Lecture (3) Electromagnetic Simple Story

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Introduction

Text Books:

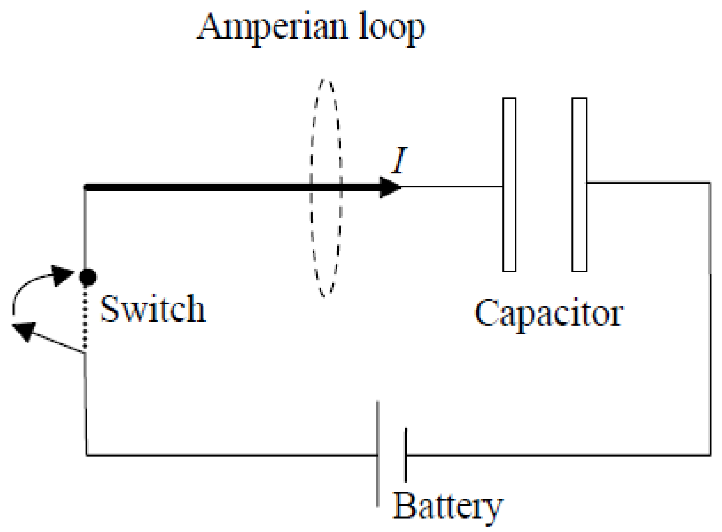
- ***College Physics, Serway / Vuille, Eight Edition, Chapters 15-18.***
- **Sadiku, *Elements of Electromagnetics, Oxford University.***
- ***Griffiths, Introduction to Electrodynamics, Prentice Hall.***
- ***Jackson, Classical Electrodynamics*, New York: John Wiley & Sons.**
- ***Open sources: MIT open courses, ….***

Maxwell's equations

Point Form	*Integral Form*
$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$	$Q/\epsilon = \oint \mathbf{E} \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{B} = 0$	$\psi = \oint \mathbf{B} \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$	$\oint \mathbf{E} \cdot \mathbf{dl} = -\epsilon_{ind}$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$

Displacement current

- *Capacitors block DC currents. No conduction current!!!*
- *Why do A.C. currents pass? Displacement current!*

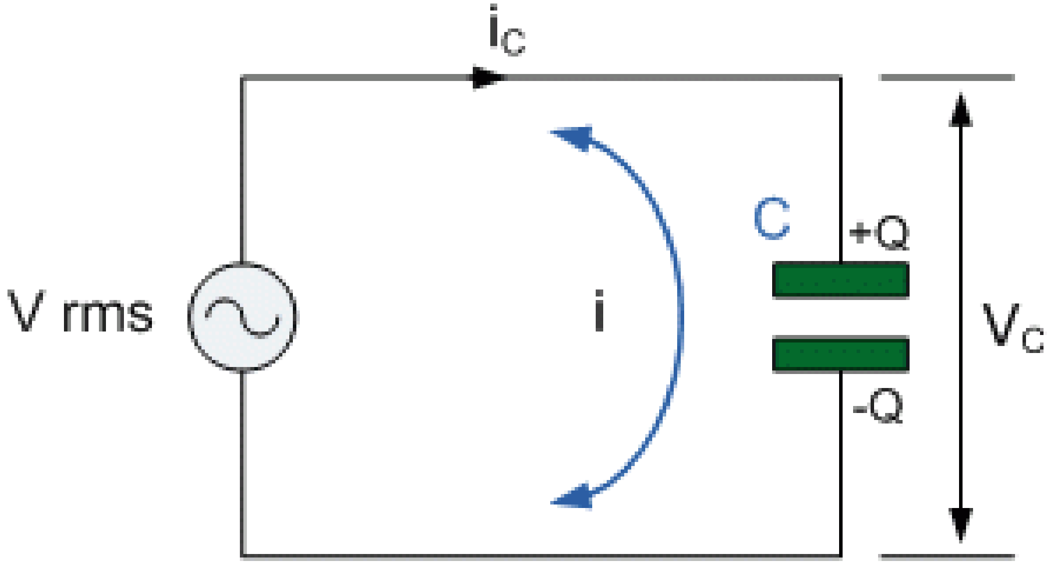


Wave Concept

- *Wave is a perturbation (energy) transfers from point to point. Energy transfers does not main particle propagation.*

Displacement current

- *Displacement current is due to the displacement of electrons.*



- *The displacement current density* $$\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

- **Current Density:** $$\mathbf{J_T} = \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's equations

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$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$	$\oint \mathbf{E} \cdot \mathbf{dl} = -\epsilon_{ind}$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \epsilon_0 \mu_0 \frac{\partial \oint \mathbf{E} \cdot d\mathbf{S}}{\partial t}$

Maxwell's equations in free space

- ***No charges, No conduction current:***

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

- ***The electric field is perpendicular to the magnetic field***

- ***Where is the source? Only a time varying fields***

Combine Curl equations for E

- ***Curl equations:*** $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\qquad$ $\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial \nabla \times \mathbf{B}}{\partial t}$$

$$\nabla \times \nabla \times \mathbf{E} = -\epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$

$$\nabla(\nabla.\mathbf{E}) - \nabla^2 \mathbf{E} = -\epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$

$$\nabla(\cancel{\nabla.\mathbf{E}}) - \nabla^2 \mathbf{E} = -\epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$

$$\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$

Wave equation

- ***Electric field wave equation*** $$\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$
- ***Similar steps:*** $$\nabla^2 \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial^2 t}$$
- ***Propagation: something moves along distance*** $\partial^2/\partial x^2$
- ***Oscillation:*** $\partial^2/\partial t^2$
- ***The constant:*** $$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \mathrm{m/s}$$

Wave equation

- ***Solution:***

$$E = A_1 e^{i(\omega t + kz)} + B_1 e^{i(-\omega t + kz)} + C_1 e^{i(\omega t - kz)} + D_1 e^{i(-\omega t - kz)}$$

- ***Euler equation:***

$$e^{ix} = cos(x) + isin(x)$$

$$e^{-ix} = cos(x) - isin(x)$$

- ***Solution: …….***

$$E = E_0 \cos(\omega t + kz) \quad E = E_0 \sin(\omega t + kz)$$

$$E = E_0 \cos(\omega t - kz) \quad E = E_0 \sin(\omega t - kz)$$

- ***Similar formula for B.***

Wave equation

- ***Example:*** $E = E_0 \cos(\omega t + kz)$
- ***Amplitude:*** E_0
- ***Oscillation frequency:*** $\nu = \omega / 2\pi$
- ***Wavelength:*** $\lambda = 2\pi / k$
- ***Velocity:*** $v = \lambda \nu$ $\quad v = \omega / k$
- ***Group Velocity:*** $v_g = \partial \omega / \partial k$

Example

- ***An electric field in free space is given by***

$$\mathbf{E} = 50\cos(10^8 t + \beta x)\mathbf{a}_y \ \text{V/m}$$

- ***Find the direction of wave propagation***

- ***Calculate*** β ***and the time it takes to travel a distance of a half wavelength***

- ***Sketch the wave at t=0, T/4, T/2.***

Example

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$$\mathbf{E} = 50\cos(10^8 t + \beta x)\mathbf{a}_y \ \ \text{V/m}$$

- ***Find the direction of wave propagation***

$$-\mathbf{a}_x$$

- ***Calculate*** β ***and the time it takes to travel a distance of a half wavelength***

$\beta = 1/3 \text{ rad/m}$ $t = 31.42\text{ns}$

- ***Sketch the wave at t=0, T/4, T/2.***

Example

(c) At $t = 0, \quad E_y = 50 \cos \beta x$

At $t = T/4, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{4\omega} + \beta x\right) = 50 \cos(\beta x + \pi/2)$

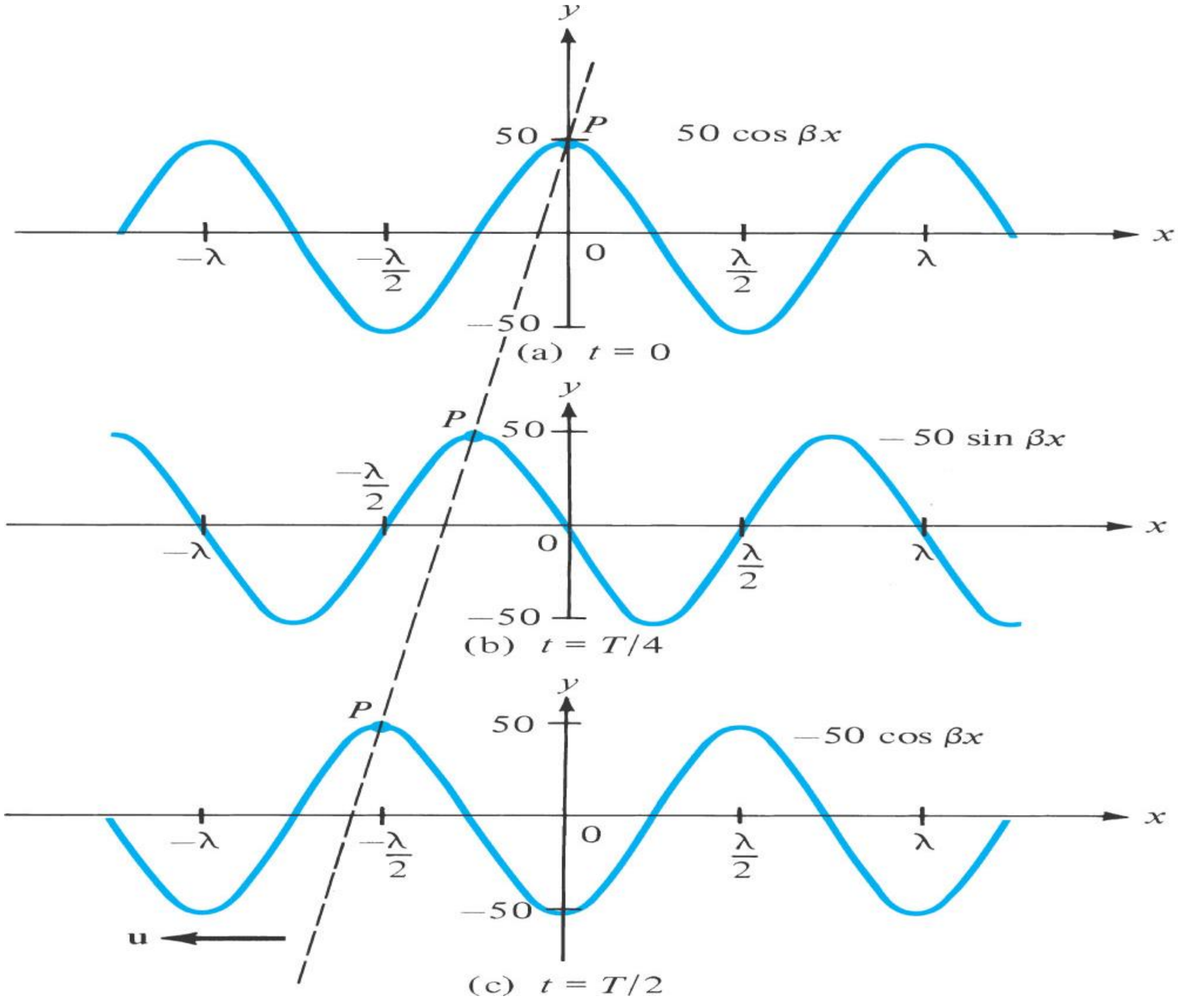
$= -50 \sin \beta x$

At $t = T/2, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{2\omega} + \beta x\right) = 50 \cos(\beta x + \pi)$

$= -50 \cos \beta x$

Example

- ***Sketch***



EM waves in Lossy Dielectrics

- *No charges, No conduction current:*

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

- *Remember:* $\mathbf{J} = \sigma \mathbf{E}$
- *Note: there is energy loss due to Ohmic heating*
- *Note: Energy dissipation in ideal coil and capacitor is zero*

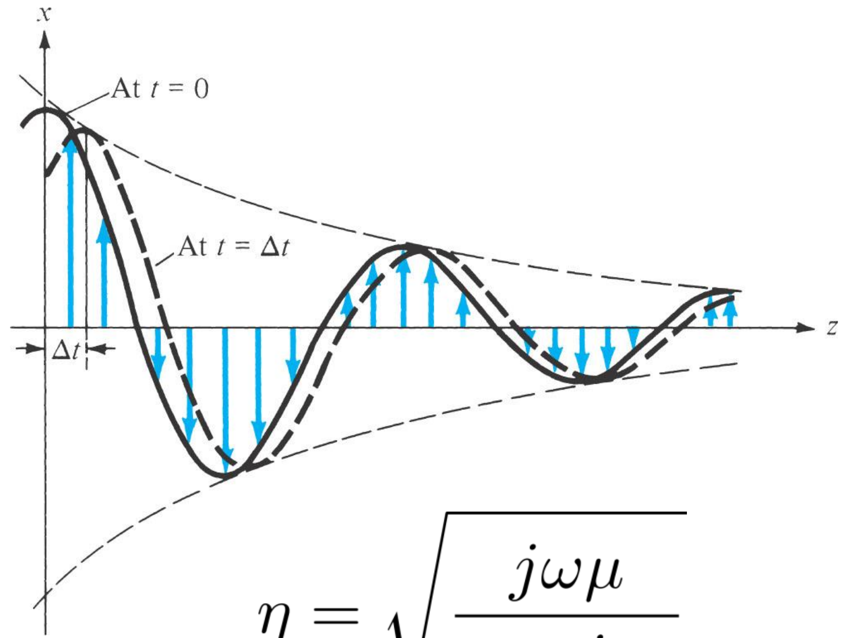
$$Energy = IVt\cos(\phi)$$

- *In lossy Dielectric material, energy damping is expected.*

EM waves in Lossy Dielectrics

- ***Solution:***

$$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$



$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1]}$$

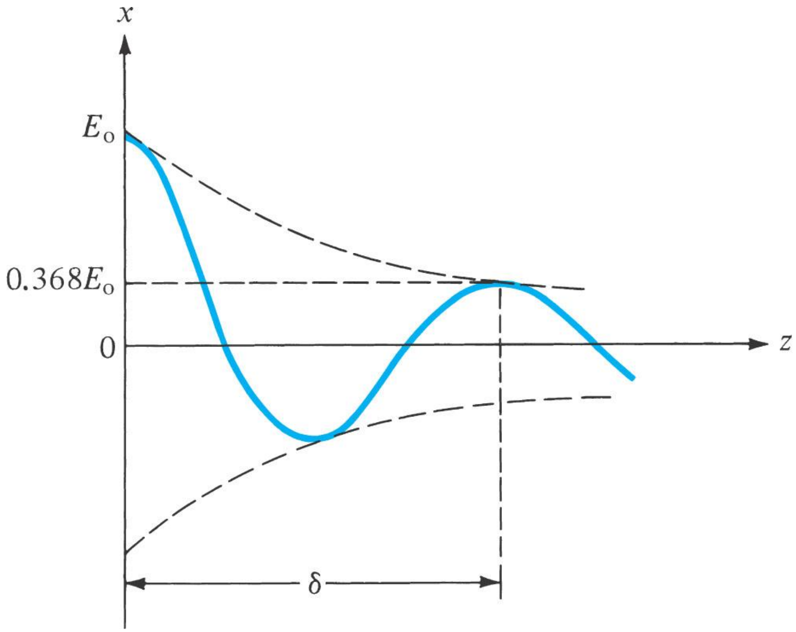
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\text{skin depth} = \frac{1}{\alpha}$$

$$\mathbf{H}(z,t) = \frac{E_0}{\eta} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_y$$

EM waves in Conductors

- ***Solution:***



$$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi\nu\mu\sigma}$$

$$\mathbf{H}(z,t) = \frac{E_0}{\eta} e^{-\alpha z}\cos(\omega t - \beta z - 45\,\mathrm{deg})\mathbf{a}_y$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} \qquad \text{skin depth} = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi\nu\mu\sigma}}$$

EM waves in Conductors

- ***Skin depth of Copper: The skin depth decreases by increasing the frequency***

Frequency (Hz)	10	60	100	500	10^4	10^8	10^{10}
Skin depth (mm)	20.8	8.6	6.6	2.99	0.66	6.6×10^{-3}	6.6×10^{-4}

- ***The diameter of High frequency signal cables?***

- **The Wi-Fi radio in your Network Box operates on the following frequencies and protocols: Models GFRG100 and GFRG110:5 GHz: 802.11a/n**
- https://support.google.com/fiber/answer/2732316?hl=en

Plasma Generation

- ***General notes:***

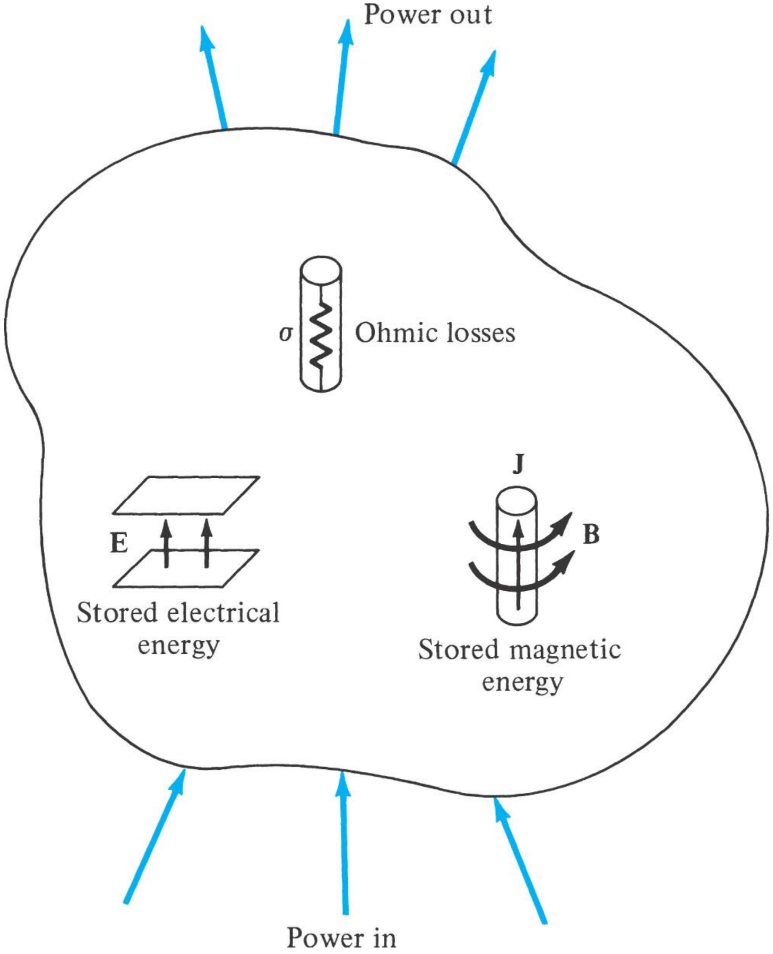
- ***Microwave can produce plasma, but when the plasma density becomes large enough the microwave can not propagate into the plasma bulk. The energy is deposited on the plasma surface. In some situation the energy is given as a surface wave.***

- ***Note:*** $k^2c^2 = \omega^2 - \omega_e^2$ $\delta = 1/\alpha = c/\omega_e$

- ***Similar situation is the production of plasma via intense optical lasers.***

- ***Plasma mirror! When does the plasma work as a mirror?***

Energy Loss


Power out
σ
Ohmic losses
J
B
E
Stored electrical
energy
Stored magnetic
energy
Power in

Energy Loss

- ***Using Maxwell´s equations***

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

- ***By Dotting by E,***

$$\mathbf{E} \cdot \nabla \times \mathbf{B} = \mathbf{E} \cdot \mu_0 \mathbf{J} + \mathbf{E} \cdot \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

- ***Note:*** $\mathbf{J} = \sigma \mathbf{E}$ $\quad$ $\mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t}$

- ***Then***

$$\mathbf{E} \cdot \nabla \times \mathbf{B} = \mu_0 \sigma E^2 + \frac{1}{2} \epsilon_0 \mu_0 \frac{\partial E^2}{\partial t}$$

Energy Loss

$$\mathbf{E}\cdot\nabla\times\mathbf{B}=\mu_0\sigma E^2+\frac{1}{2}\epsilon_0\mu_0\frac{\partial E^2}{\partial t}$$

- ***But***

$$\mathbf{E}\cdot\nabla\times\mathbf{B}=\mathbf{B}\cdot\nabla\times\mathbf{E}-\nabla\cdot\mathbf{E}\times\mathbf{B}$$

- ***Then***

$$\mathbf{B}\cdot\nabla\times\mathbf{E}-\nabla\cdot\mathbf{E}\times\mathbf{B}=\mu_0\sigma E^2+\frac{1}{2}\epsilon_0\mu_0\frac{\partial E^2}{\partial t}$$

- ***Note:***

$$\nabla\times\mathbf{E}=-\frac{d\mathbf{B}}{dt}$$

Energy Loss

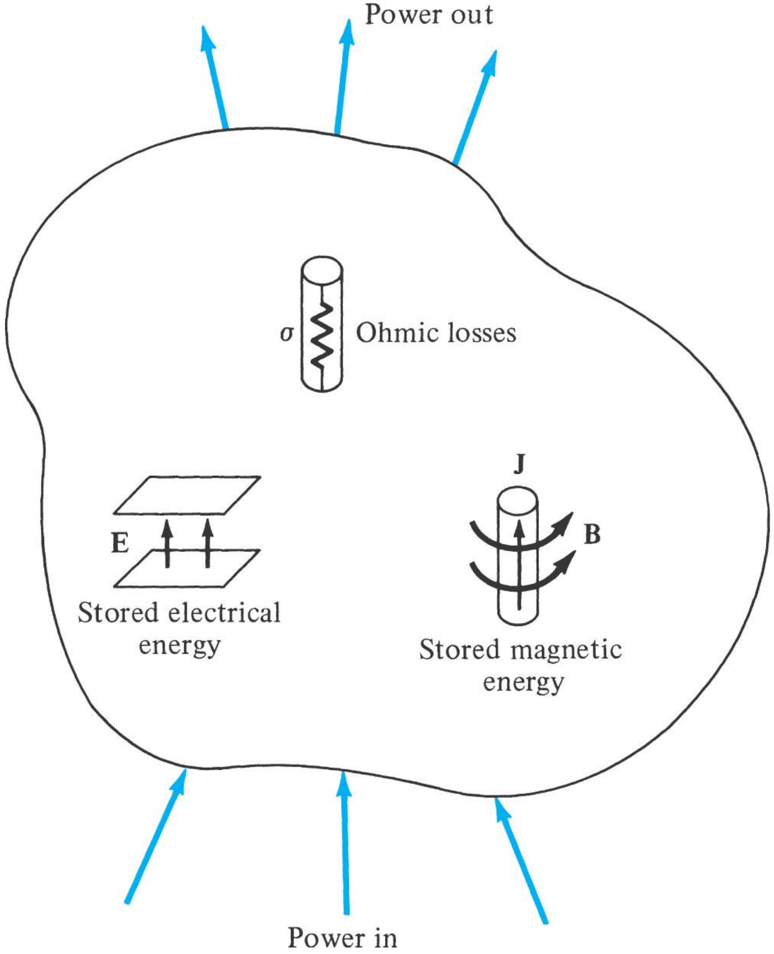
$$(\mathbf{B}\cdot-\frac{\partial\mathbf{B}}{\partial t})-\nabla\cdot\mathbf{E}\times\mathbf{B}=\mu_0\sigma E^2+\frac{1}{2}\epsilon_0\mu_0\frac{\partial E^2}{\partial t}$$

$$(\frac{-1}{2}\frac{\partial B^2}{\partial t})-\nabla\cdot\mathbf{E}\times\mathbf{B}=\mu_0\sigma E^2+\frac{1}{2}\epsilon_0\mu_0\frac{\partial E^2}{\partial t}$$

$$-\nabla\cdot\mathbf{E}\times\mathbf{B}=\mu_0\sigma E^2+\frac{1}{2}\epsilon_0\mu_0\frac{\partial E^2}{\partial t}+(\frac{1}{2}\frac{\partial B^2}{\partial t})$$

- ***Energy loss = Ohmic heat + (Electric +Magnetic) energies stored***
- ***Poynting Vector*** $\mathbf{E}\times\frac{\mathbf{B}}{\mu 0}$ ***is measured in W/m^2***

RLC Circuits

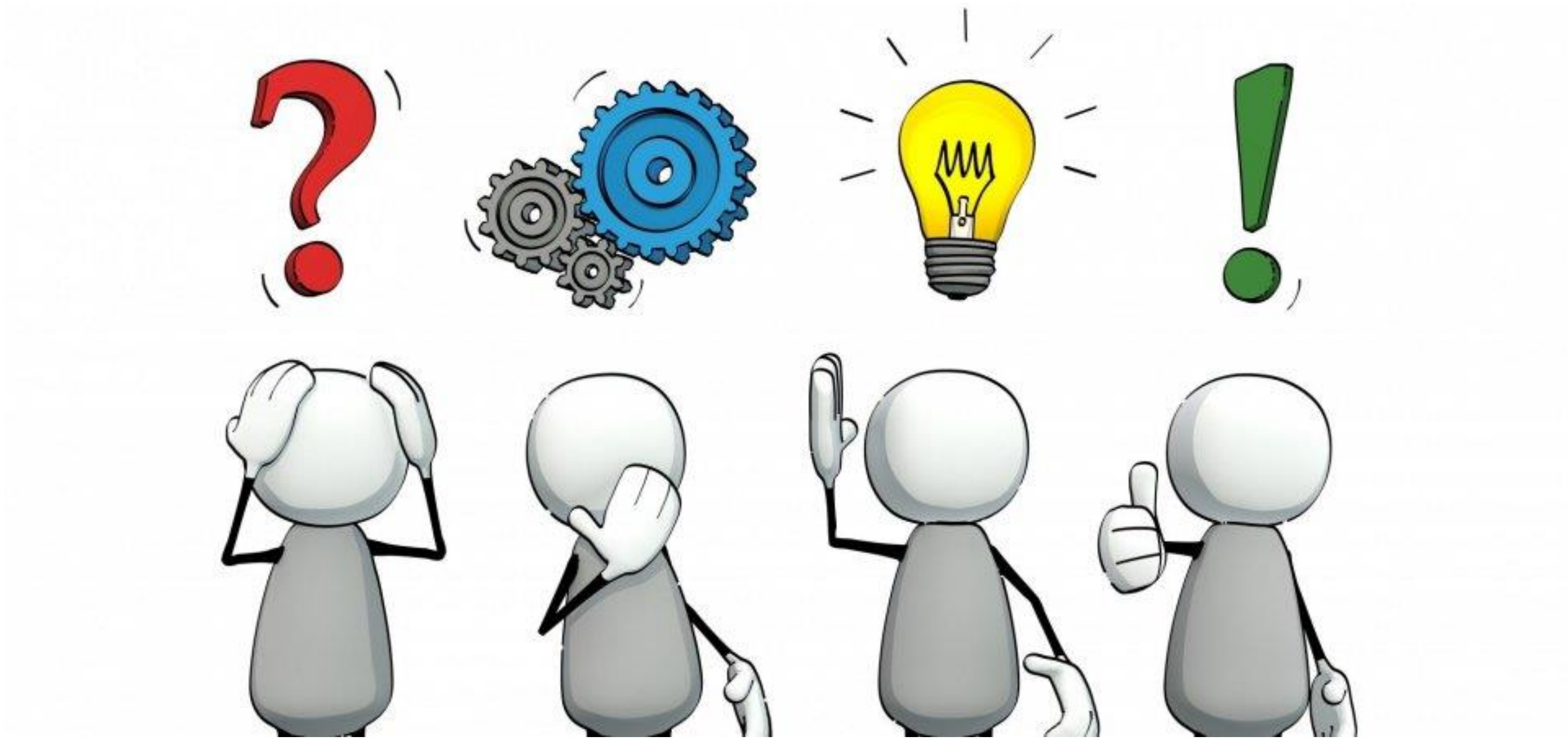


Open Discussion

- ***Success : Reflection + Refraction + Interference ….***

- ***Failure: Photo electric effect + Compton scattering due to particle nature of EM waves***

- ***Relativity: Last Chapter in Griffiths Book***



Thanks!