Lecture (1) Electrostatics
Simple Story

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Introduction

<u>Text Books:</u>

- ***College Physics, Serway / Vuille, Eight Edition, Chapters 15-18.***
- **Sadiku, *Elements of Electromagnetics, Oxford University.***
- ***Griffiths, Introduction to Electrodynamics, Prentice Hall.***
- ***Jackson, Classical Electrodynamics,* New York: John Wiley & Sons.**
- ***Open sources: MIT open courses, ….***

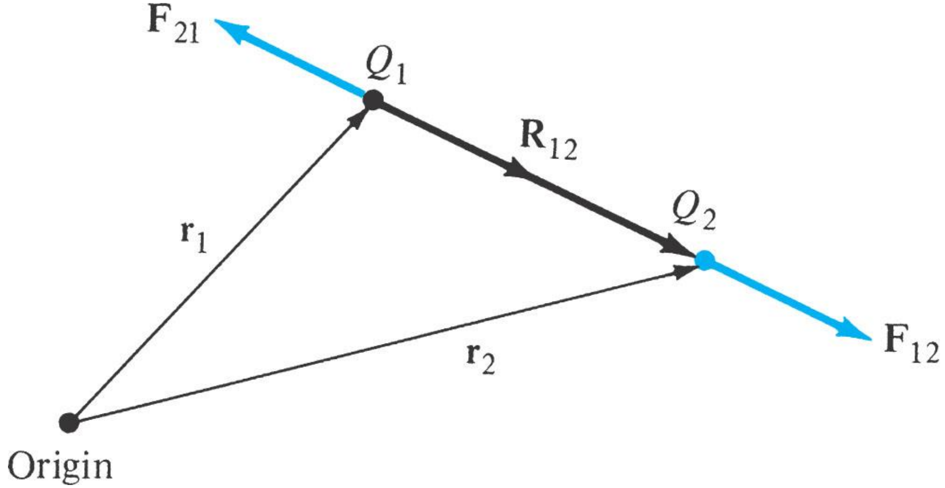
Electric Charges

- **After running a plastic comb through your dry hair, you will find that the comb attracts bits of paper.**

- ***Positive charges repel each others & Negative Charges repel each others***
- ***Positive and negative charges attract each others***

Electric Force

- ***Coulomb's law: The electric force is directly proportional to the product of the charges and inversely proportional to squared distance between the charges***



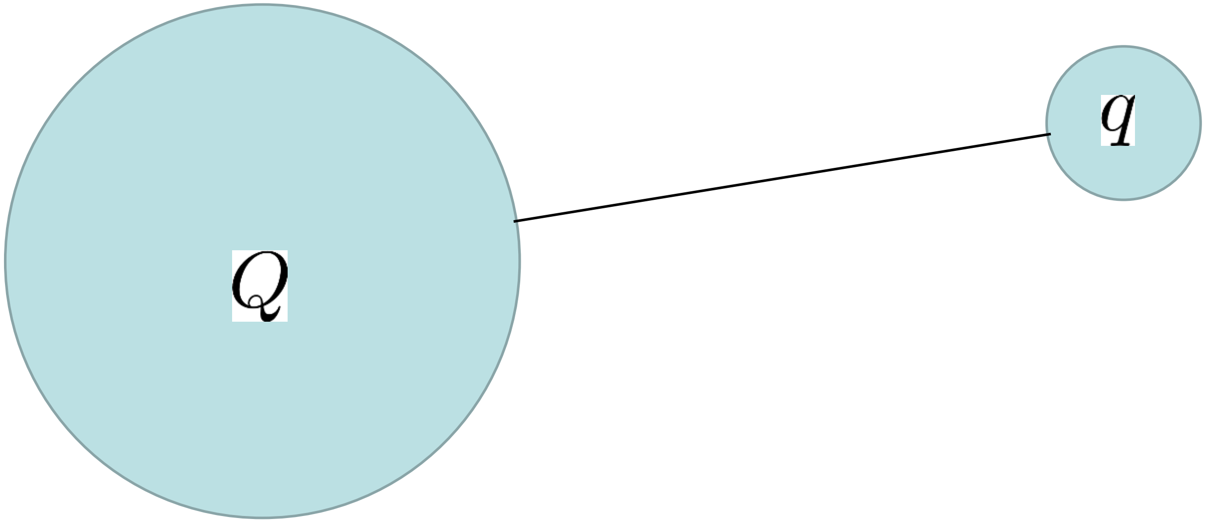
$$\vec{F} = k\frac{Q_1Q_2}{r^2}\vec{a}_r$$

$$\mathbf{F} = k\frac{Q_1Q_2}{r^2}\mathbf{a}_r$$

$$r = |\mathbf{R}_{12}| \quad k = \frac{1}{4\pi\epsilon}$$

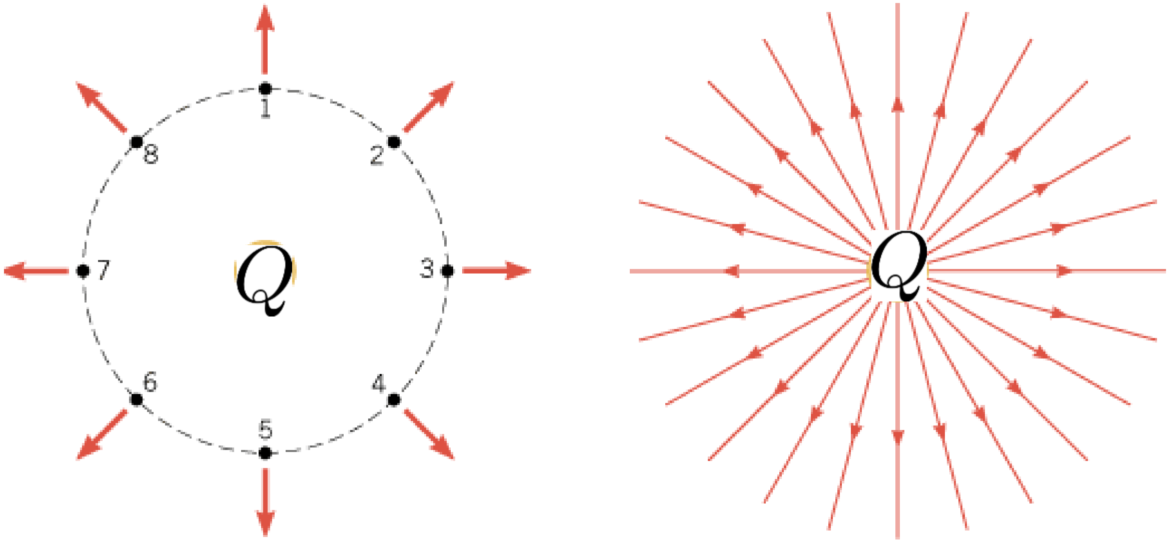
- ***The proportionality constant depends on the medium between the charges.***
- $k_{\text{Vacuum}} = 9 \times 10^9 \text{m/F}$ $\qquad$ $k_{\text{Water}} \approx 1.11 \times 10^8 \text{m/F}$

The area around a charge



- ***Examine the area around a charge using a test charge***

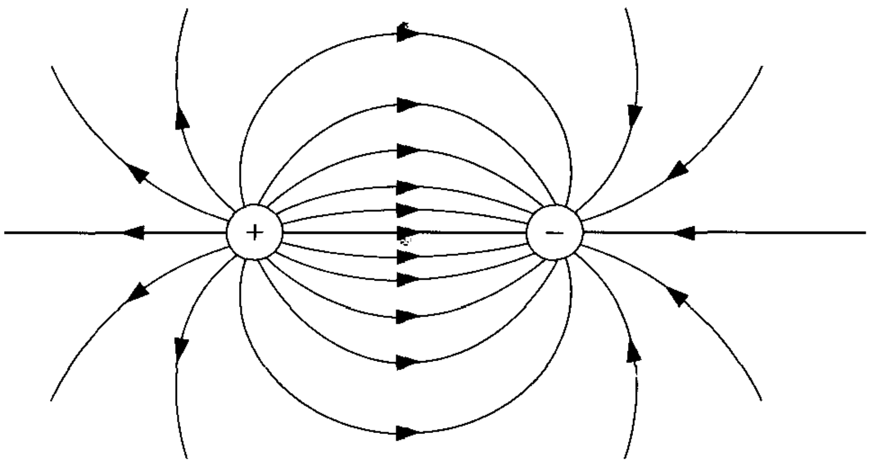
The area around a charge

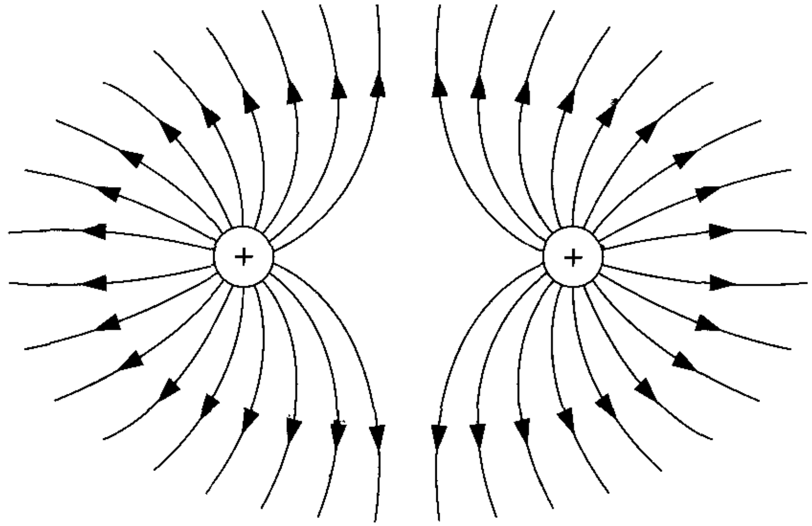


(a) (b)

- *The electric field of a charge is the force affect the test charge*
- $$\mathbf{E} = \frac{\mathbf{F}}{q} = k\frac{\frac{Qq}{r^2}}{q}\mathbf{a}_r = k\frac{Q}{r^2}\mathbf{a}_r$$

non-uniform electric field





- ***What about the divergence and the curl of the electric field?***

- ***The electric field lines does not form circular shapes, then the curl of the field is zero***

$$\nabla \times \mathbf{E} = 0 \qquad \qquad \nabla \cdot \mathbf{E} = Value$$

Field lines

- ***Field lines are imaginary lines that show the direction of the electric field***

- ***Electric flux density is the number of electric flux lines per unite area.***

$$\mathbf{D} = \frac{\psi}{\mathbf{S}} \qquad \psi = \oint \mathbf{D}.d\mathbf{s}$$

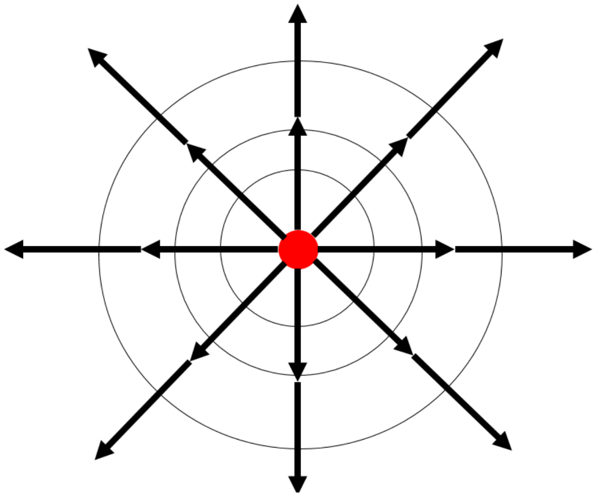
$$\mathbf{D} \propto \mathbf{E} \qquad \mathbf{D} = \epsilon\mathbf{E} \qquad \psi = \epsilon \oint \mathbf{E}.d\mathbf{s}$$

- ***The intensity of the field is related to the number of the lines and the density of the lines.***

Field lines of a point charge

- ***The lines are uniform and perpendicular to a closed spherical surface***
-

$$\psi = \epsilon \oint \mathbf{E}.d\mathbf{s}$$

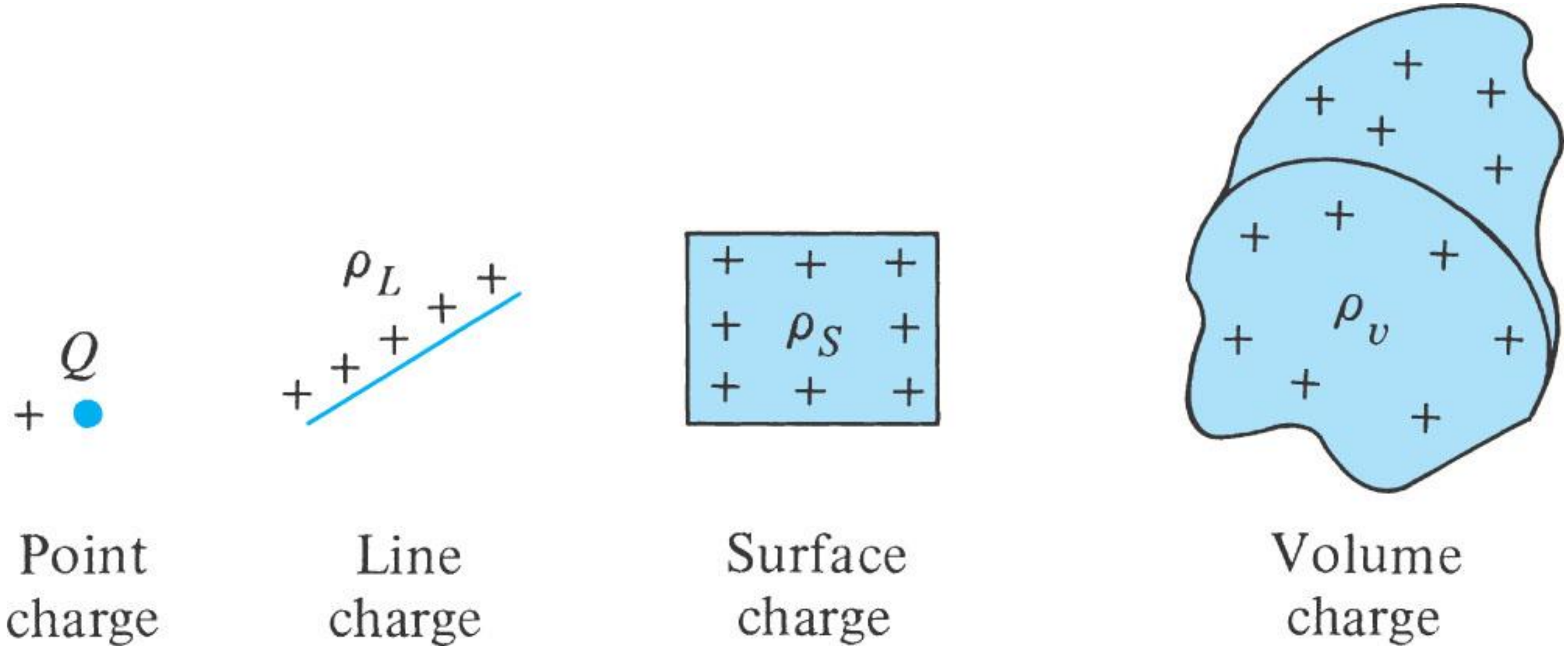


$$\psi = \epsilon_0 \; E \; S$$

$$\psi = \epsilon_0 \frac{Q}{4\pi\epsilon_0 r^2} \; 4\pi r^2$$

- $\psi = Q$ $\qquad Q = \epsilon \oint \mathbf{E}.d\mathbf{s}$ ***Gauss's law***

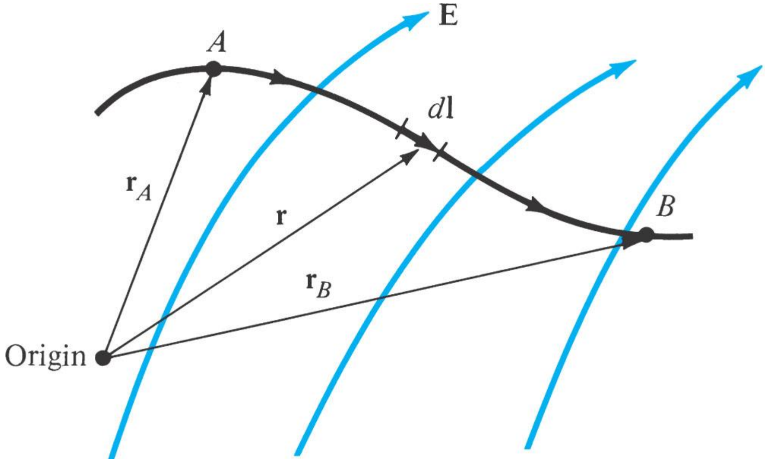
Gauss's law



$$Q = \epsilon \oint \mathbf{E}.d\mathbf{s}$$

Potential Energy

- ***When a charge move in an electric field;***



$$dW = -\mathbf{F}\cdot d\mathbf{l} \quad dW = -q\mathbf{E}\cdot d\mathbf{l}$$

$$W = -q\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

- ***The negative sign mean that the field is done by the field.***
- ***The amount of energy depends only on the A & B and does not depend on the path between A & B.***
- ***The work done in a closed path is zero.***
- ***The potential is the energy done per unite charge***

$$V = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

Potential vs Electric Field

- ***The gradient of the potential***

$$V = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad dV = -d\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad E = -dV/dl$$

$$\mathbf{E} = -\nabla V \qquad \nabla\cdot\mathbf{E} = -\nabla^2 V$$

- ***Remember that*** $Q = \epsilon \oint \mathbf{E}.d\mathbf{s} \qquad \oint \mathbf{E}\cdot d\mathbf{s} = \int \nabla\cdot\mathbf{E}\; d\tau$

$$Q/\epsilon = \int \nabla\cdot\mathbf{E}\; d\tau$$

$$\nabla\cdot\mathbf{E} = \frac{\rho_v}{\epsilon} \qquad \nabla^2 V = -\frac{\rho_v}{\epsilon}$$

Maxwell's equations

- ***Stock's Theory***

$$\oint \mathbf{E} \cdot d\mathbf{s} = \int \nabla \cdot \mathbf{E} \, d\tau$$

- ***Green's Theory***

$$\oint (\nabla \times \mathbf{E}).d\mathbf{S} = \oint \mathbf{E} \cdot d\mathbf{l}$$

- ***Maxwell's equations***

Point Form

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$$

$$\nabla \times \mathbf{E} = 0$$

Integral Form

$$Q/\epsilon = \int \nabla \cdot \mathbf{E} \, d\tau$$

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0$$

Capacitors and energy storage

- *Static charges are accumulated on the capacitor plates.*

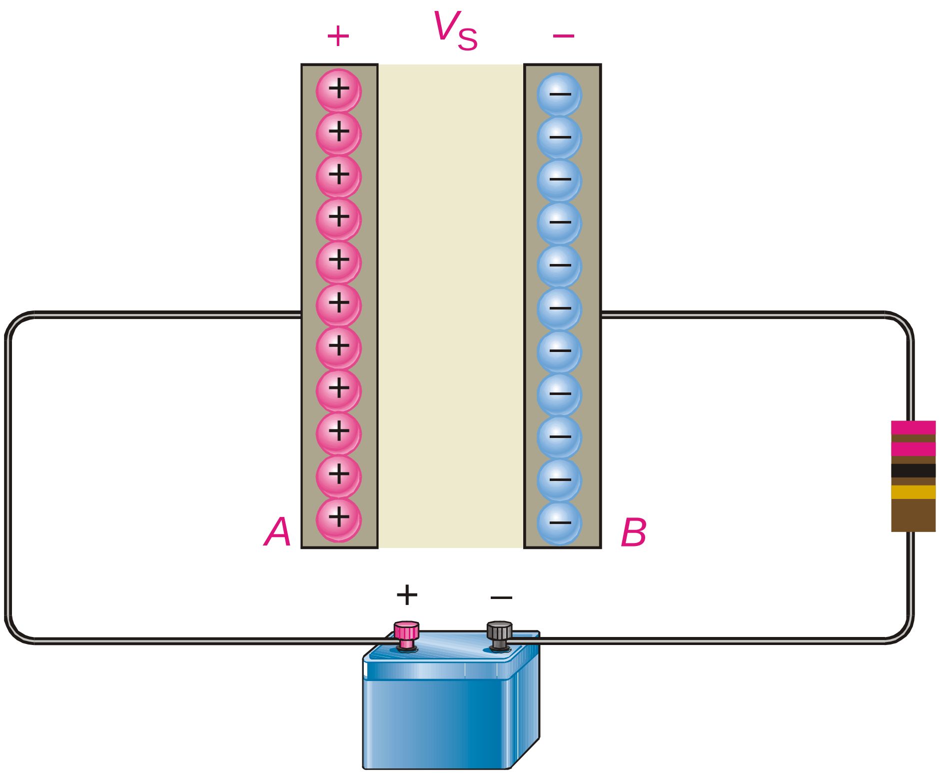
- *The energy stored*

$$Energy = \frac{1}{2}CV^2$$

- *The charged capacitor may work as a battery.*

The discharge time depends on

$$\tau = RC$$



Current density

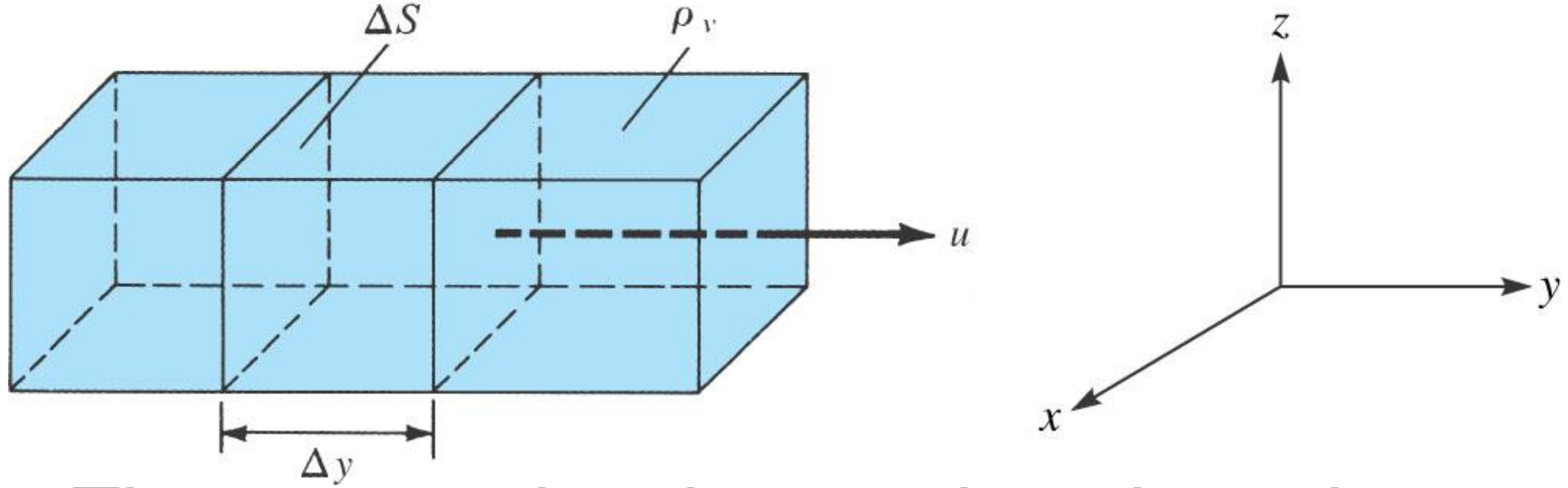
- The **current** (in amperes) through a given area is the electric charge passing through the area per unit time. 1 A = 1 C/s

$$I = \frac{dQ}{dt}$$

Current density $J = \frac{\Delta I}{\Delta S}$ or $I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$

- What is the unit of **J**? Is it scaler quantity?

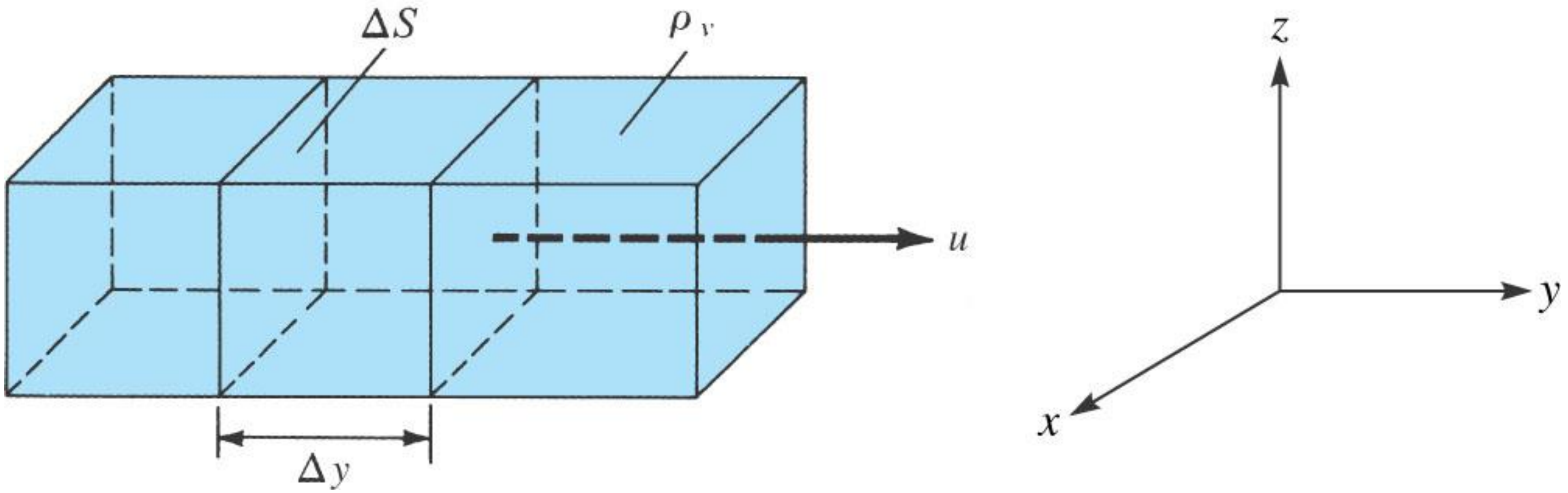
Current density



- The current density passing along the conductor is $J_y \propto \rho_v$ and $J_y \propto u_y$
- By using physical units, we can show that

$$J_y = \rho_v \, u_y$$

Current density



$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta V}{\Delta t} = \frac{\rho_v \Delta S \Delta y}{\Delta t} = \rho_v \Delta S \, u_y$$

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$$

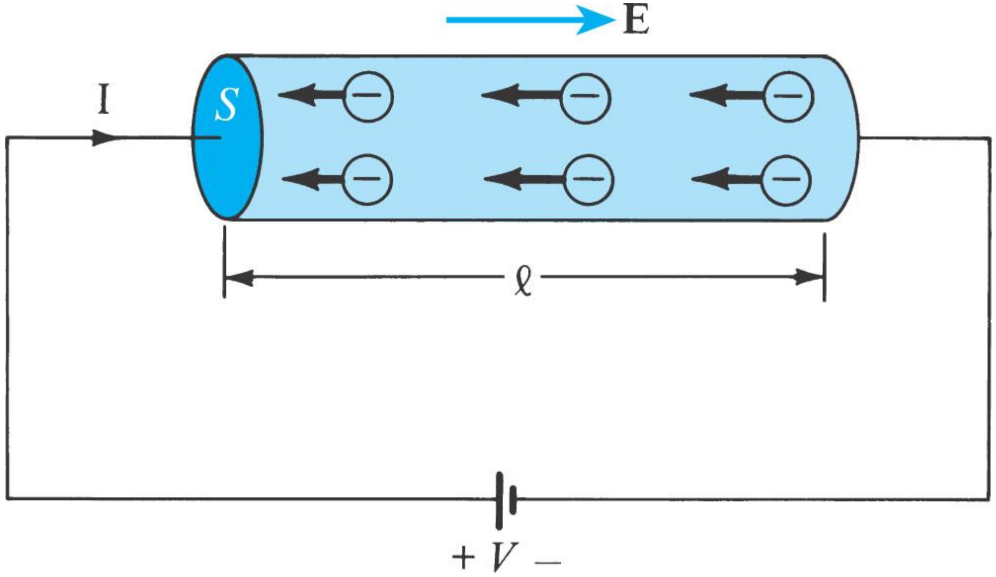
Current density

- In general: $\boldsymbol{J} = \rho_V \boldsymbol{u}$ this equation is the general form of both **convection** and **conduction** current density
- In conductors: the charge density ρ_v is defined as

$$\rho_v = \frac{\Delta Q}{\Delta V} = -n\,e$$

Where n is the electron density and e is the electron charge. Why does negative charge exist in the equation?

Conduction Current density



- Electrons move opposite to the field direction

$$\boldsymbol{F} = -e\boldsymbol{E}$$

From momentum $$\mathbf{F} = \frac{m\,d\boldsymbol{u}}{dt} = \frac{m\,(\boldsymbol{u}-\mathbf{0})}{(\tau-0)} = -e\boldsymbol{E}$$

τ: the average time interval between collisions – The time required to reach speed of $\boldsymbol{u}$.

Generalized Ohm's law

- Therefore, $\boldsymbol{u} = -\frac{e\tau}{m}\boldsymbol{E}$

If there are n electrons per unit volume

$$\rho_V = -ne$$

So

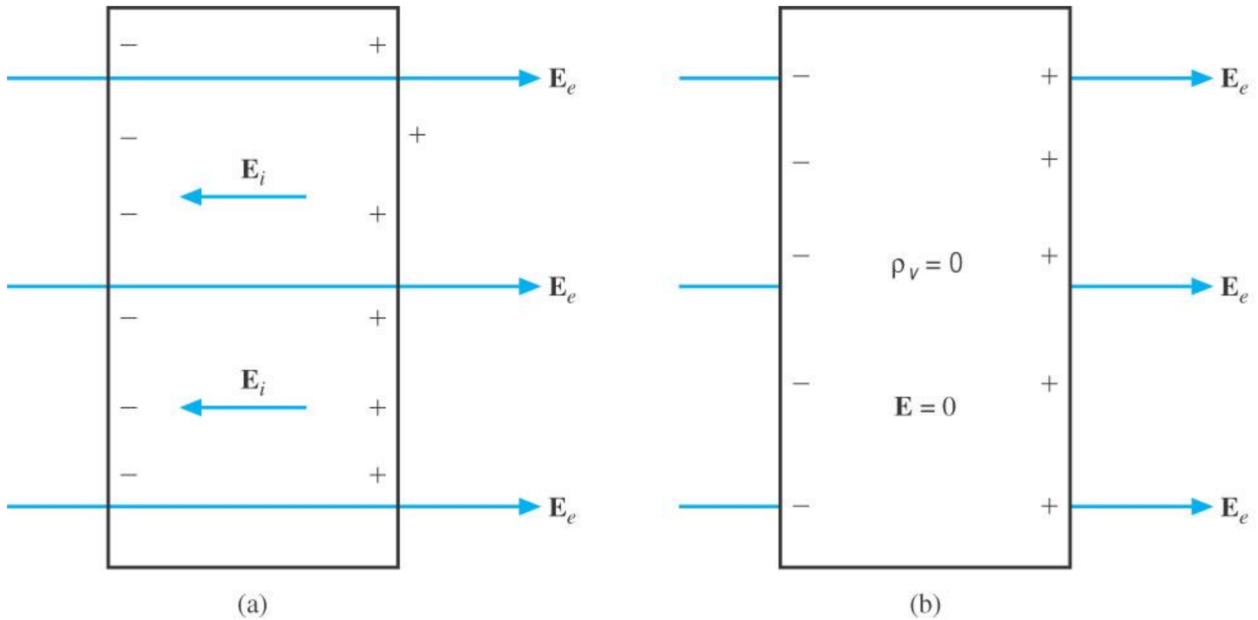
$$\boldsymbol{J} = \rho_V \boldsymbol{u} = \frac{ne^2\tau}{m}\boldsymbol{E}$$

or

$$\boldsymbol{J} = \sigma\boldsymbol{E}$$

Conductors

- A perfect conductor ($\sigma = \infty$) cannot contain E-field within it.

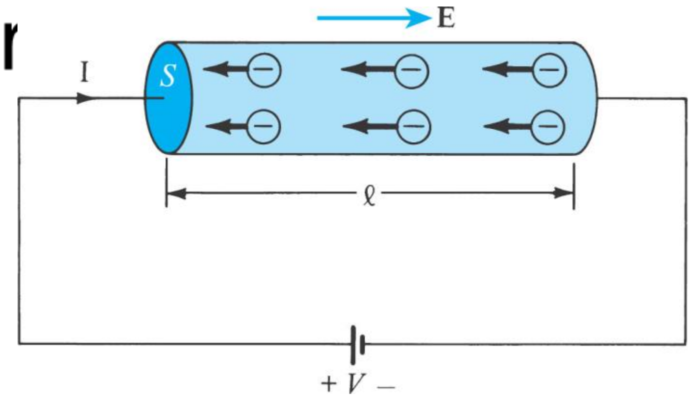


E_e is applied external field, E_i is internal field--isolated conductor: V= constant

Ohm's law

- For a conductor whose ends are maintained at potential difference V

$\boldsymbol{E} \neq 0$ inside the conductor



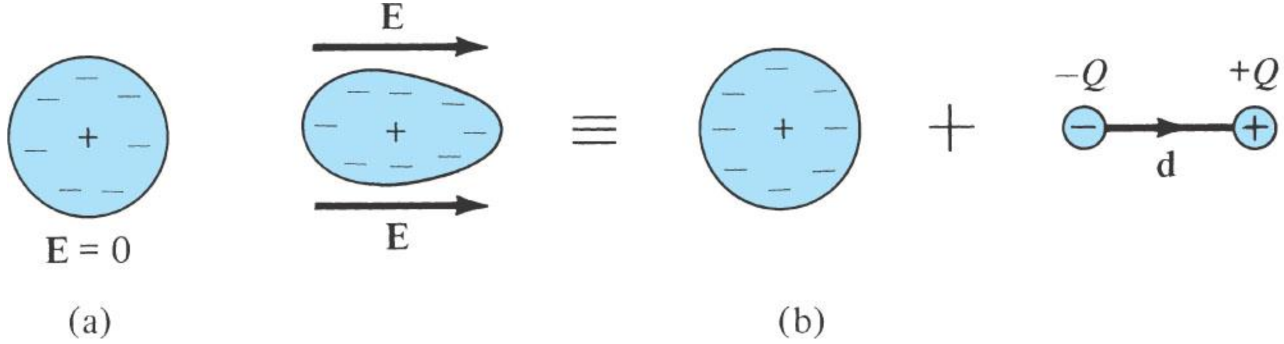
$$E = \frac{V}{l}$$

$$J = \frac{I}{S} = \sigma E = \sigma \frac{V}{l}$$

$\frac{I}{S} = \sigma \frac{V}{l} \rightarrow R = \frac{V}{I} = \frac{l}{\sigma S}$ or $R = \frac{\rho_c l}{S}$

Where $\rho_c = 1/\sigma$ is the resistivity

Polarization in Dielectrics

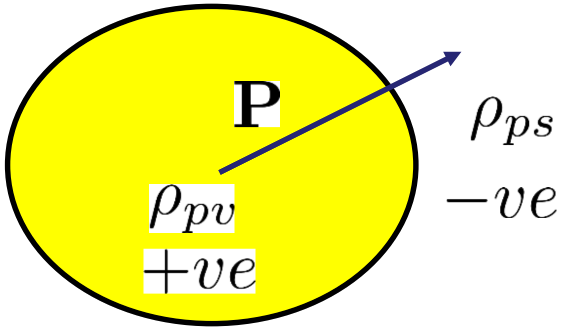


When an electric field is applied to a dielectric material, the positive and negative charges are displaced from their equilibrium. In the polarized state, the electron cloud is distorted.

Dipole moment: $\mathbf{p} = Q\boldsymbol{d}$

Polarization in dielectrics $\boldsymbol{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} Q_k \boldsymbol{d}_k}{\Delta V}$

This is **dipole moment per unit volume**

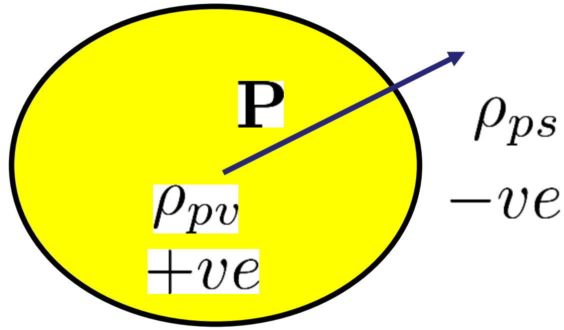


- The polarization leads to the accumulation of a negative charge on the body surface:

$$Q_b = \oint \rho_{ps} \, ds$$

- The polarization leads to the accumulation of a positive charge inside the body

$$-Q_b = \int_v \rho_{pv} \, dv$$



- From Gauss theory

$$\oint \mathbf{P} \cdot d\mathbf{s} = \int_v \nabla \cdot \mathbf{P} dv$$

$$Q_b = \oint \rho_{ps} \; ds = - \int_v \rho_{pv} \; dv$$

- Then the charge surface density

$$\rho_{ps} = \mathbf{P} \cdot a_n$$

- The volume charge density

$$\rho_{pv} = -\nabla \cdot \mathbf{P}$$

Displacement field

- When an electric field affect a dielectric, a polarization gives rise; therefore

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

- D is the displacement field. In free space, the electric displacement field is equivalent to flux density ($C \cdot m^{-2}$).
- In a dielectric material the displacement field is greater than its value in vacuum for the same electric field due to polarization.

Susceptibility

- Susceptibility is the ability of material to be polarized or the sensitivity of the material to the external electric field.
- Linear materials: materials where the polarization increases linearly with increasing the external electric field

$$\mathbf{P} \propto \mathbf{E} \quad \longrightarrow \quad \mathbf{P} = \chi_e \epsilon_0 \mathbf{E}$$

- Therefore:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \longrightarrow \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \chi_e \epsilon_0 \mathbf{E}$$

$$\mathbf{D} = (1 + \chi_e) \epsilon_0 \mathbf{E} \quad \longrightarrow \quad \mathbf{D} = \epsilon \mathbf{E}$$

Dielectric constant

- **Dielectric constant is the ratio between the permittivity of the dielectric to that of free space**

$$\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$

- **By increasing the external field affect a dielectric material, free charges may generated and the matter becomes conductor; such as arc discharge and lightening.**
- **The dielectric strength is the maximum electric field that a dielectric can stay without electrical breakdown**

Properties of dielectric materials

- **Linear material:** when the displacement field and the polarization vary linearly with the applied field
- **Homogeneous material:** The conductivity and the dielectric constant do not depend on the space coordinate; i.e., independent of x,y,z.
- **Isotropic materials:** when the material has the same properties in all directions, i.e., the electric field and the displacement field have the same directions.

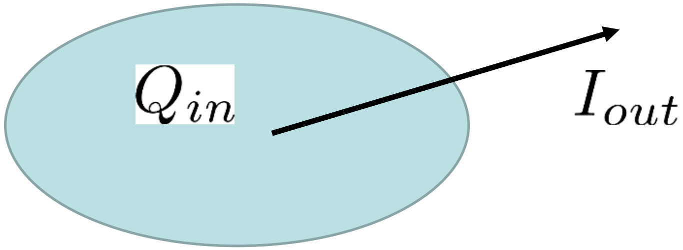
Dielectric tensor

- **Anisotropic material:** **When the dielectric constant depends on direction. The dielectric constant or the susceptibility has nine component.**
- $$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$
- **A dielectric with ($D = \epsilon E$) is linear ϵ does not change with the applied field, homogeneous if ϵ does not change from point to point, and isotropic if ϵ does not change with directions**

Continuity equation

- When a current coming out a body, then

$$I_{out} = \oint \mathbf{J} \cdot d\mathbf{s} = \frac{-dQ_{in}}{dt}$$



- From Gauss theory

$$\oint \mathbf{J} \cdot d\mathbf{s} = \int_v \nabla \cdot \mathbf{J} \ dv \quad \Rightarrow \quad \frac{-dQ_{in}}{dt} = \int_v \nabla \cdot \mathbf{J} \ dv$$

$$Q_{in} = \int_v \rho_v dv \quad \Rightarrow \quad -\frac{d}{dt}\int_v \rho_v dv = \int_v \nabla \cdot \mathbf{J} \ dv$$

$$-\frac{d}{dt}\rho_v = \nabla \cdot \mathbf{J}$$

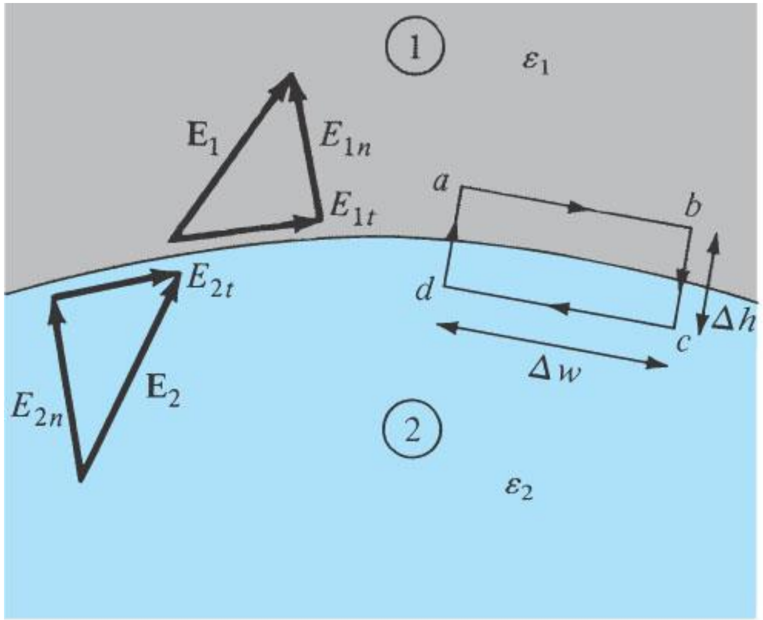
Continuity equation

- **From the continuity equation** $-\frac{d}{dt}\rho_v = \nabla \cdot \mathbf{J}$
- **Use** $\mathbf{J} = \sigma\mathbf{E}$**, then** $-\frac{d}{dt}\rho_v = \sigma\nabla \cdot \mathbf{E}$
- **Use** $\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon_0}$**, then** $-\frac{d}{dt}\rho_v = \sigma\frac{\rho_v}{\epsilon_0}$
- **Then** $\rho_v = \rho_{v0}\ e^{\frac{-\sigma}{\epsilon}t}$, **where** $T_r = \frac{\epsilon}{\sigma}$ **is relaxation time; the time it takes a charge placed in the interior of a material to drop to 0.37 of its initial value.**

Boundary Conditions

- From the integral form at D-D interface

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0 \qquad\qquad Q = \oint \mathbf{D}.d\mathbf{s}$$



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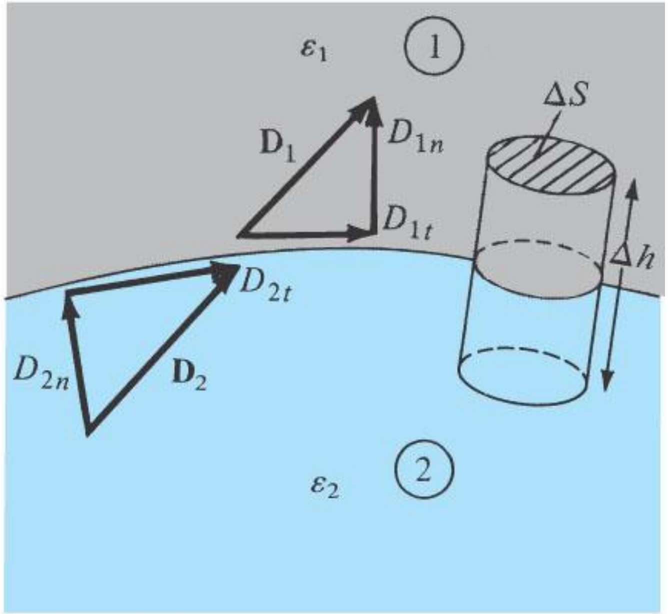


(b)

Boundary Conditions

- **When the free charge density at the interface=0**

$$E_{1t} = E_{2t} \qquad \frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

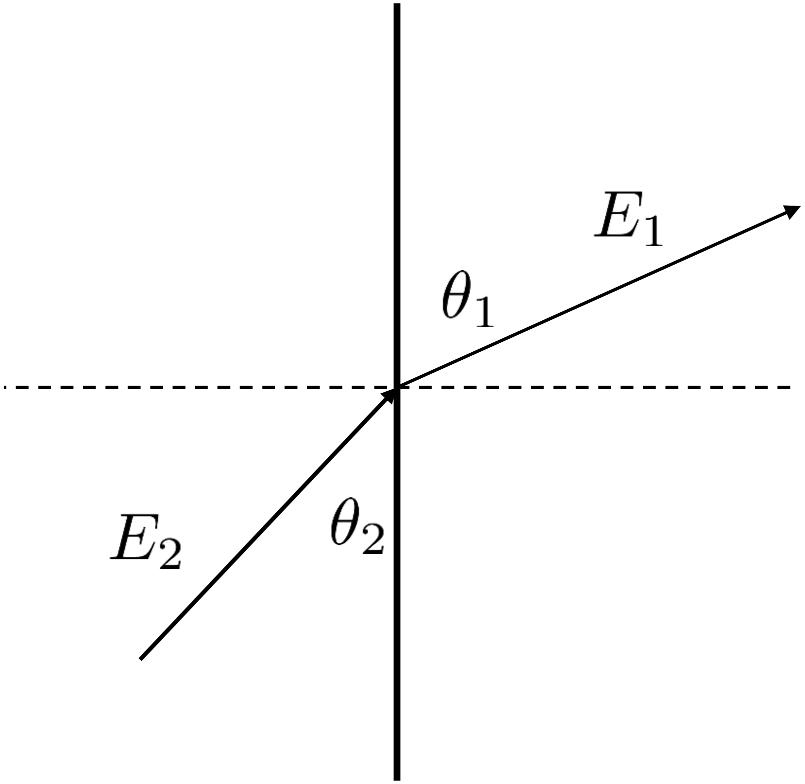
$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n} \qquad D_{1n} = D_{2n}$$

- **Snell's Law**

$$E_1 \sin(\theta_1) = E_2 \sin(\theta_2)$$

- **Refraction of the electric field**

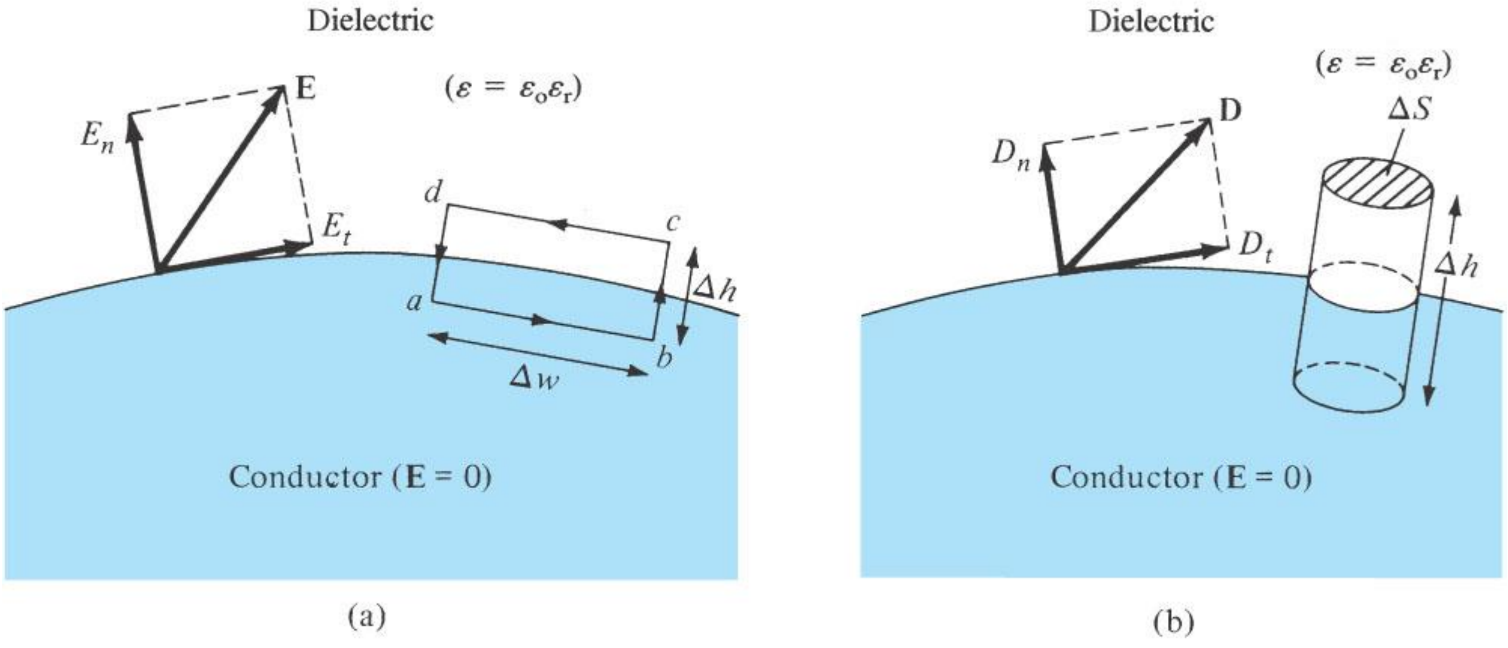
$$\frac{\tan(\theta_1)}{\tan(\theta_2)} = \frac{\epsilon_1}{\epsilon_2}$$



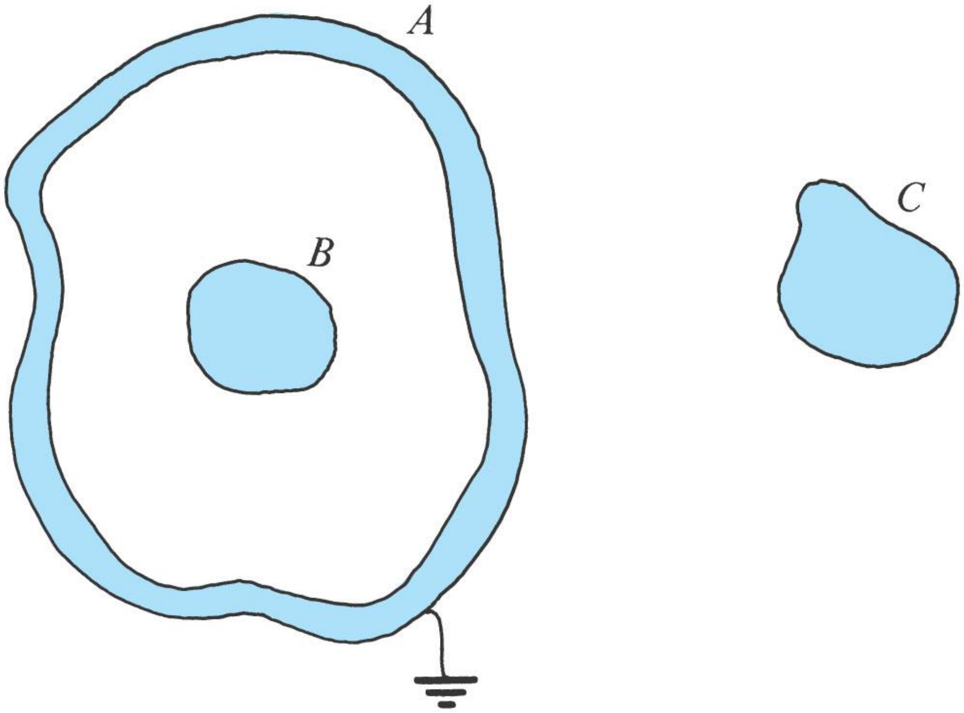
Boundary Conditions

- From the integral form at D-C interface

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0 \qquad\qquad Q = \oint \mathbf{D}.d\mathbf{s}$$

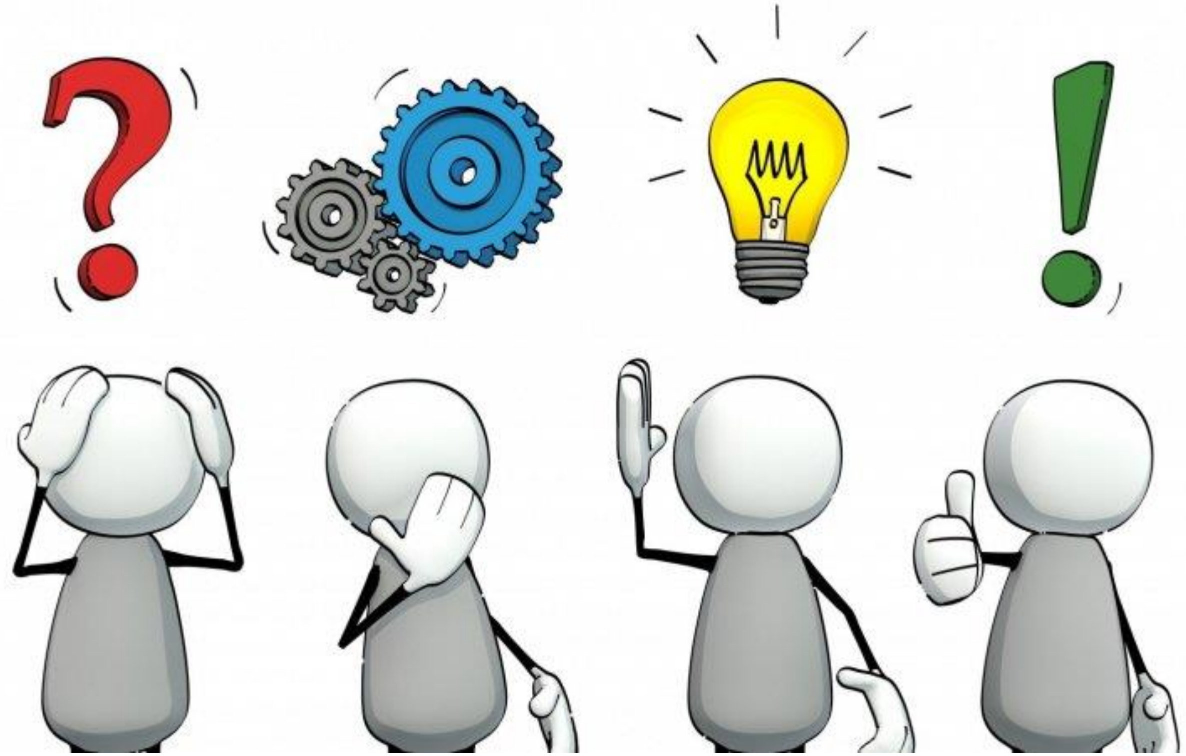


Electrostatic Shielding



- The field of C is shielded and there is no effect of c on B. TV cable?

- **Is there a difference between the field around a charge in free space and the field around a charge in a plasma?**
- **What is the distance required to shield the potential of a charge in a plasma?**
- **Can static charges make a plasma? Hint: Lightning!**
- **Is the plasma quasineutral?**



Thanks!