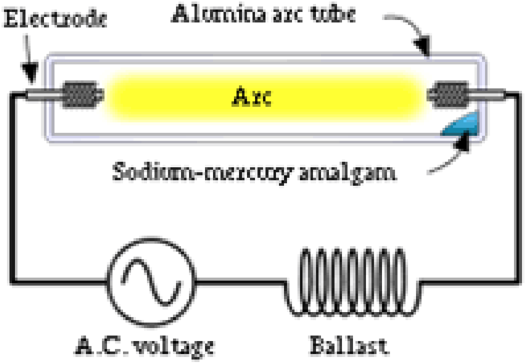
- **Plasma is the dominant constituent of the universe.**

- **It is the fourth state of matter.**
- **The word of plasma seems to be a misnomer. It comes from a Greek word which means something molded or fabricated because of behavior, it does not tend to conform to external influences**
- **Plasma is simply a system of charged particles such as electrons and ions and excited neutral species.**

The Plasma State

- A plasma is an electrically neutral ionized gas.
- The Sun is a plasma, there is no life without the plasma.
- The space between the Sun and the Earth is “filled” with a plasma.
- The Earth is surrounded by a plasma.
- A stroke of lightning forms a plasma
- Over 99% of the Universe is a plasma.





Good Start

- **Introduction to plasma physics Books: Good Overview**

- **Principles of Plasma Physics for Engineers and Scientists, Umran Inan and Marek Gołkowski, Cambridge University Press 2011.**

- **INTRODUCTION TO PLASMA PHYSICS AND CONTROLLED FUSION, Francis F. Chen, Springer Science 1984.**

- **Plasma dynamics, R.O. Dendy.**

- **Physics of Plasmas, L.C. Woods.**

- **Fundamentals of Plasma Physics, Paul M. Bellan.**

- **The Physics of Plasmas, Richard Fitzpatrick.**

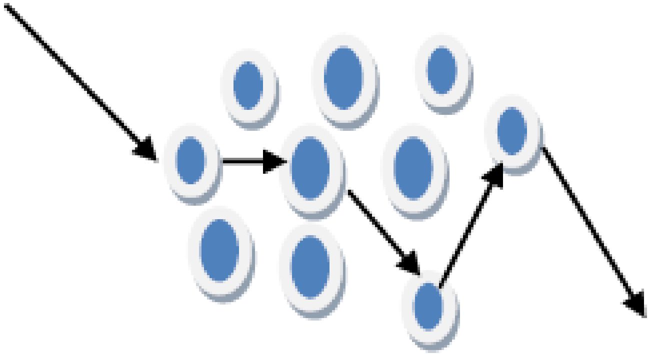
Good Start

- **Advanced Level: more specific topics (examples)**

- **IONOSPHERES Physics, Plasma Physics, and Chemistry, ROBERT W. SCHUNK and ANDREW F. NAGY.**
- **Spectroscopy of low temperature plasma, Vladimir N. Ochkin.**
- **Plasma Waves, D Gary Swanson.**
- **Plasma Medicine, Alexander Fridman, Gary Friedman.**
- **Handbook of Plasma Processing Technology: Fundamental, Etching, Deposition and Surface Interactions (Materials Science and Process Technology) by Stephen M. Rossnagel, William D. Westwood, Jerome J. Cuomo.**

Plasma VS neutral gas?

- In neutral gas, such as, ordinary air, there is no net electromagnetic force. The molecule motion is determined via only the gravitational force, which is negligible. Therefore, the molecule trajectories follow the zigzag motion.

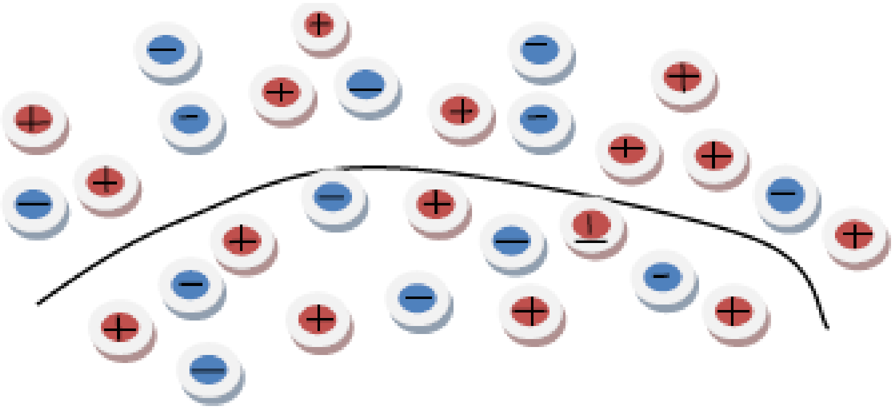


- Gravitational Force:

$$\mathbf{F_g} = \frac{G m_1 m_2}{r^2}$$

Plasma VS neutral gas?

- Plasma has charged particles which move around and generate electromagnetic fields. The motion of any charged particle in the plasma depends on its properties such as, charge, mass, and velocity and the electromagnetic field affect on it.



- Electric Force: $$\mathbf{F_E} = \frac{KQ_1Q_2}{r^2}$$

- Which is bigger and dominant for small charged particle?

Plasma VS neutral gas?

- ***1.1***: **The Saha's law describes the equilibrium between the populations of two successive *n* ionization states;**

- $$\frac{n_m}{n_k} = \frac{g_m\, exp\left(-\frac{E_m}{K\,T}\right)}{g_k\, exp\left(\frac{-E_k}{K\,T}\right)}$$

- **Where n_m and n_k are the population of highly ionized stage and the lower ionized stage, respectively. g_m and g_k are degeneracy of states *m* and *k*, respectively. E_m and E_k are the ionization energy of the ionized stage m and k.**

- **For neutral gas**

$$\frac{n_i}{n_{neutral}} \approx 10^{-122}$$

Plasma VS neutral gas?

- **Degree of ionization**

$$\alpha = \frac{n_i}{n_{neutral} + n_i}$$

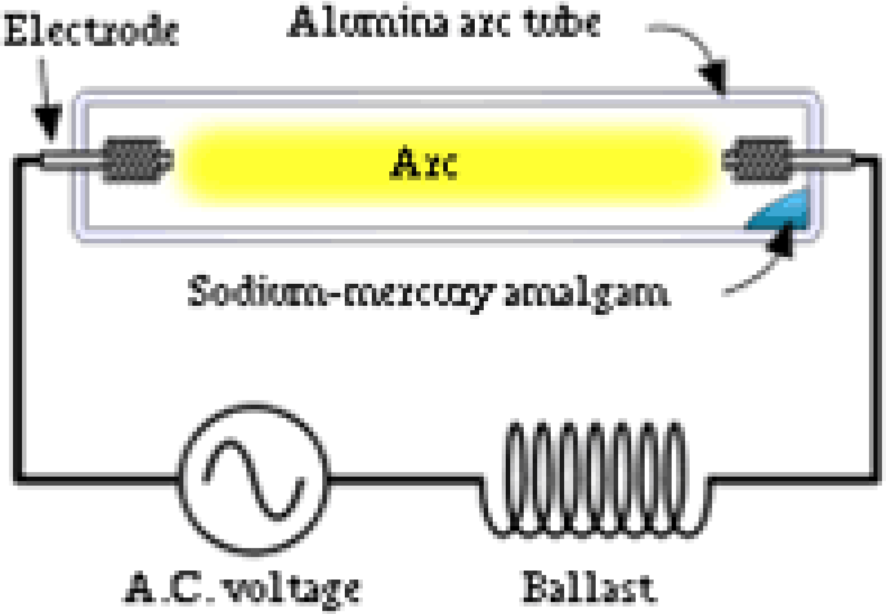
- **For fully ionized plasma** $\alpha = 1$

- **For partially ionized plasma** $0 < \alpha < 1$

- **For neutral gas** $\alpha = 0$

Plasma Generation

- The plasma could be generated via different ways as
  - Intense Laser beam
  - Microwave
  - Electric energy = Potential difference as in gas discharges



Electron Current

- Free electrons may eject from the cathode. The electric field will accelerate free electrons toward the anode. Ionization of neutral atoms and molecules may take place when the kinetic energy of electrons is greater than the ionization of neutral atoms and molecules.

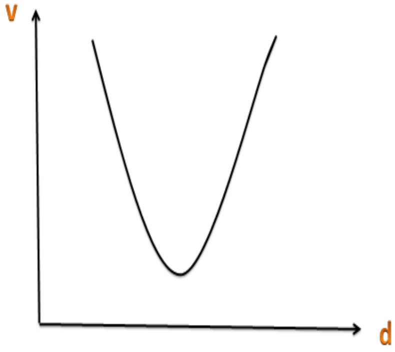
$$m\frac{dv}{dt} = -eE \qquad m\frac{dv}{dt} = e\frac{d\phi}{dt}\frac{1}{v}$$

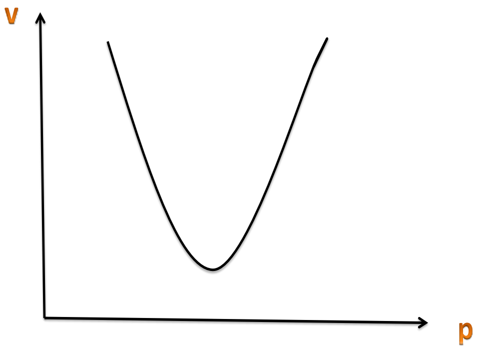
$$E = -\frac{d\phi}{dx} \qquad mvdv = ed\phi$$

$$m\frac{dv}{dt} = e\frac{d\phi}{dx} \qquad \frac{1}{2}mv^2 = e\phi$$

$$m\frac{dv}{dt} = e\frac{d\phi}{dt}\frac{dt}{dx} \qquad J = \frac{I}{A} = n_e ev = n_e e\sqrt{\frac{2e\phi}{m}}$$

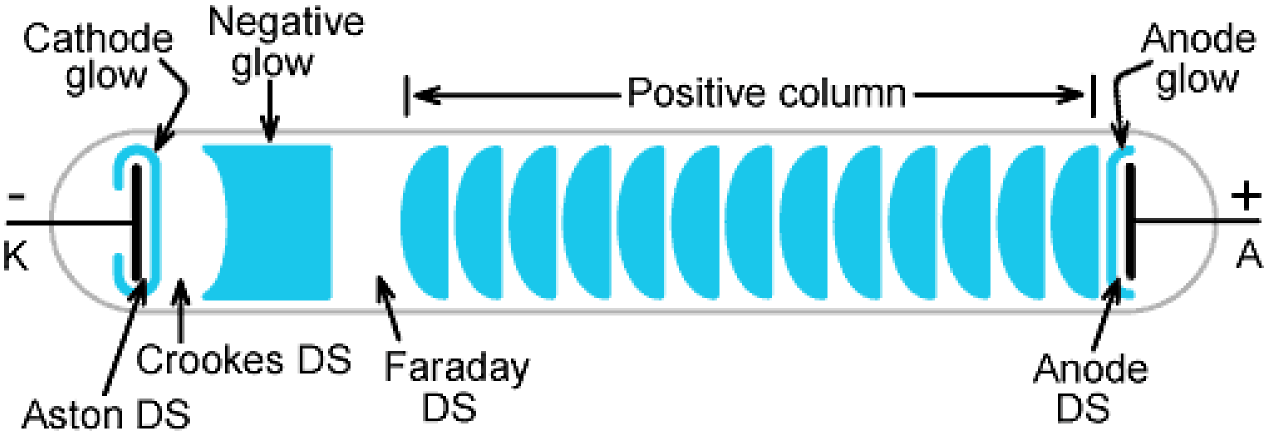
Pachen's Law





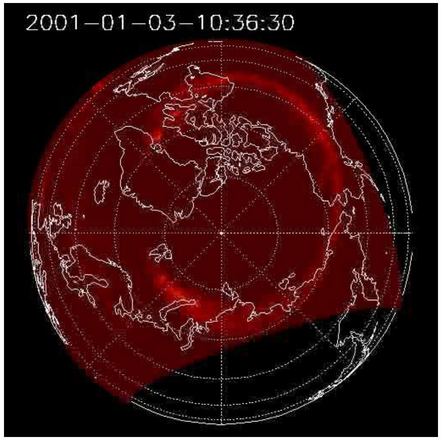
- The gas pressure and the distance between the electrodes affect the voltage required to achieve discharge.
- $v_{min} \propto pb = kpd.$
- Considering secondary electrons: $V_{BD=} Bpd/\ln[\frac{Apd}{\ln\left(\frac{1}{\gamma}\right)}]$
- Where *A* and *B* are gas dependent constants, and *γ* is the second Townsend ionization coefficient.

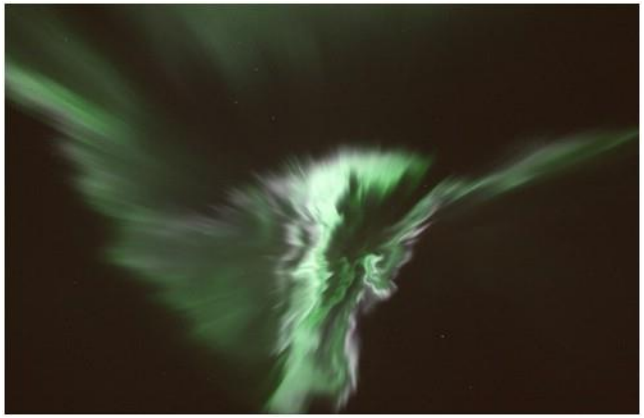
Normal and abnormal discharges

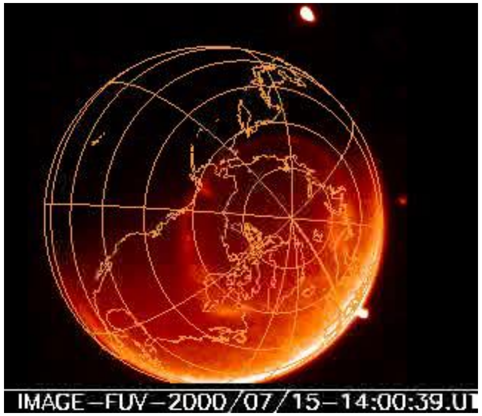


- **Glow discharges are classified as normal or abnormal discharges**
- **The normal discharge has a constant current density at the cathode. When the power to the discharges varied, only the area over which they glow exists varies. After the entire cathode is enveloped within the glow, the current density increases. This is known as the abnormal or anomalous glow discharge. The abnormal discharge requires relatively large increases in the applied voltage to cause small increases in the current density. As the current density increases, it is accompanied by a decrease in cathode sheath thickness, therefore, leading to a higher electric field and voltage drop in the sheath, which increases the positive ion energy.**

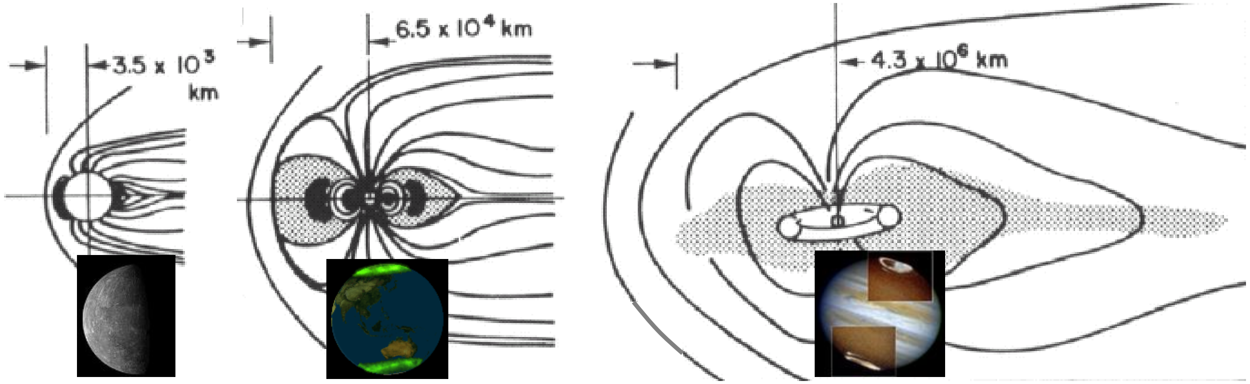
Auroral Displays: Direct Manifestation of Space Plasma Dynamics







Stellar wind coupling to planetary objects is ubiquitous in astrophysical systems

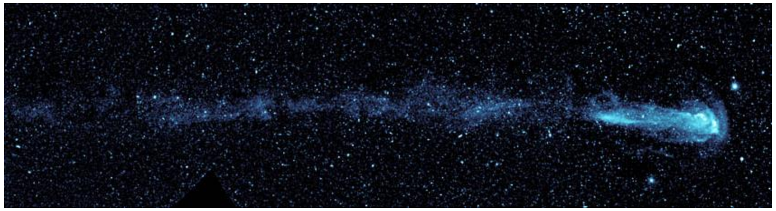


SUBSTORM RECURRENCE: MERCURY: 10 min EARTH: 3.75 hrs JUPITER: 3 weeks

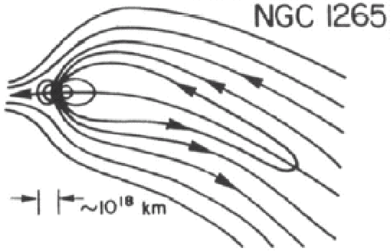
Magnetized wind coupling to stellar and galactic systems is common thoughout the Universe

ASTROSPHERE

Mira (a mass shedding red giant)
and its 13 light-year long tail



GALACTIC CONFINEMENT



The Motion of Charged Particles

- Equation of motion

$$m\frac{d\vec{v}}{dt} = q\vec{E} + q\vec{v}\times\vec{B} + \vec{F}_g$$

- SI Units
  - mass (m) - kg
  - length (l) - m
  - time (t) - s
  - electric field (E) - V/m
  - magnetic field (B) - T
  - velocity (v) - m/s
  - F_g stands for non-electromagnetic forces (e.g. gravity) - usually ignorable.

- B acts to change the motion of a charged particle only in directions perpendicular to the motion.
  - Set E = 0, assume B along z-direction.

$$m\dot{v}_x = qv_y B$$

$$m\dot{v}_y = -qv_x B$$

$$\ddot{v}_x = \frac{q\dot{v}_y B}{m} = -\frac{q^2 v_x B^2}{m^2} = -\Omega_c^2 v_x$$

$$\ddot{v}_y = -\frac{q^2 v_y B^2}{m^2} = -\Omega_c^2 v_y$$

$$\Omega_c = \frac{|q|B}{m}$$

- Solution is circular motion dependent on initial conditions. Assuming at t=0: $v_x = 0; v_y = v_\perp$

$$v_x = v_\perp \sin(\pm\Omega_c t)$$

and
$$v_y = v_\perp \cos(\pm\Omega_c t)$$

$$x - x_0 = \mp \frac{v_\perp}{\Omega_c} \cos(\Omega_c t)$$

$$y - y_0 = \pm \frac{v_\perp}{\Omega_c} \sin(\Omega_c t)$$

  - Equations of circular motion with angular frequency Ω_c (cyclotron frequency or gyro frequency). Above signs are for positive charge, below signs are for negative charge.
    - If q is positive particle gyrates in left-handed sense
    - If q is negative particle gyrates in a right-handed sense

- Radius of circle (r_c) - cyclotron radius or Larmor radius or gyro radius.

$$v_\perp = \rho_c \Omega_c$$

$$\rho_c = \frac{mv_\perp}{qB}$$

  - The gyro radius is a function of energy.
  - Energy of charged particles is usually given in electron volts (eV)
    - Energies in space plasmas go from electron Volts to kiloelectron Volts (1 keV = 10^3 eV) to millions of electron Volts (1 meV = 10^6 eV)
    - Cosmic ray energies go to gigaelectron Volts (1 GeV = 10^9 eV).
- The circular motion does no work on a particle

$$\vec{F} \cdot \vec{v} = m \frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{d(\frac{1}{2}mv^2)}{dt} = q\vec{v} \cdot (\vec{v} \times \vec{B}) = 0$$

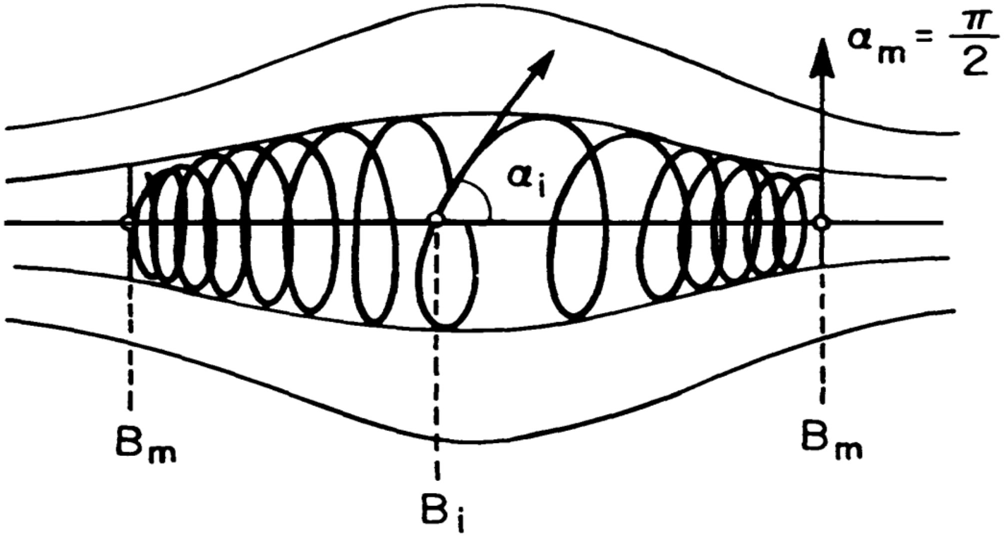
Only the electric field can energize particles!
Particle energy remains constant in absence of E !

In a Magnetic Mirror:

- The force is along $\vec{B}$ and away from the direction of increasing B.
- Since $E_{\parallel} = 0$ and kinetic energy must be conserved

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_{\parallel}^2 + v_{\perp}^2)$$

  a decrease in $v_{\parallel}$ must yield an increase in $v_{\perp}$
- Particles will turn around when $B = \tfrac{1}{2}mv^2 / \mu$
- The loss cone at a given point is the pitch angle below which particles will get lost: $\sin^2\alpha_i = B_i / B_m = 1/R_m$

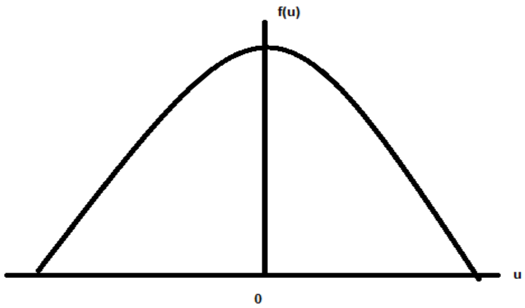


Disadvantages of the single particle model

- **It is challenging to solve the equation of motion of all plasma particles (not feasible):**
- **Because the number of particles is too much.**
- **The equation of motion must be solved in a self-consistent way with electric and magnetic fields.**
- **Maxwell’s equations must be solved for each particle.**

Distribution function

- Because plasma particles collide together, the speed of the particles has a distribution.



- Boltzmann distribution: number of gas particles with speed is related to energy of gas and to the rate at which they collide with each other and boundary surfaces.
- For probability distribution, the normalized number of particles per unit volume with speed V is
- $f(v) = A \ Exp\left[\frac{-mv^2}{2KT}\right]$

Average Speed

- **Average speed:**

$$<v> = \bar{v} = u = \frac{v_1 + v_2 + v_3 + \dots}{1 + 1 + 1 + \dots}$$

$$<v> = \bar{v} = u = \frac{n_1 v_1 + n_2 v_2 + n_2 v_3 + \dots}{n_1 + n_2 + n_3 + \dots}$$

$$n = \int f(v) dv$$

$$u = \frac{\int v f(v) dv}{\int f(v) dv}$$

Average Energy

- **Average speed:**

$$< E > = \bar{E} = \frac{E_1 + E_2 + E_3 + ...}{1 + 1 + 1 + ...}$$

$$< E >= \bar{E} = \frac{n_1 E_1 + n_2 E_2 + n_2 E_3 + ...}{n_1 + n_2 + n_3 + ...}$$

$$n = \int f(v) dv$$

$$< E >= \frac{\int \frac{1}{2} m v^2 f(v) dv}{\int f(v) dv}$$

Temperature

- **The temperature is a macroscopic quantity:**

$$< E >= \frac{\int \frac{1}{2} m v^2 f(v) dv}{\int f(v) dv}$$

- **If we assume Boltzmann distribution:** $f(v) = A \exp(-\frac{mv^2}{2k_B T})$
- **If** $y^2 = \frac{mv^2}{2k_B T}$ **then for one dimension**
- $$< E >= \frac{k_B T \int y^2 \exp(-y^2) dy}{\int \exp(-y^2) dy} = \frac{k_B T}{2}$$
- **For 3 dimensions** $< E >= \frac{3k_B T}{2}$

Temperature

- **The temperature is measured in Kelvin, Celsius, Fahrenheit,…**
- **However, in plasma community we could use the energy unit . This is not correct sometimes!!!!!!!!!!!!!**

- **So when we say the temperature of the plasma is 1 eV, this means that the temperature is 11605 K.**

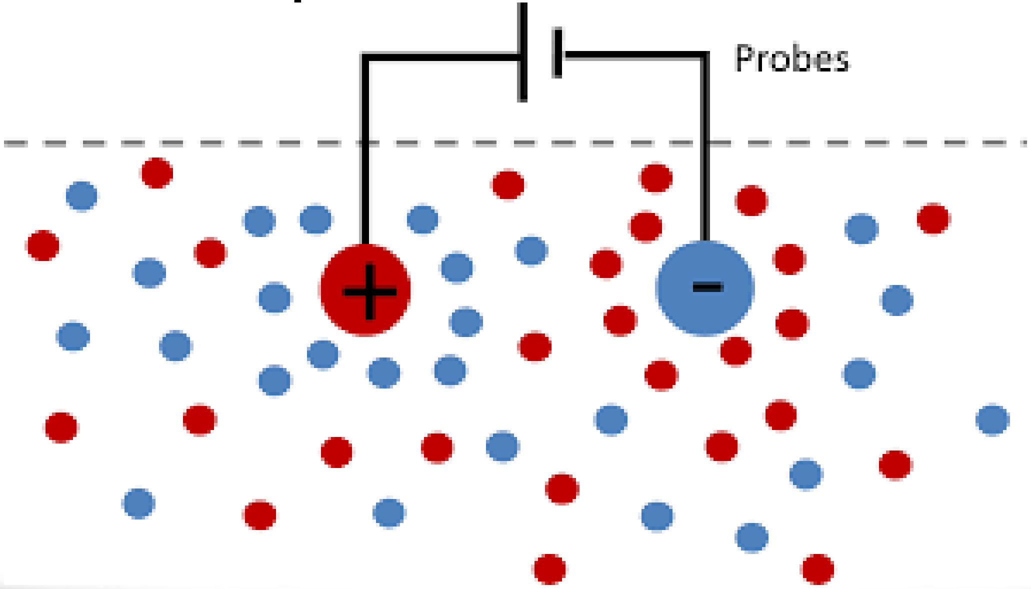
$$\mathrm{eV} = 11605K$$

- **Calculate the energy of a plasma in 1D and 3 D if its temperature is 5 eV.**

- **In magnetized plasma, the plasma may have two temperatures, Why?**

Debye Shielding

- Try to put an electric field inside plasma by inserting two charged balls connected to a battery. The balls attract particles of the opposite charge and immediately a cloud of ions would surround the negative ball and cloud nof electrons would surround the positive ball. If the temperature is finite, the edge of the cloud then occurs at the radius where the electric potential energy is approximately equal to the thermal energy KT of the particles. The shielding is not complete, where a potential of the order of KT/e can leak into the plasma and cause finite electric fields to exit there.



Debye Shielding

- $\varepsilon_0 \nabla^2 \emptyset = \varepsilon_0 \frac{d^2\emptyset}{dx^2} = -e(n_i - n_e)$
- If the density far away is n_∞
- $n_\infty = n_i$ $\qquad\qquad n_e = n_\infty e^{e\emptyset/kT}$
- Where $e\ \emptyset$ is the potential energy and kT is the kinetic energy. Then,
- $\varepsilon_0\, \nabla^2 \emptyset = -e(n_i - n_e) = -e\left(n_i - n_\infty e^{e\emptyset/kT}\right)$

Debye Shielding

- $\varepsilon_0 \nabla^2 \emptyset = e n_i \left(e^{\emptyset/kT} - 1\right)$
- $\varepsilon_\circ \frac{d^2\emptyset}{dx^2} = e n_\infty \left[1 + \frac{e\emptyset}{kT} + \frac{1}{2}\left(\frac{e\emptyset}{kT}\right)^2\right]$
- $\varepsilon_\circ \nabla^2 \varphi = e n_i \left(1 + \frac{e\emptyset}{kT} - 1\right)$
- $\varepsilon_\circ \nabla^2 \emptyset = \frac{e^2 n_i \emptyset}{kT}$
- $\nabla^2 \emptyset = \frac{e^2 n_i \emptyset}{\epsilon_\circ kT}$

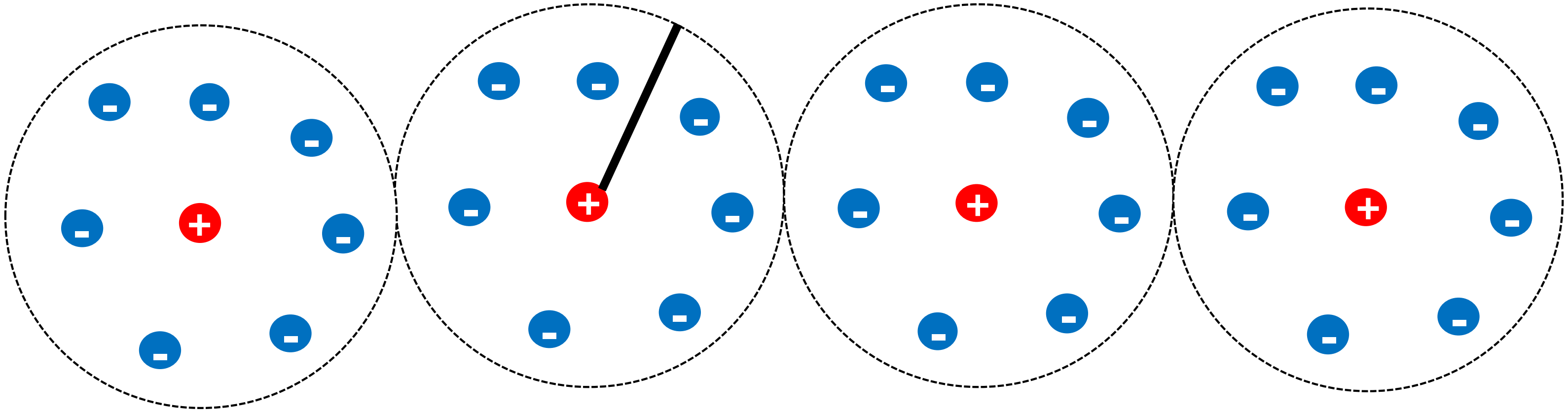
Debye Shielding

- Assume that $\emptyset = e^{\propto x}$, then
- $\nabla\emptyset = \alpha e^{\alpha x}$
- $\nabla^2\emptyset = \alpha^2 e^{\alpha x}$
- So $\propto^2 \ e^{\propto x} = \dfrac{e^2 n_i e^{\propto x}}{\varepsilon_\circ kT}$
- $\propto = \sqrt{\dfrac{e^2 n_i}{\varepsilon_\circ kT}}$
- $\lambda_D = \boldsymbol{Debye\ length} = \dfrac{1}{\alpha} = \sqrt{\dfrac{\varepsilon_o kT}{e^2 n_i}}$

Quasineutral spheres

- The Debye radius is the distance at which the potential of an ion charge is decreased by 0.37 of its value in free space.

$$r \approx \lambda_{\mathrm{D}}$$



- In ideal plasma the number of charges in the Debye sphere must be large.

Plasma parameters

- **Debye length is much smaller than the plasma dimensions**

$$\lambda_D \ll L$$

- **So the plasma consists of quasineutral spheres, therefor, the plasma is defined as quasineutral exhibits collective behavior.**
- **Number of particles in Debye Sphere is greater than 1.**
- **From the definition of plasma density**

$$n = \frac{N}{V} = \frac{\text{number of prticles}}{\text{Volume}}$$

- **In a sphere**

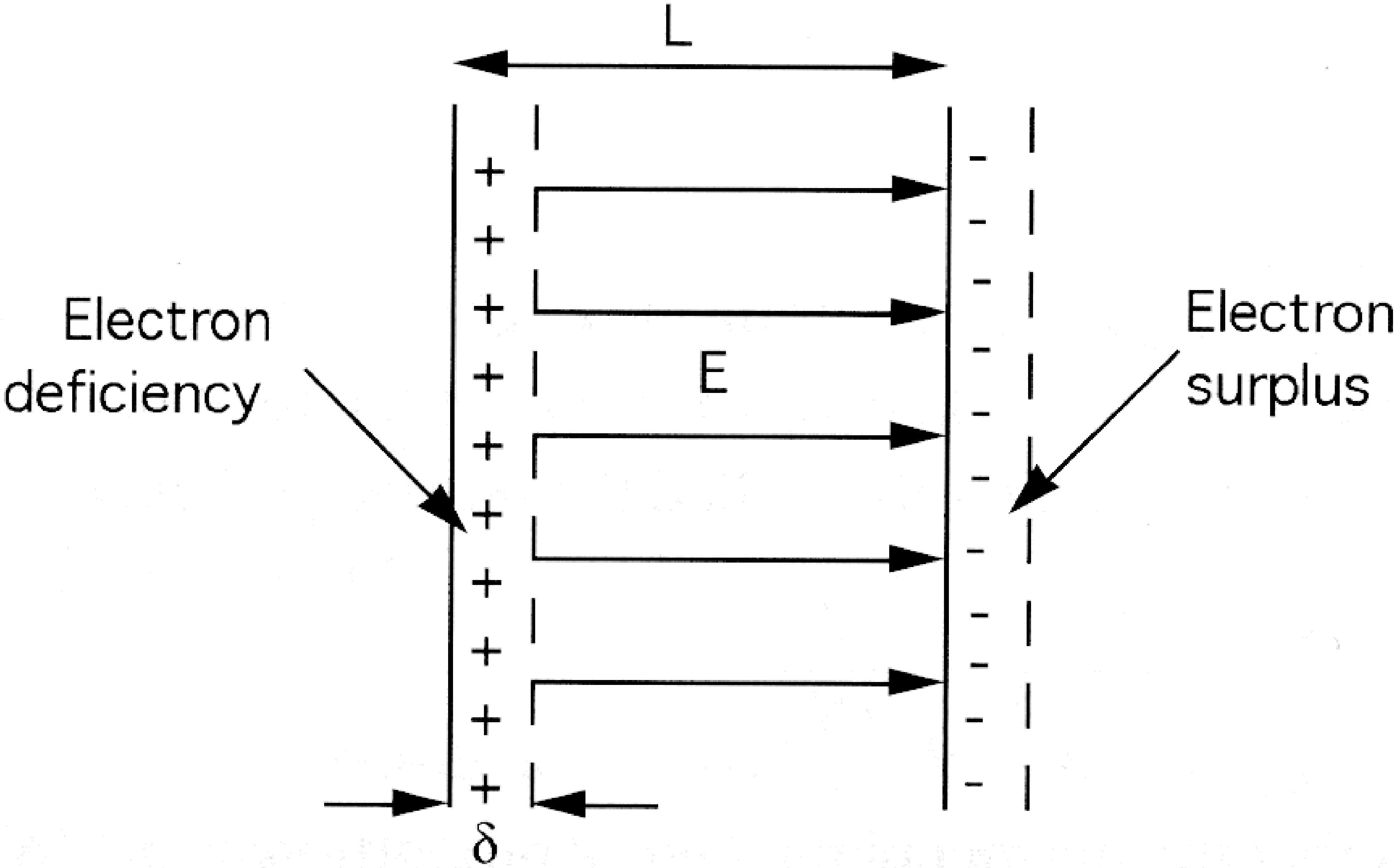
$$N = n * (4/3)\pi r^3$$

Plasma parameters

- **The frequency of a process times the relaxation of this process must be greater than 1**

$$\omega\tau \gg 1$$

- **For example: the ionization rate must be greater than the recombination rate, otherwise, the plasma will die out.**


L
Electron deficiency
E
Electron surplus
δ

- The plasma frequency
  - Consider a slab of plasma of thickness L.
  - At t=0 displace the electron part of the slab by δ_e <<L and the ion part of the slab by δ_i <<L in the opposite direction.

$$\delta = \delta_e - \delta_i$$

  - Poisson’s equation gives

$$E = \frac{en_0}{\varepsilon_0}\delta$$

  - The equations of motion for the electron and ion slabs are

$$m_e \frac{d^2\delta_e}{dt^2} = -eE$$

$$m_{ion} \frac{d^2\delta_i}{dt^2} = eE$$

$$\frac{d^2\delta}{dt^2} = \frac{d^2\delta_e}{dt^2} - \frac{d^2\delta_i}{dt^2} = -(\frac{e^2 n_0}{\varepsilon_0 m_e} + \frac{e^2 n_0}{\varepsilon_0 m_{ion}})\delta$$

- The frequency of this oscillation is the plasma frequency

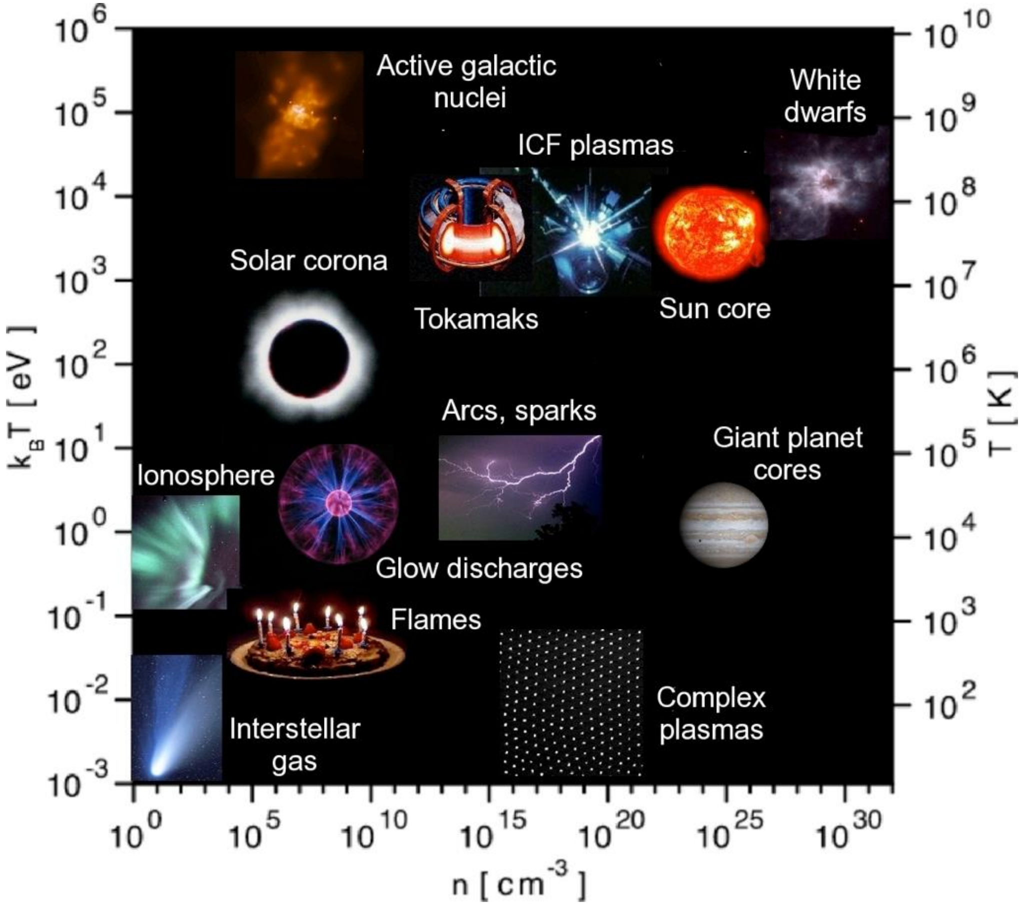
$$\omega_p{}^2 = \omega_{pe}^2 + \omega_{pi}^2$$

$$\omega_{pe}^2 = \frac{e^2 n_0}{\varepsilon_0 m_e}$$

$$\omega_{pi}^2 = \frac{e^2 n_0}{\varepsilon_0 m_{ion}}$$

- Because $m_{ion} >> m_e$ $\boxed{\omega_p \approx \omega_{pe}}$

- Useful formulas:



$$\lambda_D = 7.4m\left(\frac{T}{n}\right)^{\frac{1}{2}}; T(eV), n(cm^{-3})$$

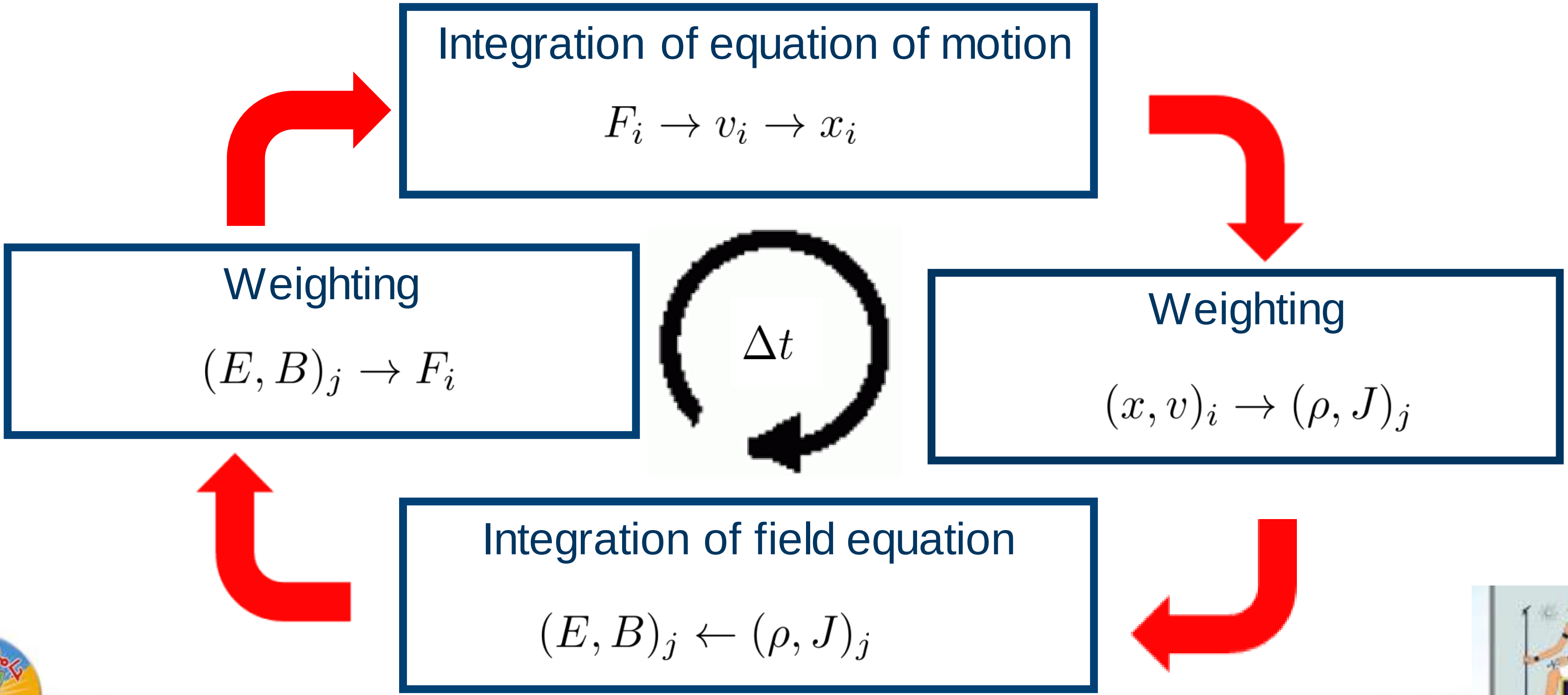
$$N_D = 1.7\times10^{9}\frac{T^{3/2}}{n^{1/2}}; T(eV), n(cm^{-3})$$

$$f_{pe} = 8.9kHz\cdot\sqrt{n}; n(cm^{-3})$$

$$f_{pi} = f_{pe} / sqrt(m_i / m_e) \Rightarrow f_{pi} / 43 = 210Hz\cdot\sqrt{n}\ (for\ \ protons)$$

Kinetic Description

- Kinetic means „of or relating to motion“.
  - It is impractical to solve the equation of motion of all plasma particles.
  - Boltzman equation is an integro-differential equation.
- Particle-in-Cell : Super particle $10^6 - 10^9$ real particles.



Integration of equation of motion

$F_i \rightarrow v_i \rightarrow x_i$

Weighting

$(x,v)_i \rightarrow (\rho,J)_j$

Integration of field equation

$(E,B)_j \leftarrow (\rho,J)_j$

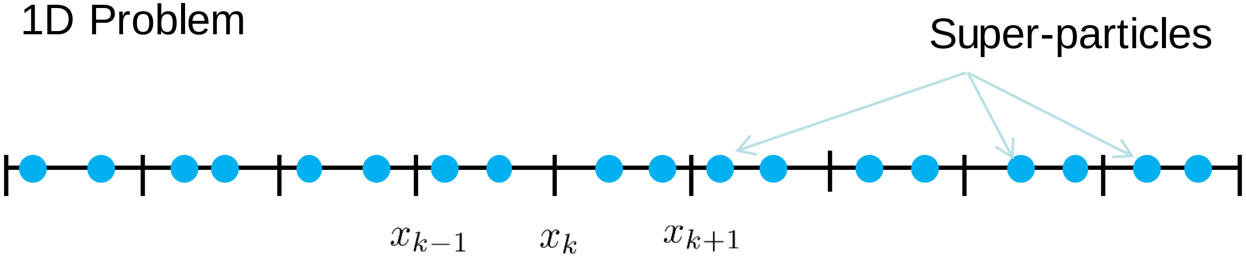
Weighting

$(E,B)_j \rightarrow F_i$

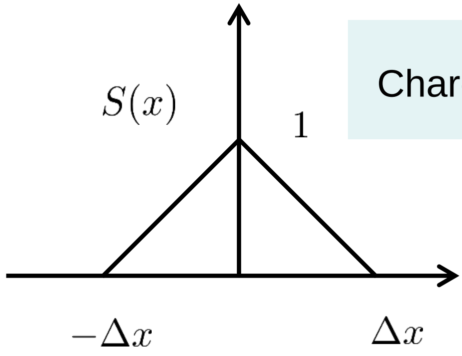
Δt

Particles

1D Problem

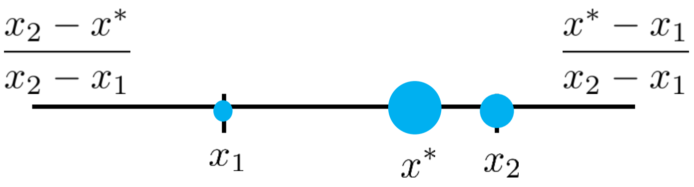


Simulation domain



Charge cloud

First order wieghting



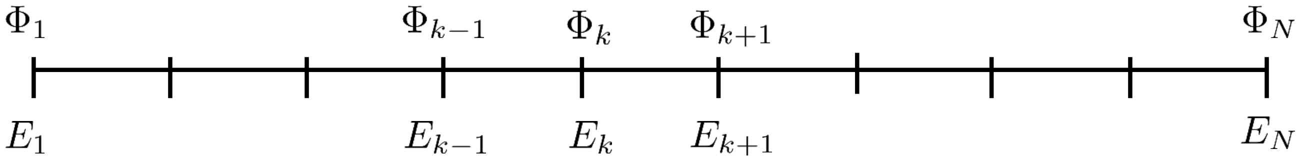
Fields

Poisson‘s eq.

$$\nabla^2\Phi = -\frac{\rho}{\epsilon} \longrightarrow \frac{\Phi_{k+1} - 2\Phi_k + \Phi_{k-1}}{\Delta x^2} = -\frac{\rho}{\epsilon}$$

Boundary condition

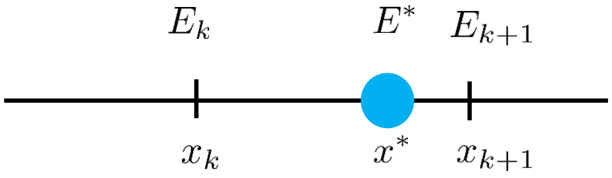
Boundary condition



$$E_1 = -\frac{\Phi_2 - \Phi_1}{\Delta x} - \rho_1 \frac{\Delta x}{2\epsilon} \qquad E_k = -\frac{\Phi_{k+1} - \Phi_{k-1}}{2\Delta x} \qquad E_N = -\frac{\Phi_N - \Phi_{N-1}}{\Delta x} + \rho_N \frac{\Delta x}{2\epsilon}$$

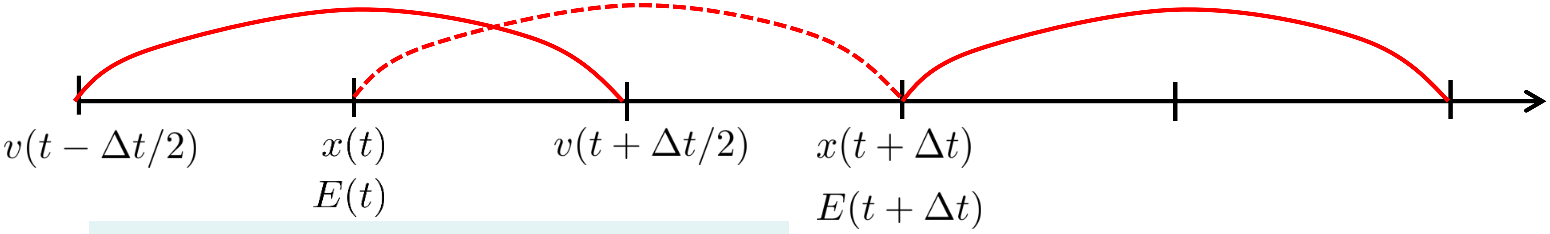
Interpolation of the fields to the particle positions

$$E^* = \frac{x^* - x_k}{\Delta x} E_{k+1} + \frac{x_{k+1} - x^*}{\Delta x} E_k$$



Moving particles

„Leapfrog“
scheme



Descritization of equation of motions

$$\frac{v(t+\Delta t/2)-v(t-\Delta t/2)}{\Delta t}=\frac{q}{m}E(t)$$

$$\frac{x(t+\Delta t)-x(t)}{\Delta t}=v(t+\Delta t/2)$$

Monte-Carlo Scheme is required for collisions

Chalenges of PIC simulation

- Numerical instabilities:
  - Accuracy criterion $\omega_p \Delta t \leq 0.2$
  - Courant criterion $v_{\max} \Delta t \leq \Delta x$
  - The computational grid has to resolve the Debye lenght $\Delta x \leq \lambda_D$

- In order to have a good statistics, a resonable high number of particles per Debye lenght must be used $N_{\mathrm{D}} \gg 1$

- Keep the probabilty for collisions small

$$P_{\mathrm{coll}} = 1 - e^{-\nu t} \leq 0.1$$

- Alternatives:
  - Implicit schems
  - Parrallilization

Fluid Models

- Look from a distance at the ensemble of particles : transport coeffecients

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = S$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right)\vec{u} = qn(\vec{E}+\vec{u}\times\vec{B}) - \vec{\nabla}\cdot\Pi + \vec{S}_c$$

…..

- Open set of equations even with Maxwell eqs.
- Provide macroscopic description.
- Transport coeffecients are functions of local (E/n) or local mean free energy.

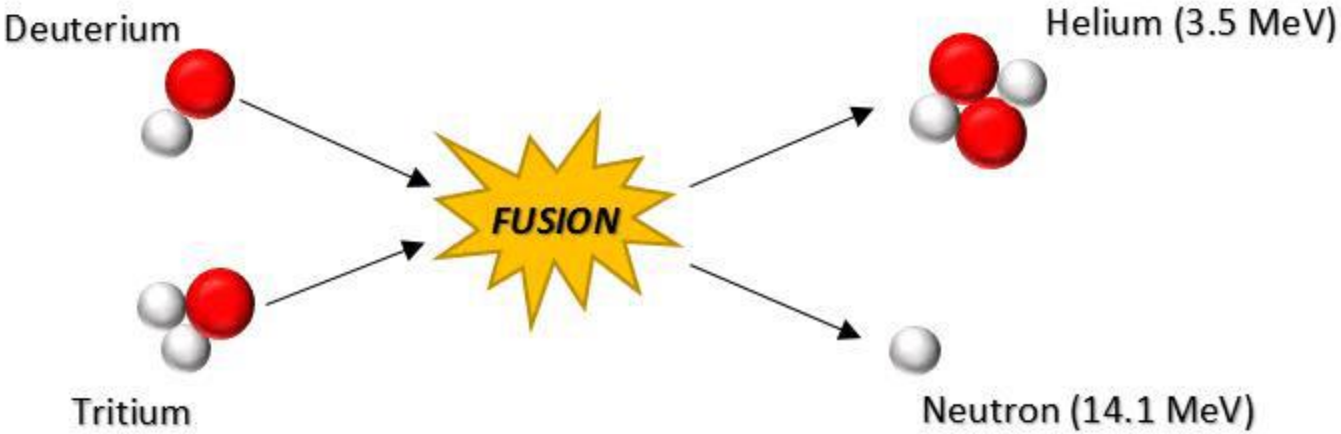
$$\frac{dE}{dx}\lambda \ll E \qquad\qquad \frac{dE}{dt}\nu^{-1} \ll E$$

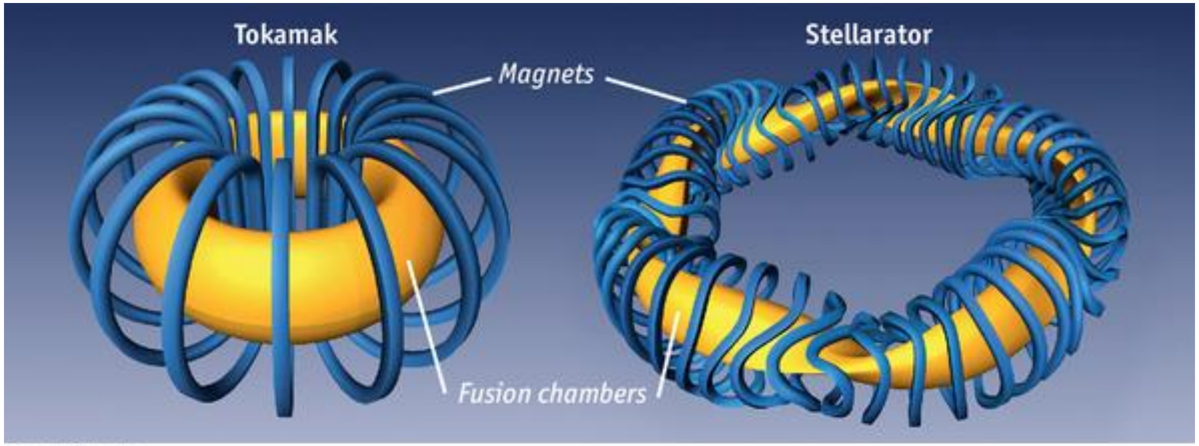
- Transport coeffecients can be derived from cross-sections, however, such calculations are zero-dimentional .

Applications

- Plasma plays an essential role in different applications such as
- gas discharges
- controlled thermonuclear fusion
- space physics
- modern astrophysics
- energy conversion and ionpropulsion
- Solid state plasma
- Gas laser
- Microelectronics
- Water treatment
- Cancer treatment
- Textile treatment

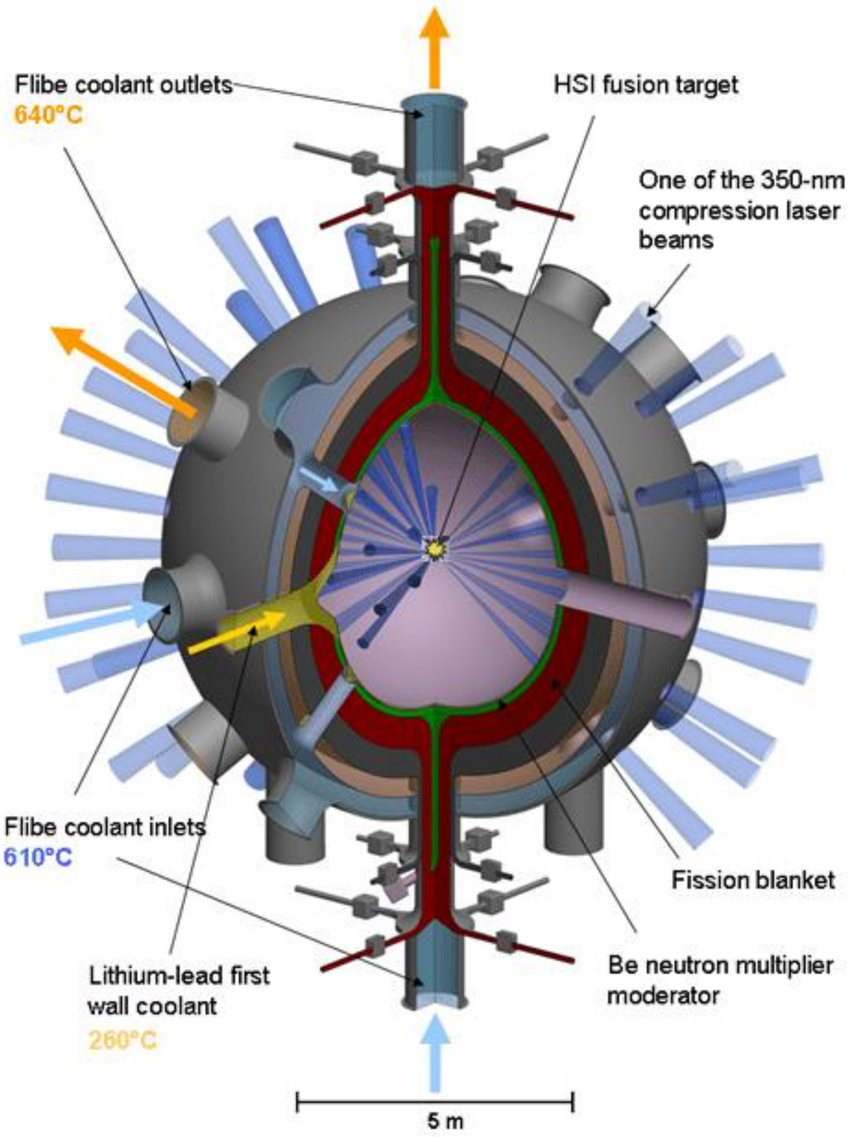
Nuclear fusion



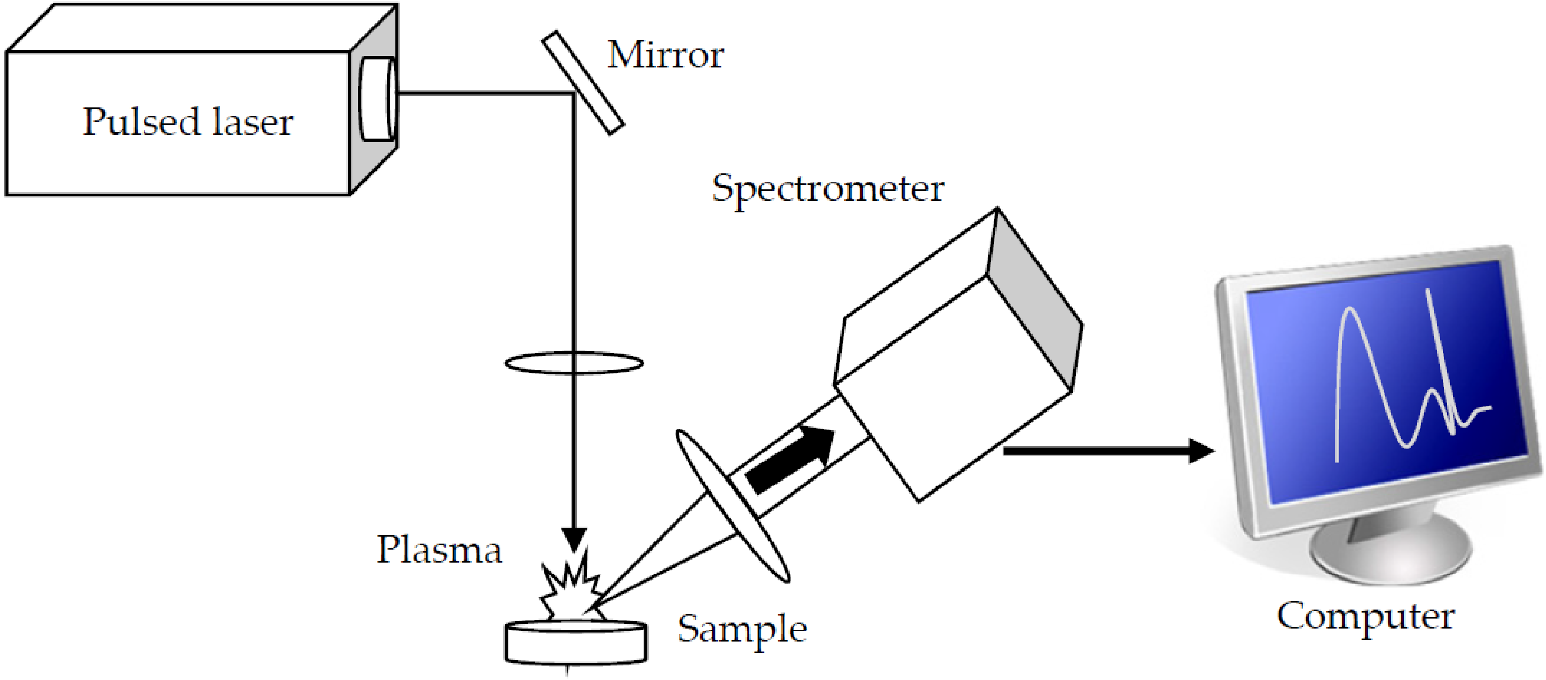


Economist.com

Inertial confinement fusion



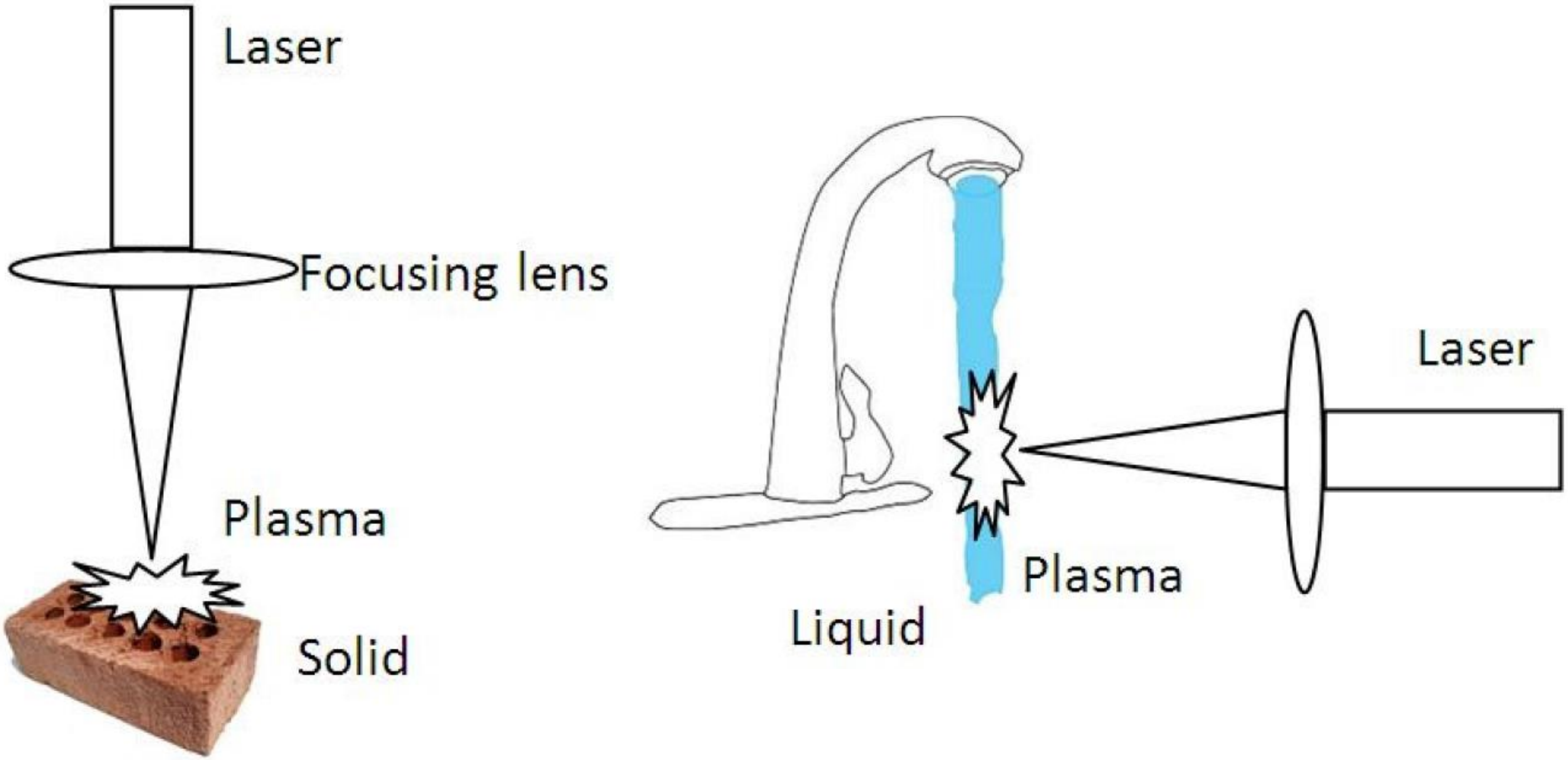
Laser induced breakdown spectroscopy



- **The conventional LIBS configuration**

InTech

LIBS for different Materials



- **Solid and liquid samples**

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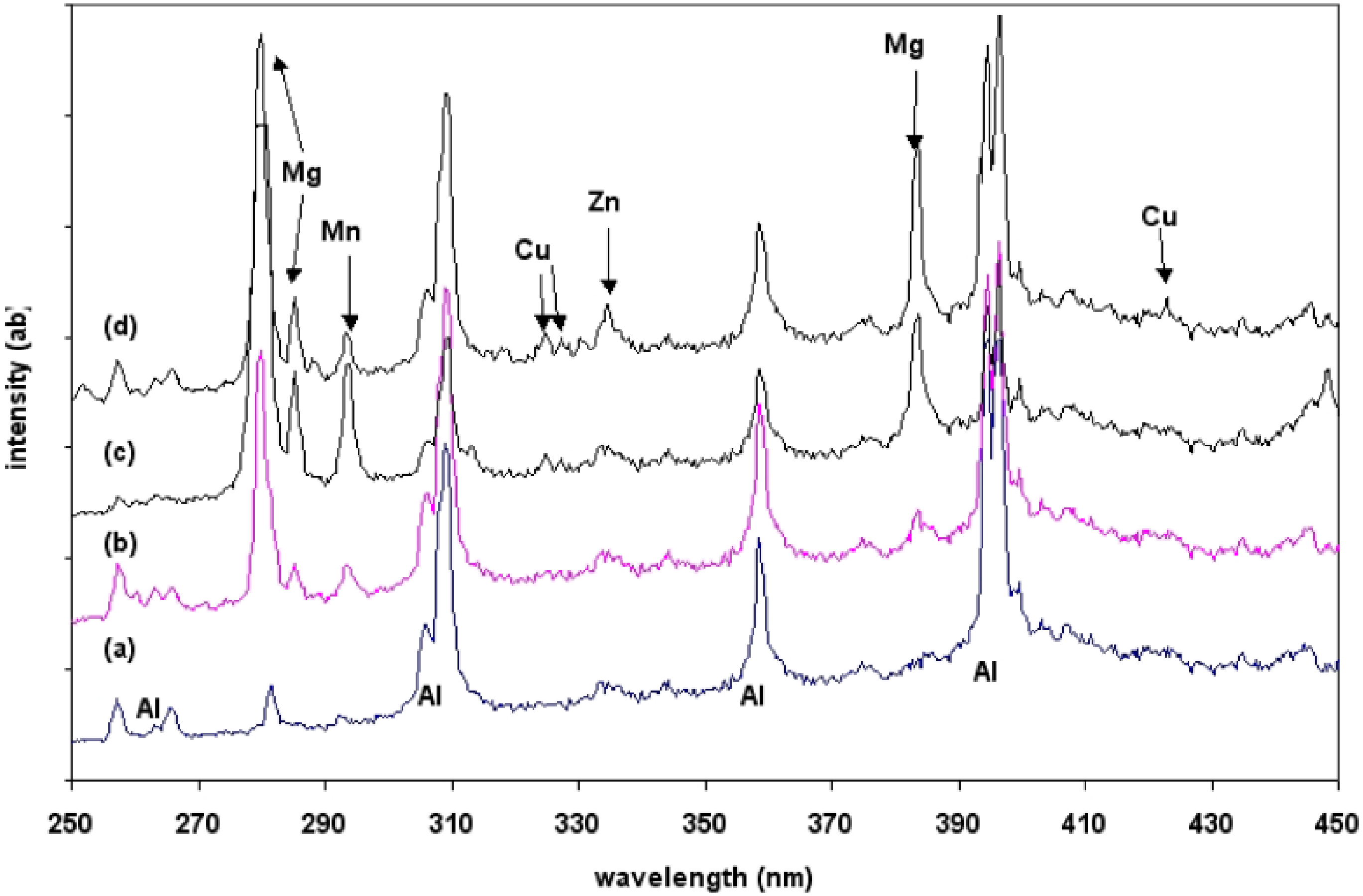
Al alloys spectrum



- **LIBS for Al alloys (a) Pure Al, (b) 3003 alloy, (c) 2024-T3 alloy, and (d) 7057-T6 alloy.**

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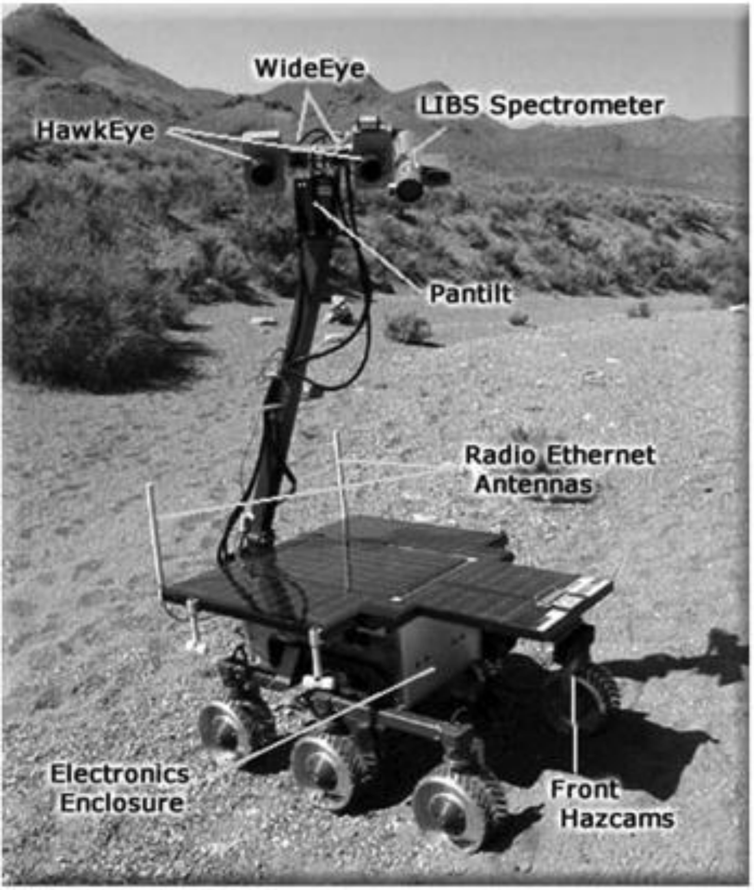
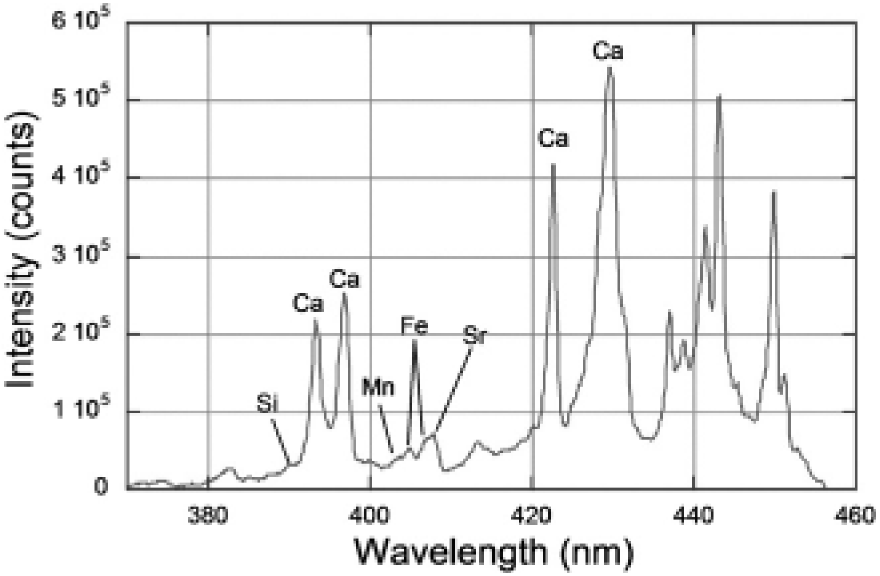
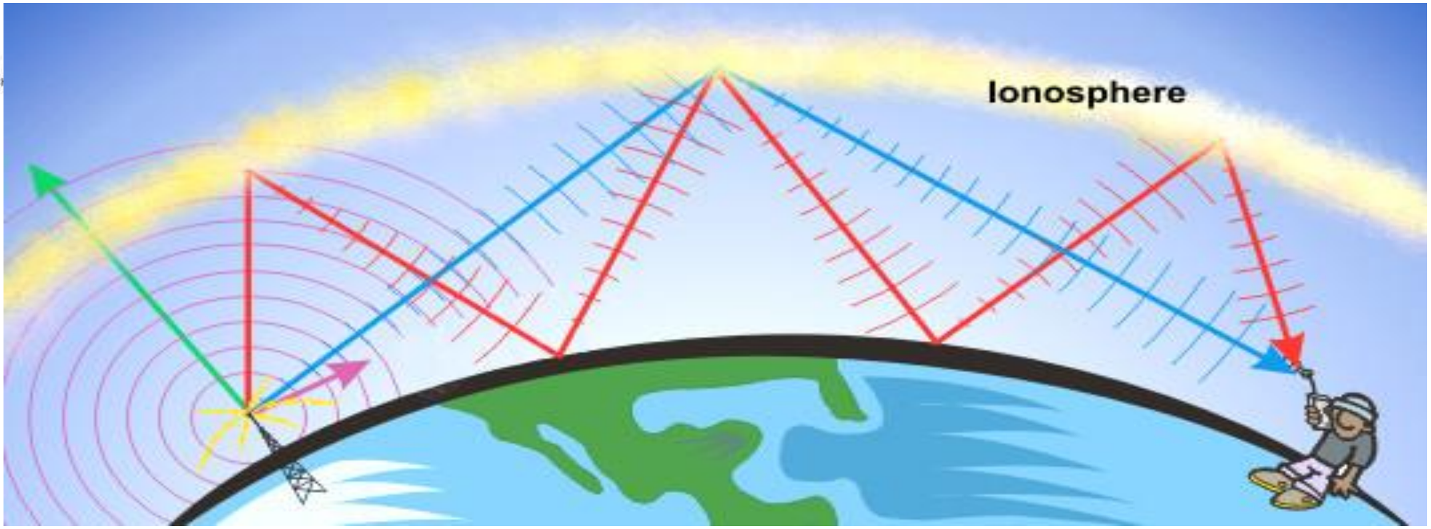
Study your samples in the field!!

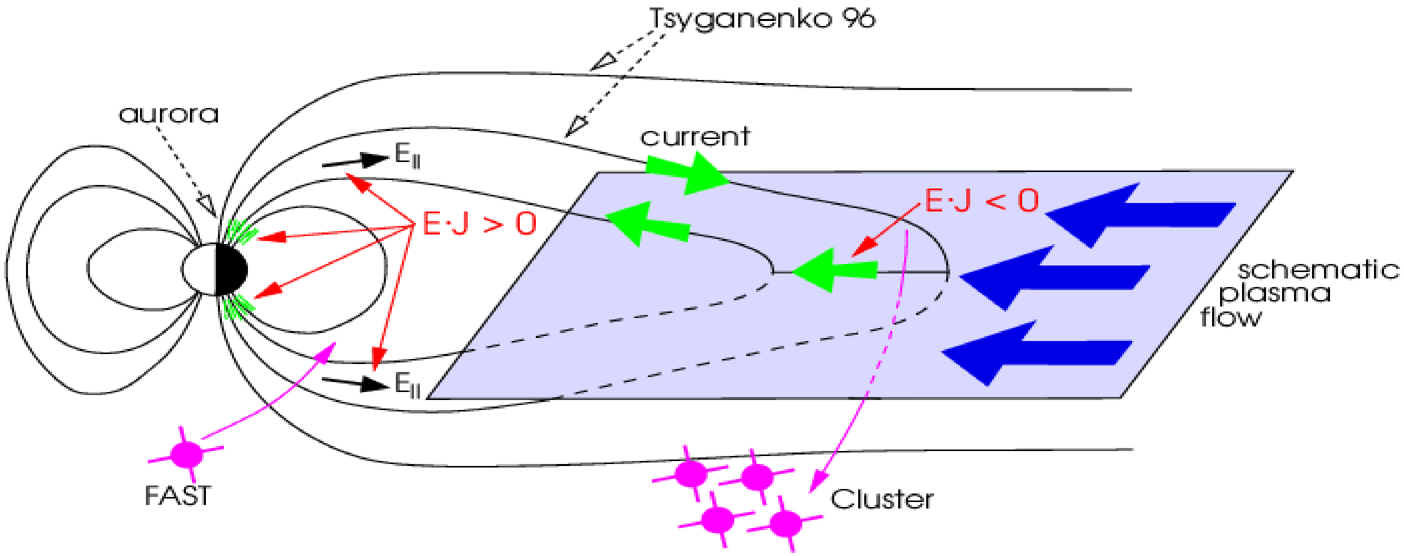




- **The k9 rover in the field (Nevada, USA).**

Space plasma

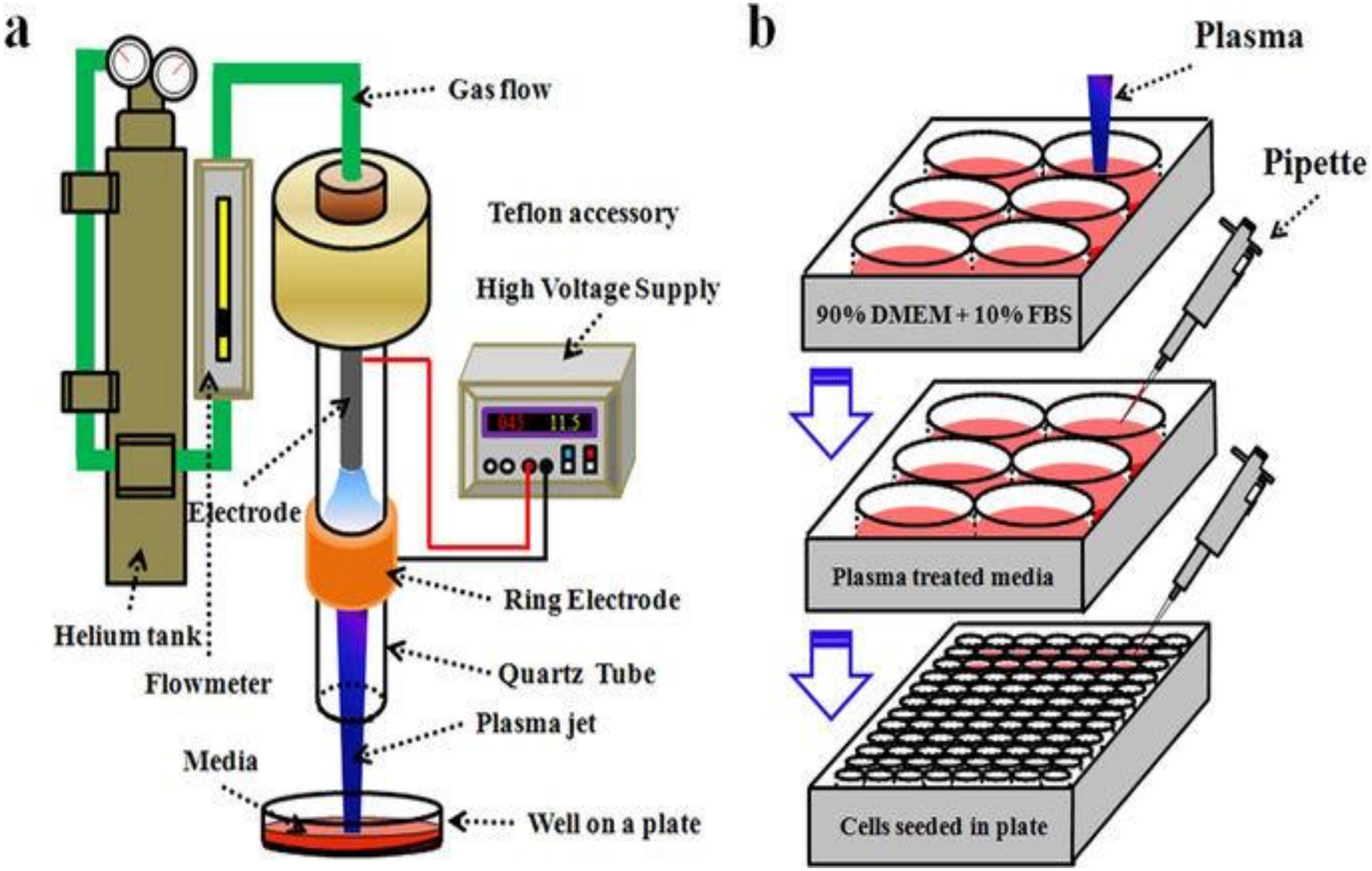




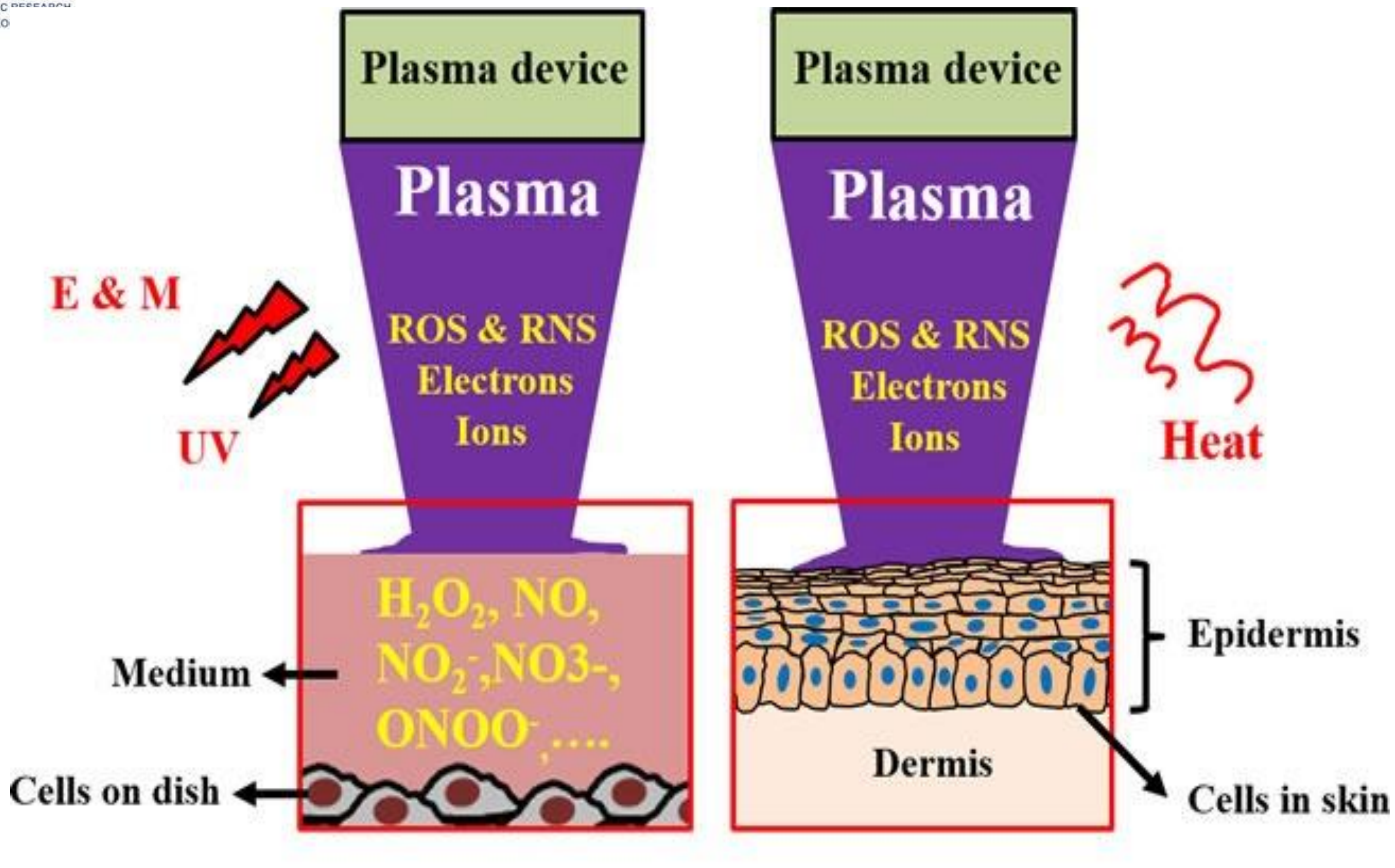
Light Sources

Plasma Propulsion

Plasma Medicine



Plasma Medicine
Cancer Treatment

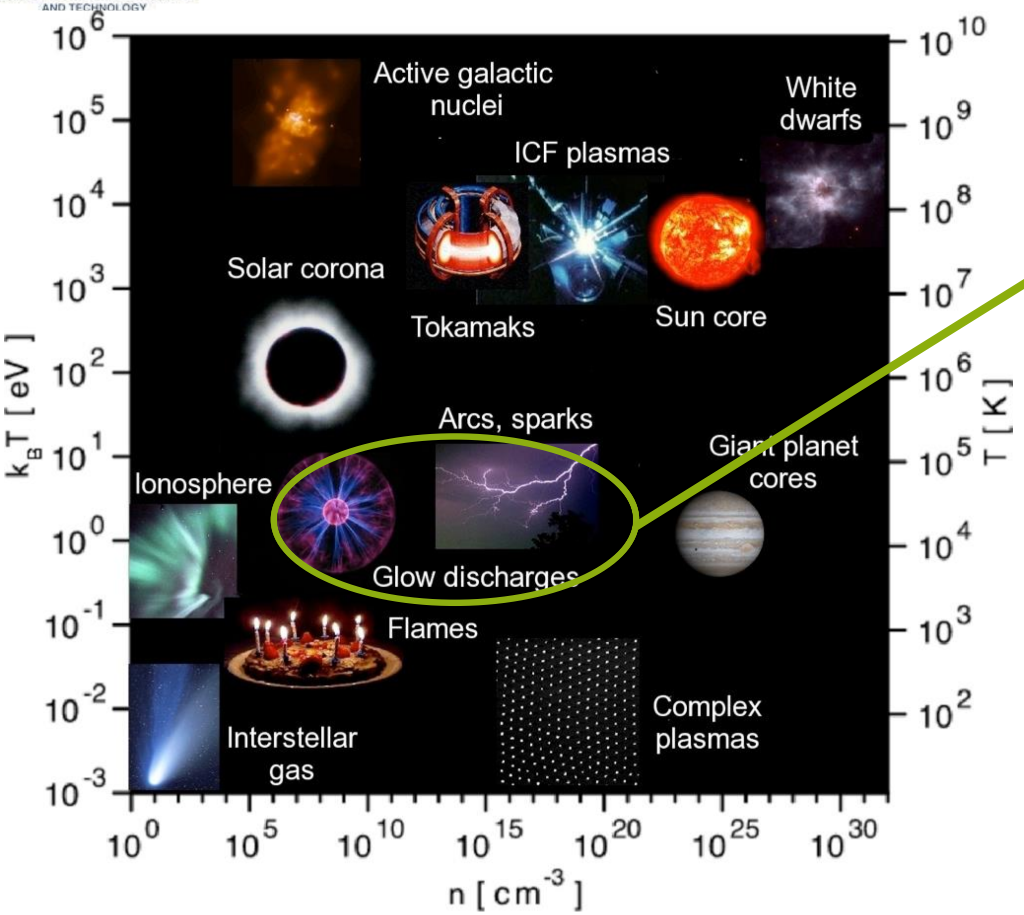


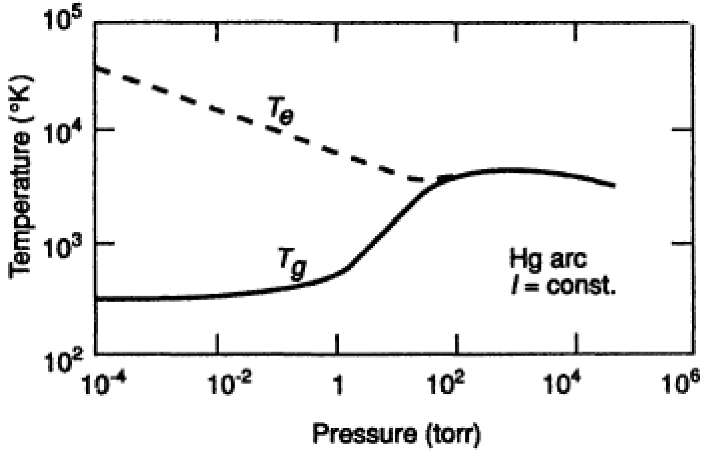
In vitro application

In vivo application

Low Temperature Plasma

- Low degree of ionization
- Neutral background 10^6 the ion and electron density
- Collisions with the background gas is dominant compared to electron ion collisions

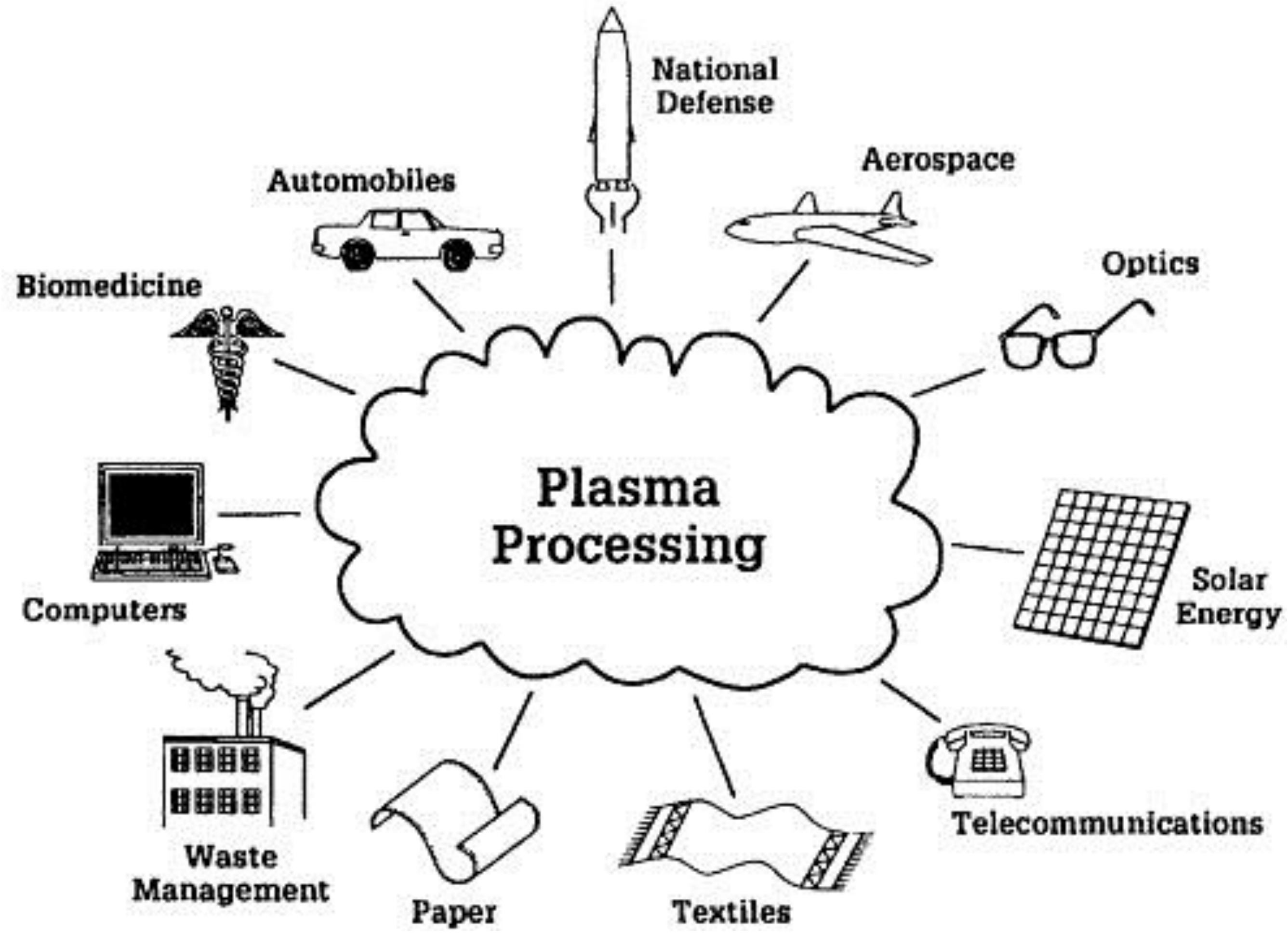




- Non-equilibrium plasmas at low pressures

$$T_e = 11000 - 60000\mathrm{K} \qquad T_e = 1 - 5\mathrm{eV} \qquad T_i = 300\mathrm{K}$$

Various applications



Thanks!

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