Nonlinear ion-acoustic waves at Venus ionosphere

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Outline

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- *Ion-acoustic waves (IAWs) in ionosphere of Venus*
- *Aim of the work*
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- *Large amplitude IAWs*
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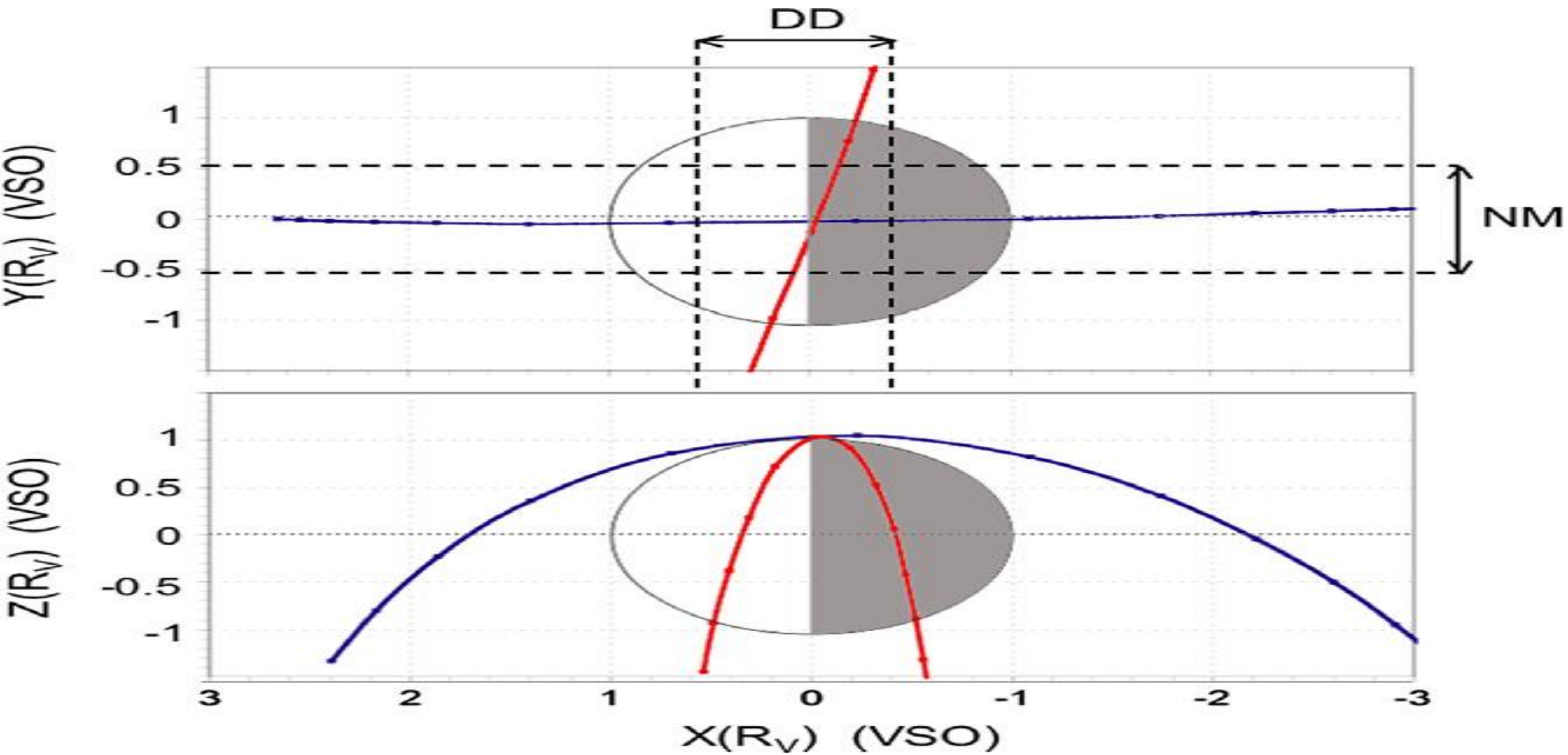
Introduction

Venus, unlike Earth, has no an intrinsic planetary magnetic field. Therefore, the solar wind interacts directly with upper atmosphere, so it's ionosphere is formed due to this interaction. This property is of essential significance in determining the characteristics of the plasma wave environment of this planet

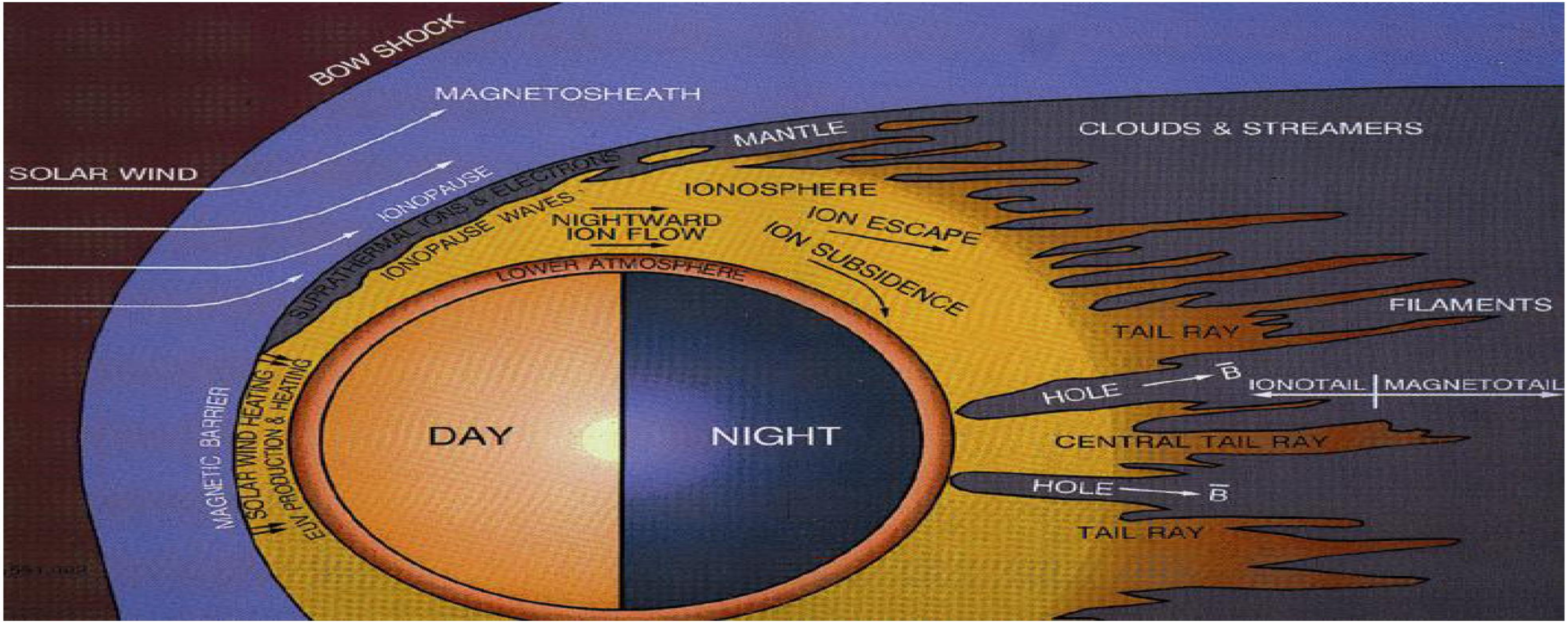
several spacecraft's had visited Venus. The most important spacecraft's which continuously investigated Venus over a time range of several years, namely the Pioneer Venus Orbiter (PVO) and Venus Express (VEX)

Two VEX orbits : Noon Midnight and Dawn-Dusk



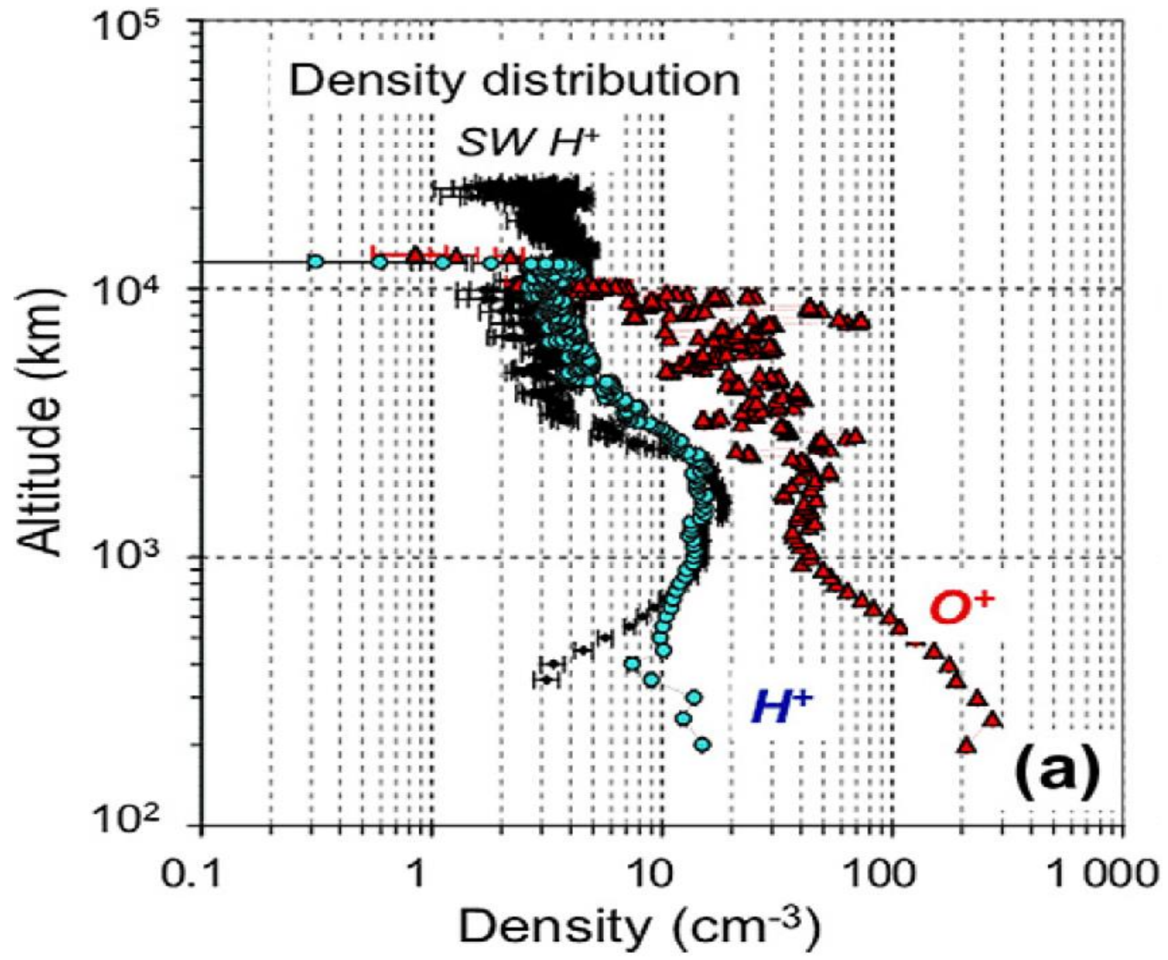
Plasma environment of Venus

A sketch of the most important plasma boundaries and interaction regions in the environment of Venus

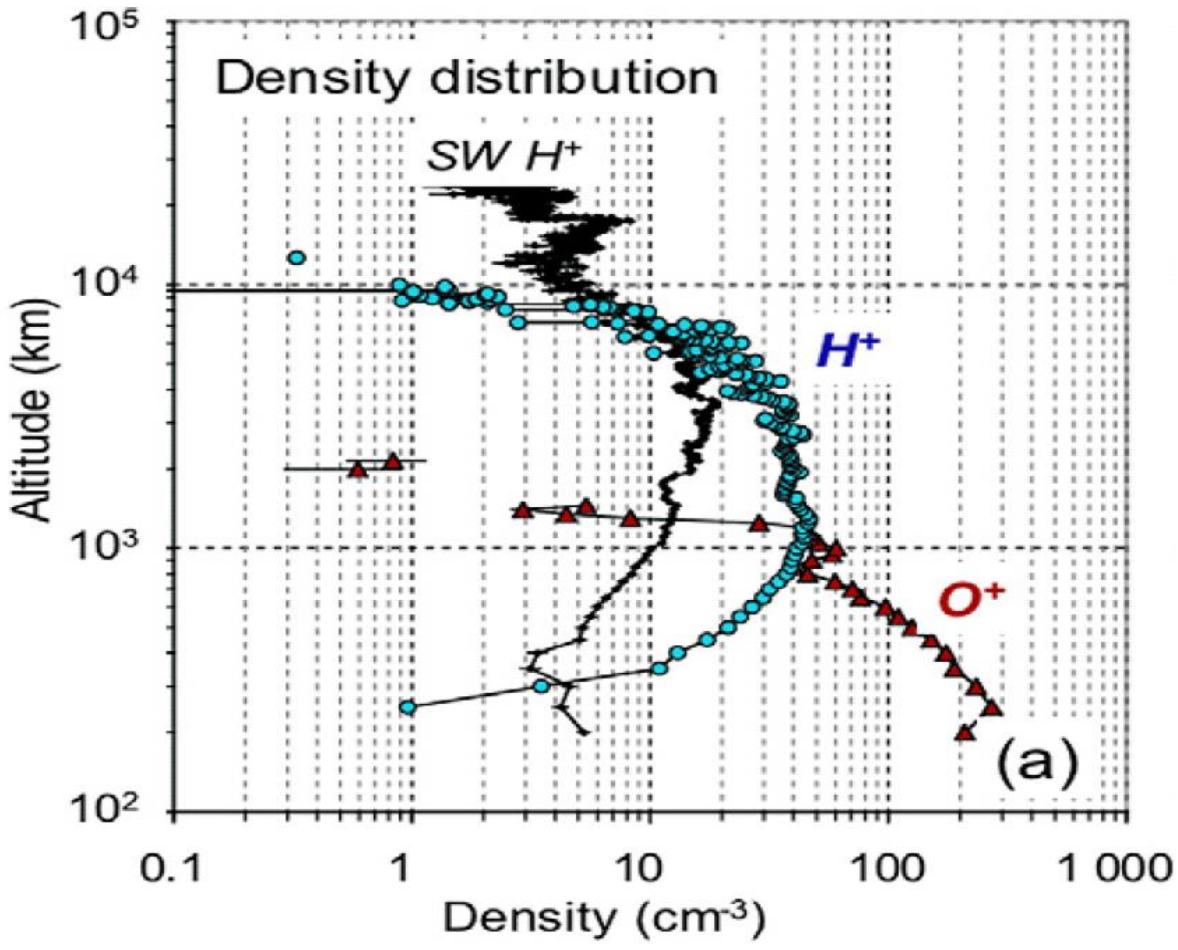


Plasma environment of Venus

Noon–Midnight meridian



Dawn–Dusk meridian



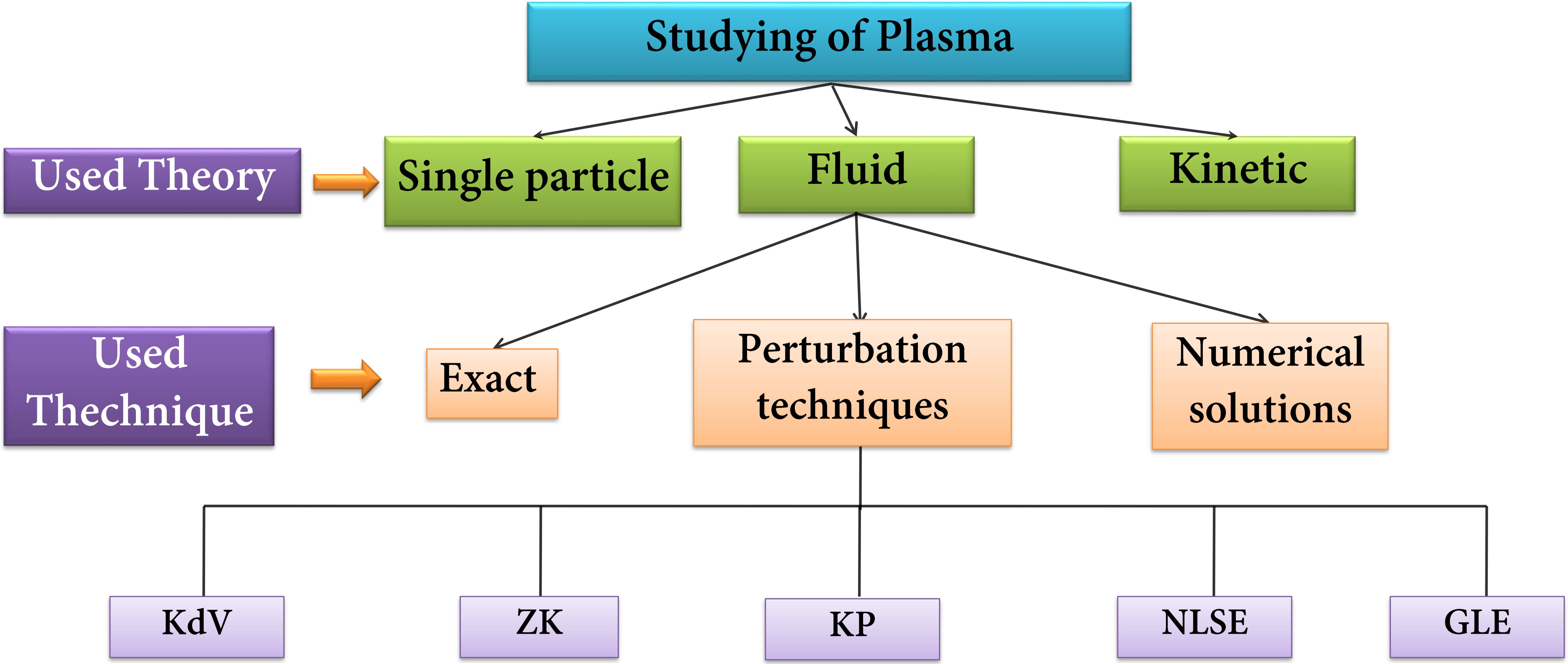
Plasma waves in ionosphere of Venus

Various ionospheric parameters such as plasma density, temperature, and the magnetic field exhibit wavelike structures in the ionosphere of Venus. These structures were originally identified as whistler-mode, but were subsequently identified as ion-acoustic waves (IAWs)

Ion acoustic waves are the low-frequency longitudinal plasma density oscillations, where electrons and ions are propagating in the phase space.

Among nonlinear structures, the ion-acoustic solitary waves (IASWs) which arise due to delicate nonlinear wave structures arise due to the balance of nonlinearity and dispersion.

Plasma models



Aim of the work

investigating the propagation conditions of the nonlinear IAWs in a plasma environment of Venus using different techniques based on the amplitude and propagating waves.

For this purpose, we use the observed data for the Noon-Midnight (NM) meridian of VEX to get the numerical values of the physical parameters in the core of ionospheric region at (altitude 100~1000 km)

Weakly nonlinear analysis for small- but finite amplitude IAWs

Fully nonlinear analysis for large amplitude IAWs

Model equations

The basic equations, based on fluid description, for a collisionless, homogeneous, unmagnetized multi-component plasma consisting of two positive ions and Maxwellian electrons are as follow for positive H^+ and O^+ ions respectively.

Continuity Equations

$$\frac{\partial n_H}{\partial t} + \frac{\partial}{\partial x}(n_H u_H) = 0$$

$$\frac{\partial n_O}{\partial t} + \frac{\partial}{\partial x}(n_O u_O) = 0$$

n_i Density of ions
u_i Velocity of ions
m_i Mass of ions
T_i Temperature of ions
P_i pressure of ions
Index i refer to H, O

Equations of Motion

$$m_H n_H \left(\frac{\partial u_H}{\partial t} + u_H \frac{\partial u_H}{\partial x}\right) = -e n_H \frac{\partial \phi}{\partial x} - \frac{\partial P_H}{\partial x}$$

$$m_O n_O \left(\frac{\partial u_O}{\partial t} + u_O \frac{\partial u_O}{\partial x}\right) = -e n_O \frac{\partial \phi}{\partial x} - \frac{\partial P_O}{\partial x}$$

Model equations

The Boltzmann distributed electrons (n_e)

$$n_e = n_{eo} e^{(\frac{e\phi}{k_B T_e})}$$

where

- k_B Boltzmann constant
- T_e electron temperature
- e electron charge
- ϕ electrostatic potential
- ϵ_o free space permittivity

Poisson's equation

$$\frac{\partial^2 \phi}{\partial^2 x} = \frac{e}{\epsilon_o}(n_e - n_H - n_O)$$

A soliton is a solitary wave which maintains its identity after it collides with another wave of the same type. A wave equation having soliton solutions has both nonlinearity and dispersion.

Small but finite amplitude IAWs

We firstly write the basic equations in dimensionless form as

Normalized Basic Equations

Continuity Equations

$$\frac{\partial n_H}{\partial t} + \frac{\partial}{\partial x}(n_H u_H) = 0$$

$$\frac{\partial n_O}{\partial t} + \frac{\partial}{\partial x}(n_O u_O) = 0$$

Equations of Motion

$$\frac{\partial u_H}{\partial t} + u_H \frac{\partial u_H}{\partial x} + 3\sigma_1 n_H \frac{\partial u_H}{\partial x} + \frac{\partial \phi}{\partial x} = 0$$

$$\frac{\partial u_O}{\partial t} + u_O \frac{\partial u_O}{\partial x} + \frac{3\sigma_2}{\mu} n_H \frac{\partial u_H}{\partial x} + \frac{1}{\mu}\frac{\partial \phi}{\partial x} = 0$$

Here, $\mu = m_O/m_H$, $\sigma_1 = T_H/T_e$ and $\sigma_2 = T_O/T_e$ where $m_O(m_H)$ and $T_O(T_H)$ are the mass and the temperature of the ions while T_e is the electron temperature.

Small but finite amplitude IAWs

Where n_H, n_O are the normalized ion number density by the unperturbed number density n_{H0}, n_{O0} respectively. $u_{H(O)}$ is the normalized ion fluid velocity by the ion acoustic speed $c_s = (k_B T_e/m_H)^{1/2}$. The time and space variables are normalized by the ion plasma period $\omega_{pi}^{-1} = (\epsilon_0 m_H/n_{H0} e^2)^{1/2}$ and the ion Debye length $\lambda_{Di} = (\epsilon_0 k_B T_e/n_{H0} e^2)^{1/2}$, respectively.

Normalized Boltzmann distribution

$$n_e = exp[\phi]$$

Normalized Poisson Equation

$$\frac{\partial^2 \phi}{\partial^2 x} = \alpha n_e - \gamma n_O - n_H$$

n_e is the normalized electron density by the equilibrium value and ϕ is the normalized electrostatic potential by $k_B T_e/e$ where $\gamma = n_{O0}/n_{H0}$, $\alpha = n_{e0}/n_{H0}$.

Small but finite amplitude IAWs

The stretched space-time coordinate

$$\xi = \varepsilon^{1/2}(x - \lambda t)$$

$$\tau = \varepsilon^{3/2} t$$

*

Where ε is a smallness parameter measuring the weakness of the amplitude or dispersion and λ is the soliton phase speed normalized by c_s

The dependent variables $n_{H(O)}$, $u_{H(O)}$, and ϕ can be expanded about their equilibrium values in power series of ε as

$$n_H = 1 + \varepsilon n_H^{(1)} + \varepsilon n_H^{(2)}$$

$$n_O = 1 + \varepsilon n_O^{(1)} + \varepsilon n_O^{(2)}$$

$$u_H = \varepsilon u_H^{(1)} + \varepsilon u_H^{(2)}$$

$$u_O = \varepsilon u_O^{(1)} + \varepsilon u_O^{(2)}$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon \phi^{(2)}$$

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Small but finite amplitude IAWs

The lowest–order equations in ε gives

$$\alpha = \gamma + 1$$

The next–order in ε provides the compatibility condition as

$$\alpha = \frac{\gamma}{\mu(\lambda^2 - 3\sigma_2/\mu)} + \frac{1}{(\lambda^2 - 3\sigma_1)}$$

Solving this equation to get the roots of phase velocity λ

The higher–order in ε order gives

$$\frac{\partial^2 \phi^{(2)}}{\partial^2 \xi} = \alpha n_e^{(2)} - \gamma n_O^{(2)} - n_H^{(2)}$$

Small but finite amplitude IAWs

The Korteweg- de Vries (KdV) equation which describe the nonlinear propagation of the IASWs waves in the final form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + AB\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{1}{2} A \frac{\partial^3 \phi^{(1)}}{\partial^3 \xi} = 0$$

The second term represent *nonlinearity term* and the third term represent *dispersion term*.

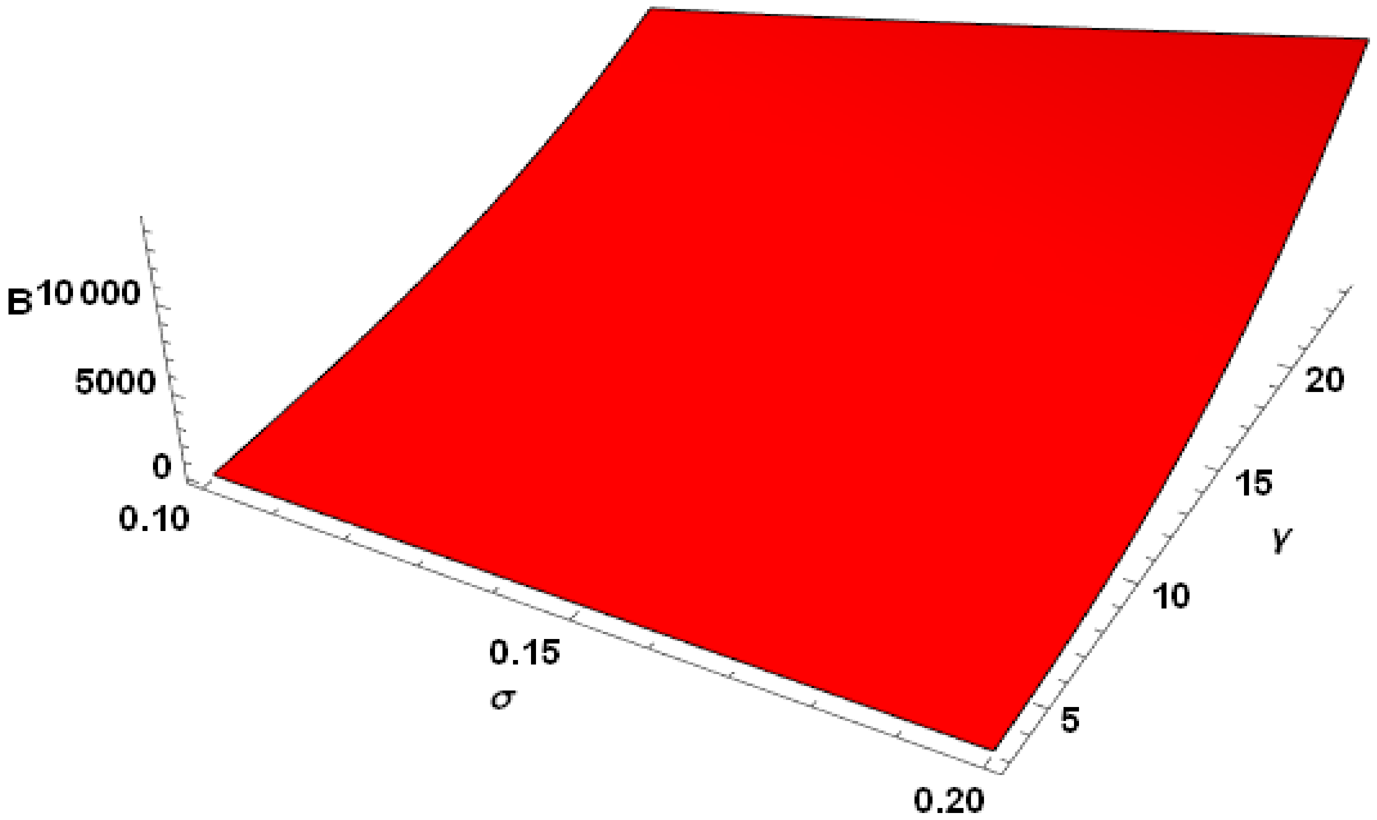
Where The coefficients A, B respectively

$$A = \left[\frac{\gamma\lambda}{(\lambda^2 - 3\sigma_2/\mu)^2} + \frac{\lambda}{(\lambda^2 - 3\sigma_1)^2}\right]^{-1}$$

$$B = \frac{1}{2}\left[\frac{3\gamma\left(\lambda^2 + \frac{\sigma_2}{\mu}\right) - \gamma\mu\left(\lambda^2 - \frac{3\sigma_2}{\mu}\right)^2}{\mu^2\left(\lambda^2 - \frac{3\sigma_2}{\mu}\right)^3} + \frac{3(\lambda^2 + \sigma_1) - (\lambda^2 - 3\sigma_1)^2}{(\lambda^2 - 3\sigma_1)^3}\right]$$

Small but finite amplitude IAWs

The sign of B could be either positive or negative depending on the plasma parameters.



This figure represents the variation of nonlinearity coefficient B versus γ and σ. It is clear that B is always positive for the given plasma system.

Small but finite amplitude IAWs

localized soliton solution of KdV equation

$$\chi = \xi - U\tau'$$

$$\tau' = \tau$$

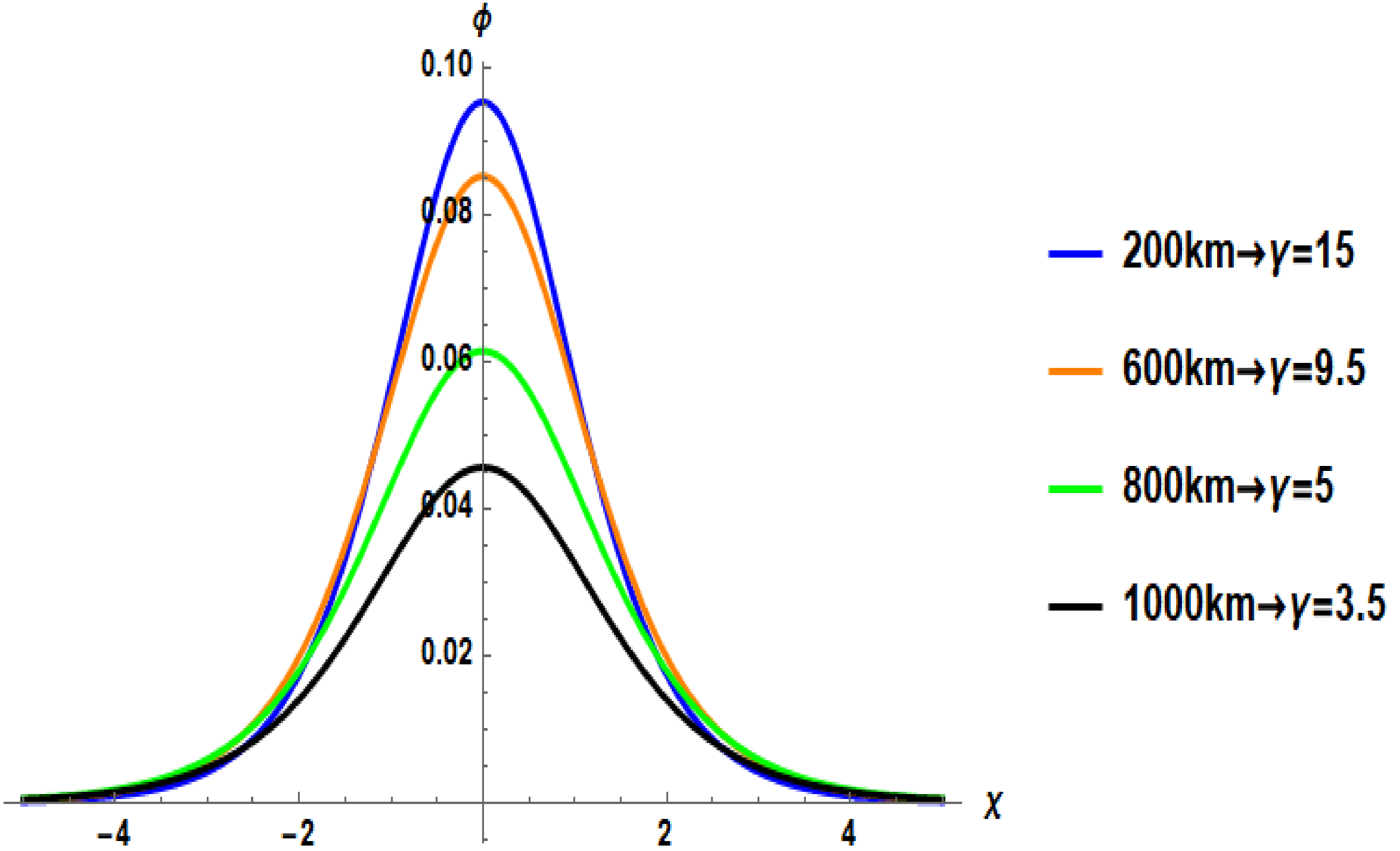
The boundary conditions $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\xi \to 0$, $d^2\phi^{(1)}/d\xi^2 \to 0$, The Solution is given by

$$\phi = \phi_m \operatorname{sech}^2\left(\frac{\chi}{W}\right)$$

where $\phi_m = 3U/AB$ and $W = \sqrt{2A/U}$ are the maximum amplitude and the width of the localized pulse, respectively, and χ is the transformed coordinate with respect to a frame moving with constant speed U.

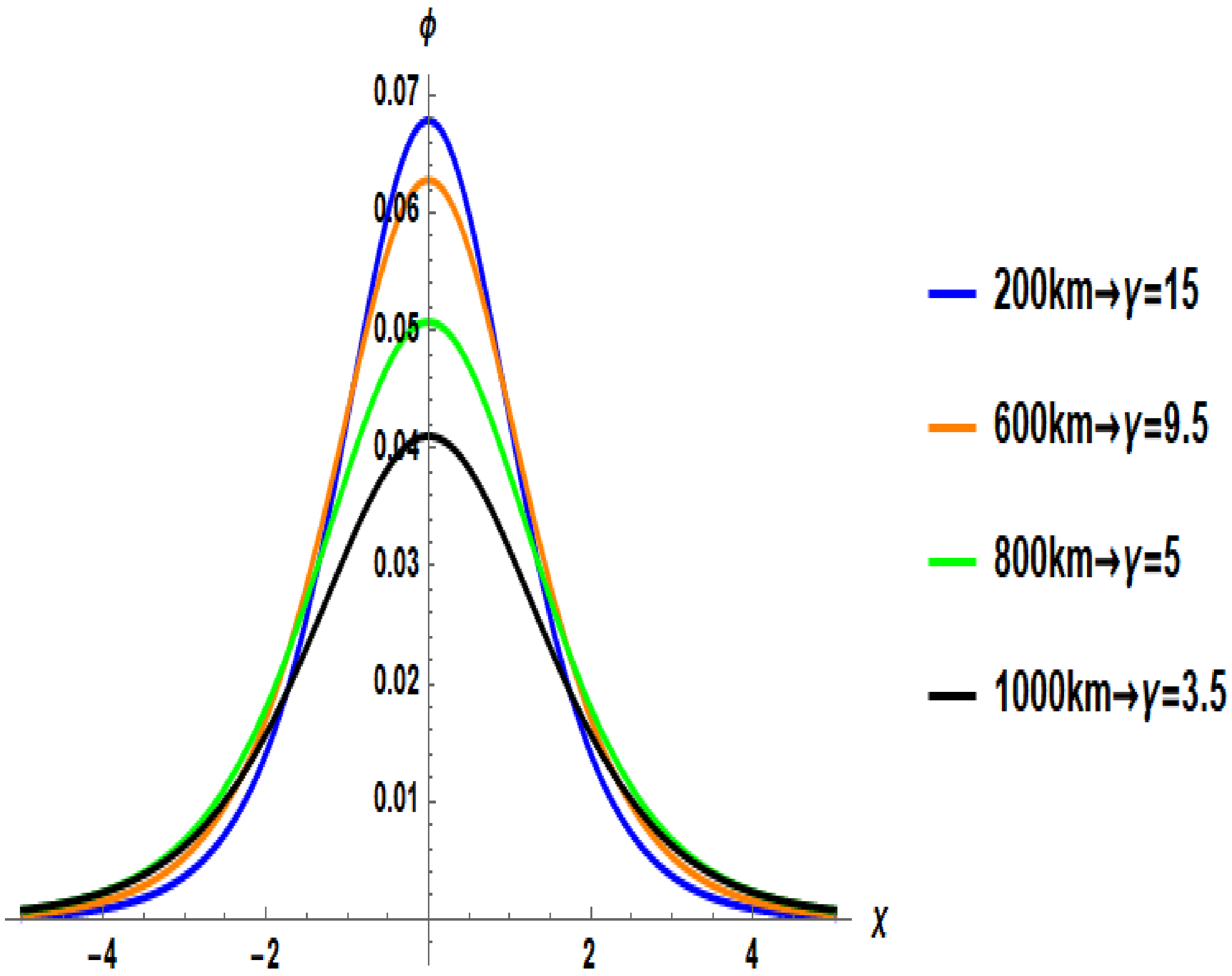
Small but finite amplitude IAWs

The effect of density ratio γ and temperature ratio σ ratios on the profiles of the IASWs for the two positive roots of phase velocity λ_1 and λ_2

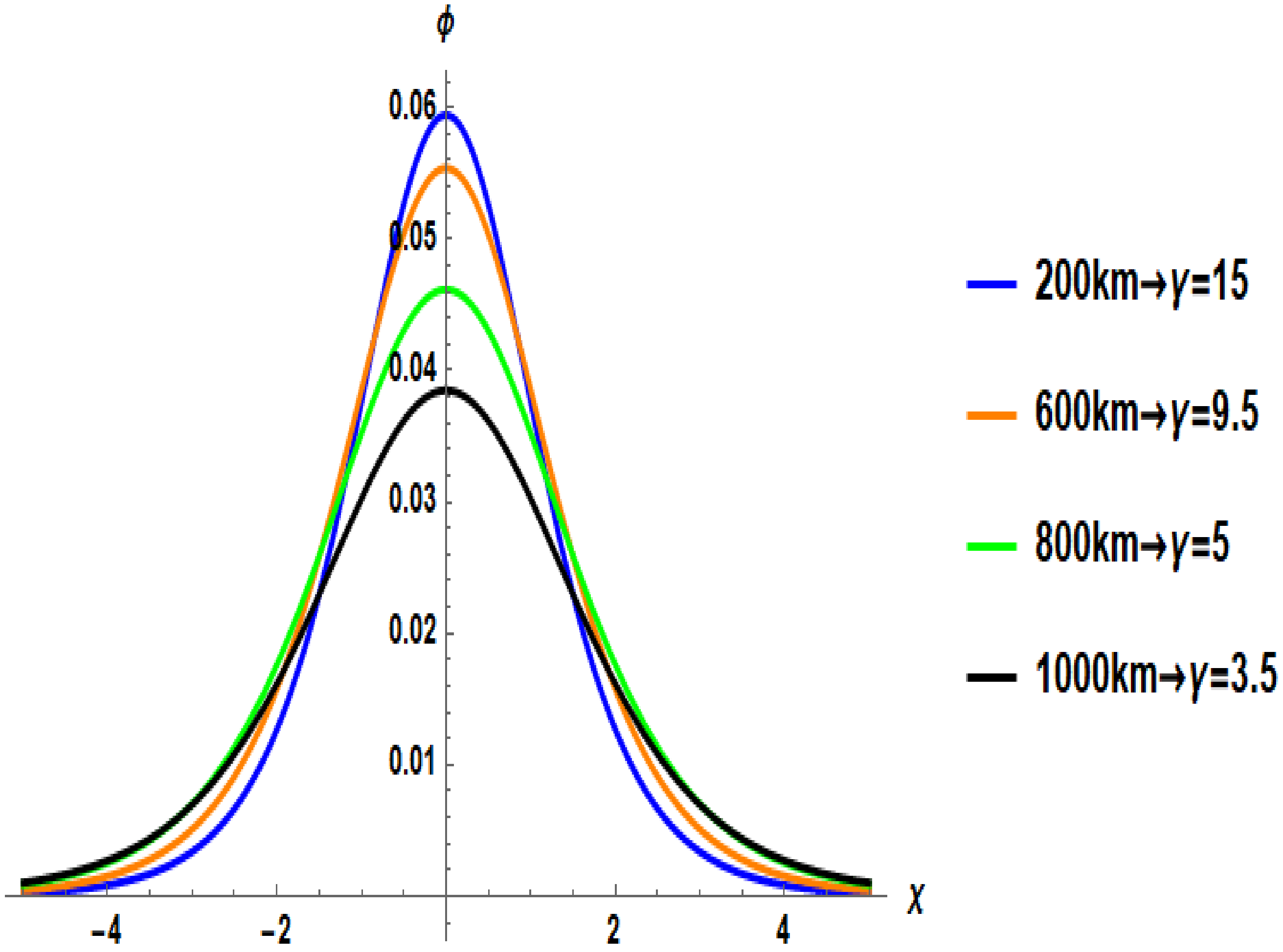


Solitary potential profiles for phase velocity λ_1 versus χ at different values of γ for $\sigma = 0.1$

Small but finite amplitude IAWs



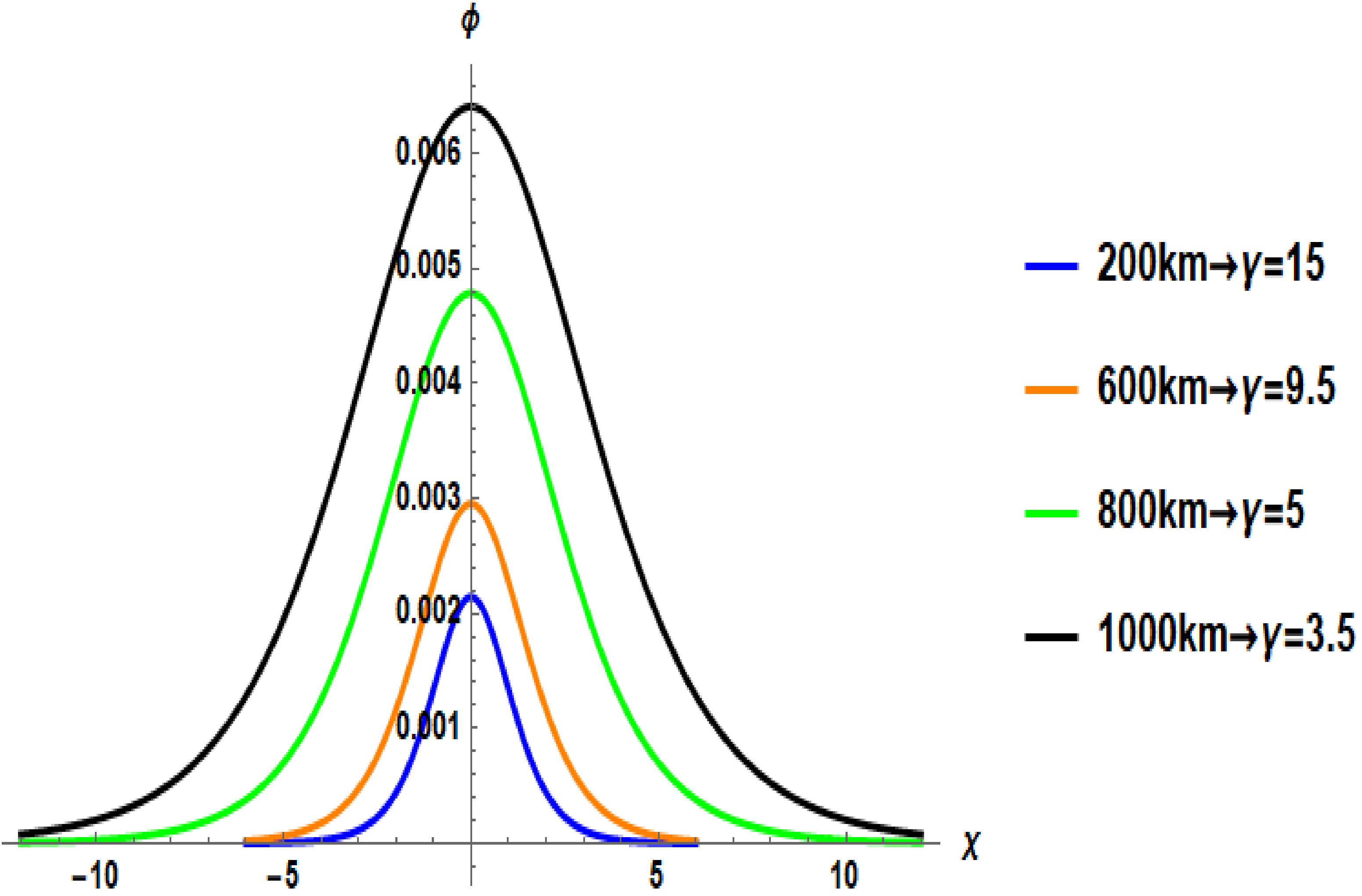
Solitary potential profiles versus χ at different values of γ for $\sigma = 0.15$



Solitary potential profiles versus χ at different values of γ for $\sigma = 0.2$

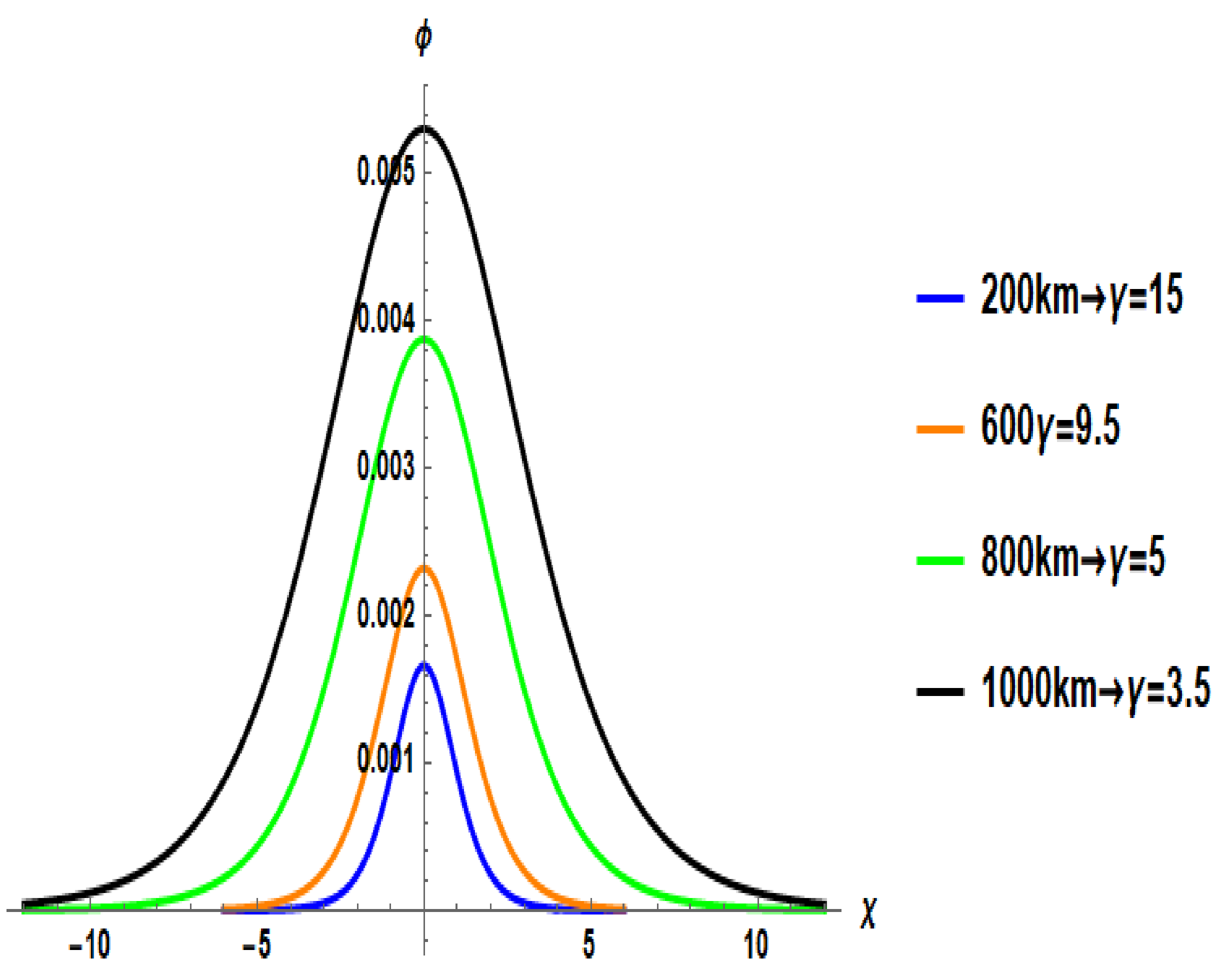
Small but finite amplitude IAWs

And for phase velocity λ_2

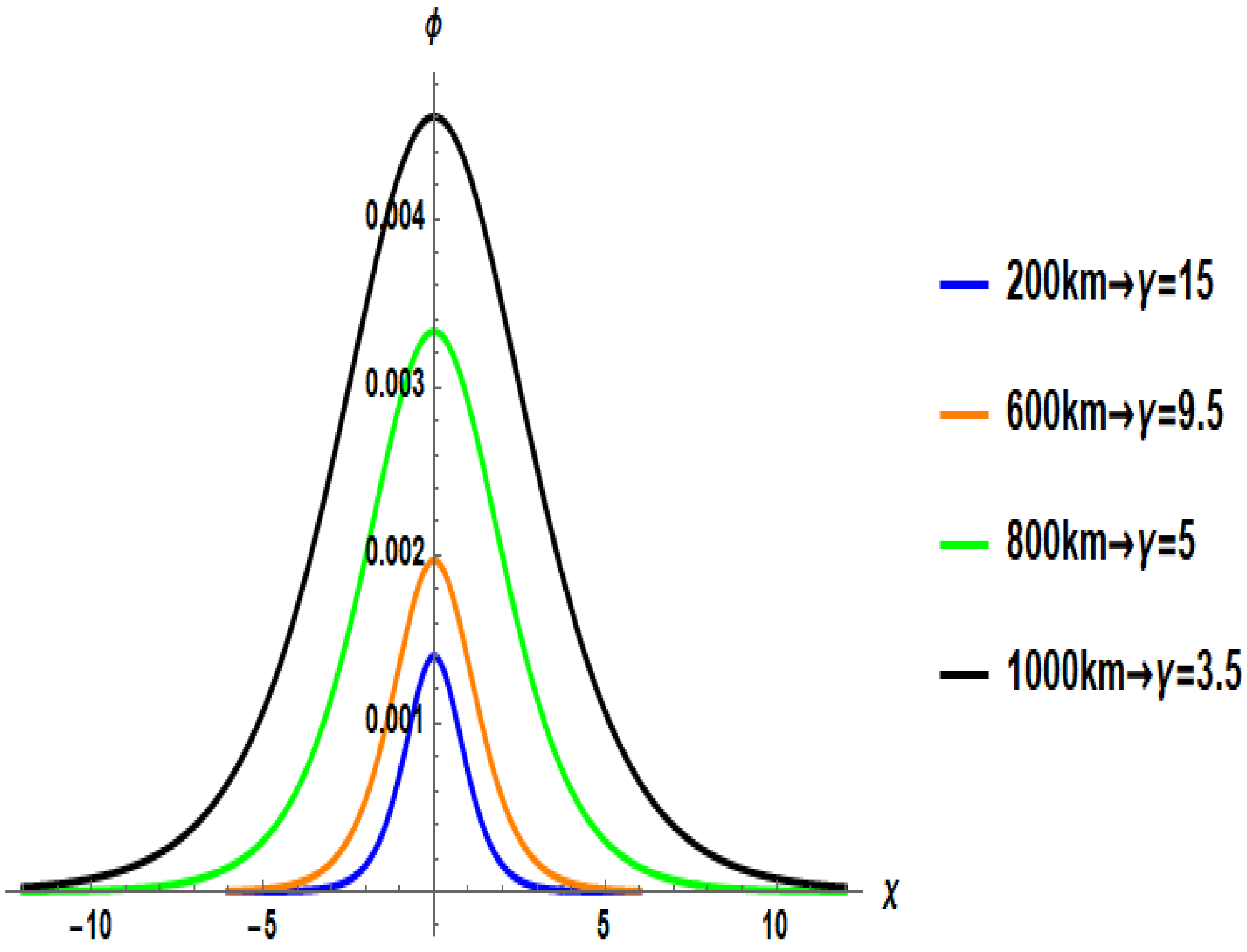


Solitary potential profiles for phase velocity λ_2 versus χ at different values of γ for $\sigma = 0.1$

Small but finite amplitude IAWs



Solitary potential profiles versus χ at different values of γ for $\sigma = 0.15$

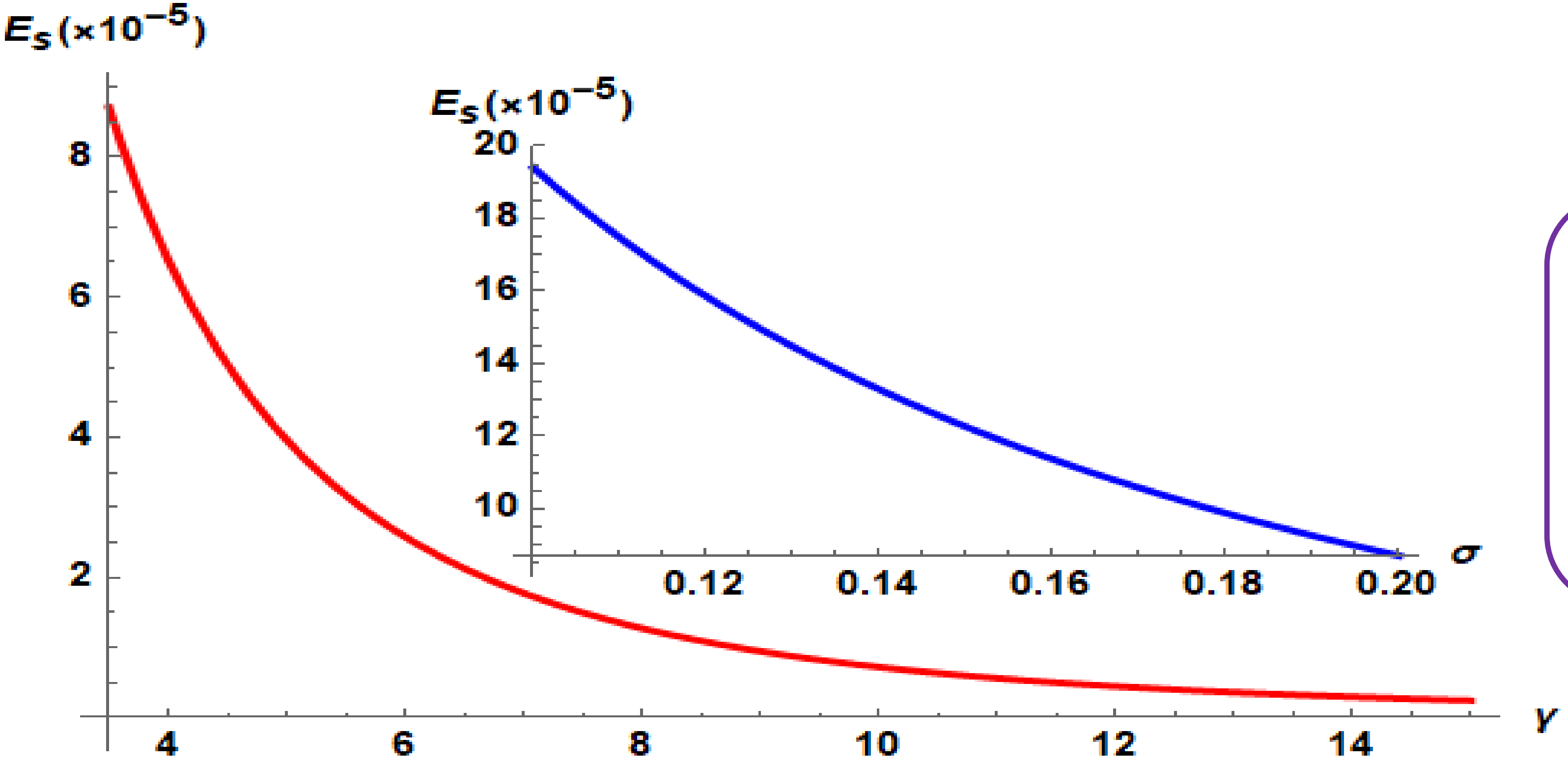


Solitary potential profiles versus χ at different values of γ for $\sigma = 0.2$

Small but finite amplitude IAWs

The soliton solution is the toolbox that we use to obtain the soliton energy

$$E_s = \frac{4}{3}\phi_m^2 W$$

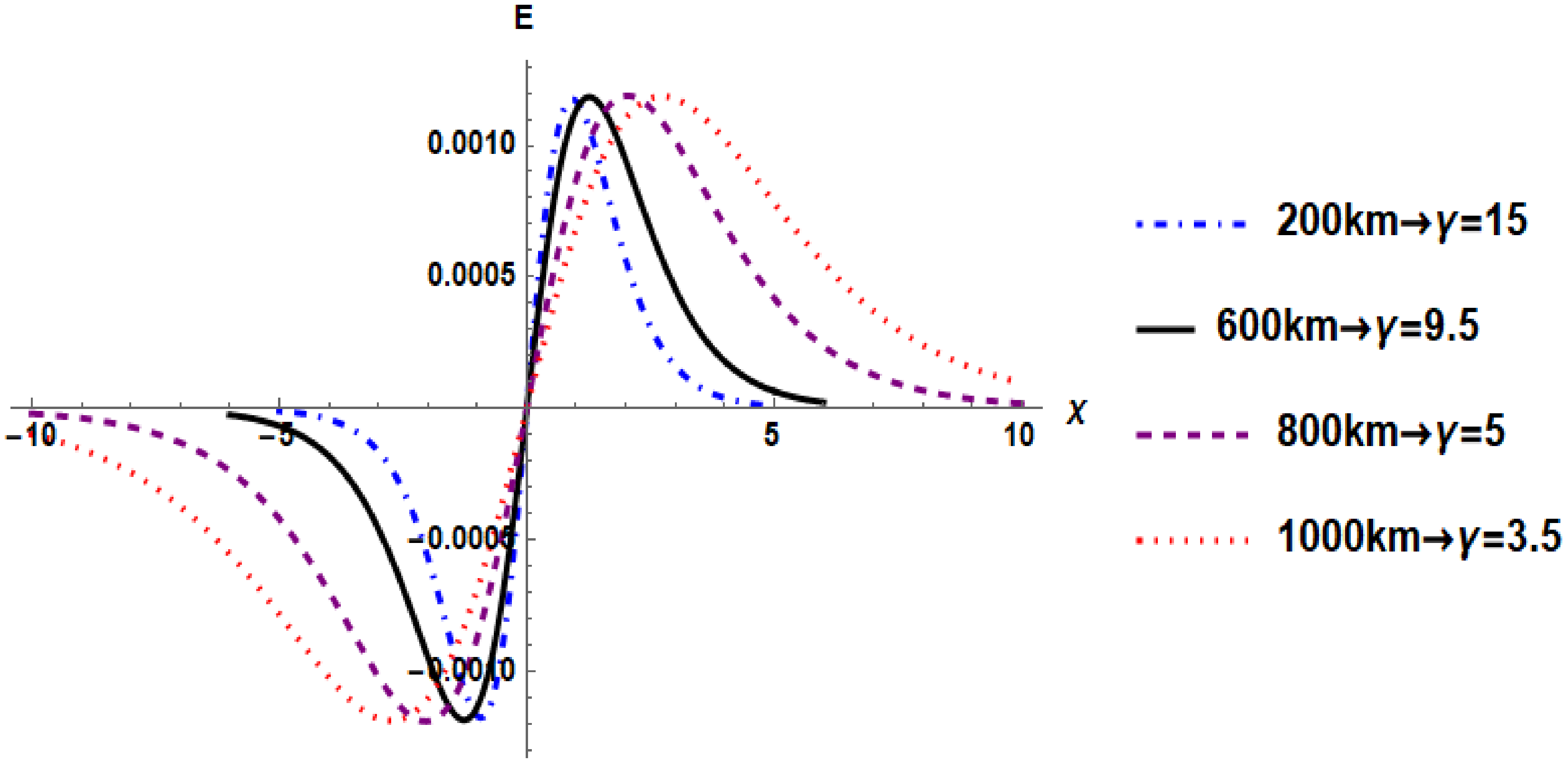


This figure represent the soliton energy against the density ratio γ and temperature ratio σ.

Small but finite amplitude IAWs

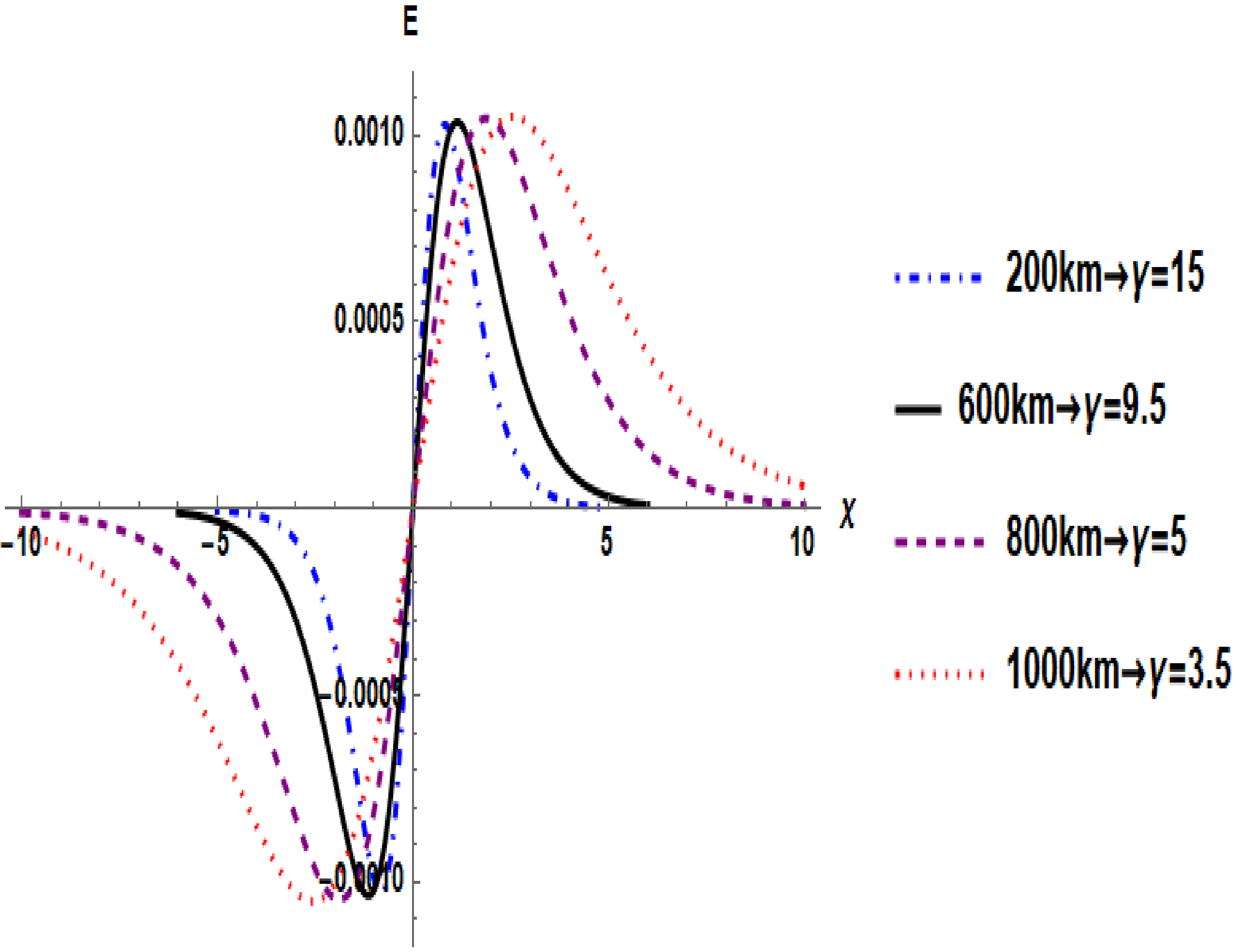
The magnitude of the normalized electric field E of the IAWs

$$E = E_o \operatorname{sech}^2\left(\frac{\chi}{W}\right)\tanh\left(\frac{\chi}{W}\right)$$

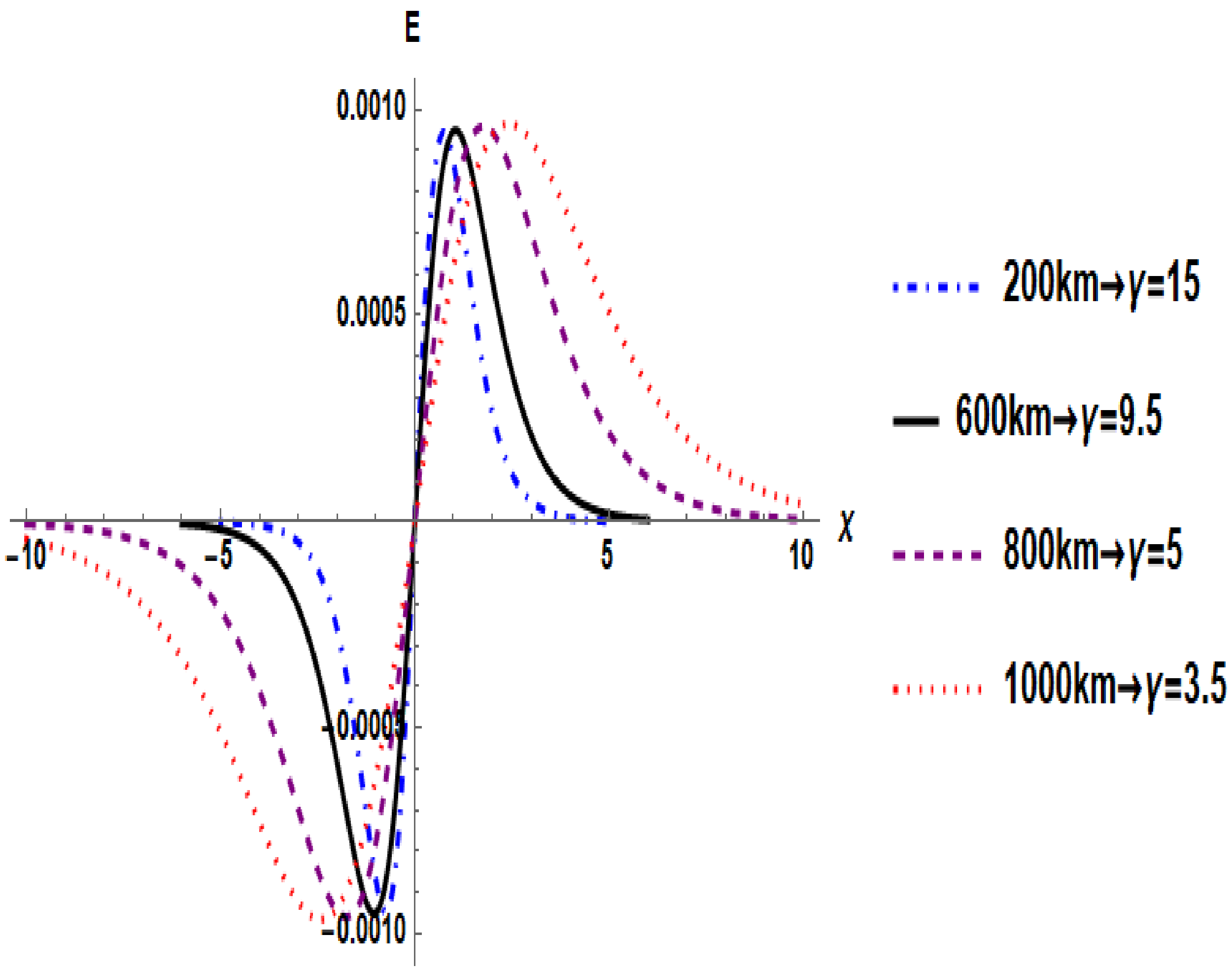


Behavior of the electric field versus χ at different values of γ for $\sigma = 0.1$

Small but finite amplitude IAWs



Behavior of the electric field versus χ at different values of γ for $\sigma = 0.15$



Behavior of the electric field versus χ at different values of γ for $\sigma = 0.2$

large amplitude IAWs

We assume that all fluid variables depend on a single variable $\eta = x - Mt$ where η is normalized by λ_{Di} and M is the Mach number normalized by c_s.

To study stationary soliton solutions, we use the steady state condition namely, $n_i \to 1$, $u_i \to 0$, $\phi \to 0$, and $d\phi/d\eta \to 0$ at $\eta \to \pm\infty$). From Poisson equation

$$\frac{d^2\phi}{d\eta^2} = \alpha e^{\phi} - \frac{1}{2\sqrt{3\sigma_1}}\left\{\left[\sqrt{\left(M+\sqrt{3\sigma_1}\right)^2 - 2\phi}\right] - \left[\sqrt{\left(M-\sqrt{3\sigma_1}\right)^2 - 2\phi}\right]\right\}$$

$$- \frac{\gamma}{2\sqrt{\frac{3\sigma_2}{\mu}}}\left\{\left[\sqrt{\left(M+\sqrt{\frac{3\sigma_2}{\mu}}\right)^2 - \frac{2\phi}{\mu}}\right] - [\sqrt{\left(M-\sqrt{\frac{3\sigma_2}{\mu}}\right)^2 - \frac{2\phi}{\mu}}]\right\}$$

large amplitude IAWs

Multiplying both sides of previous equation by $d\phi/d\eta$, integrating and imposing the appropriate boundary conditions ($\phi \to 0$ and $d\phi/d\eta \to 0$ at $\eta \to \pm\infty$), gives

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + V(\phi) = 0$$

This relation provides an energy integral of an oscillating particle of unit mass with a velocity $d\phi/d\eta$ and position ϕ in a (Sagdeev-like) potential $V(\phi)$.

$$V(\phi) = \frac{1}{\sqrt{108\sigma_1}}\left\{R_1^3\left[1-\left(1-\frac{2\phi}{R_1^2}\right)^{3/2}\right] - R_2^3\left[1-\left(1-\frac{2\phi}{R_2^2}\right)^{3/2}\right]\right\}$$
$$+\frac{\gamma\mu}{\sqrt{\frac{108\sigma_1}{\mu}}}\left\{R_3^3\left[1-\left(1-\frac{2\phi}{\mu R_3^2}\right)^{3/2}\right] - R_4^3\left[1-\left(1-\frac{2\phi}{\mu R_4^2}\right)^{3/2}\right]\right\} - \alpha\{e^{\phi}-1\}$$

$R_1 = M + \sqrt{3\sigma_1}$ $R_2 = M - \sqrt{3\sigma_1}$ $R_3 = M + \sqrt{3\sigma_1/\mu}$ $R_4 = M - \sqrt{3\sigma_1/\mu}$

large amplitude IAWs

Therefore, the existence of solitary wave solution is possible, if the Sagdeev potential satisfies the following conditions:

(i) $V''(\phi) \leq 0$ at $\phi = 0$. This condition yields the inequality

$$\alpha + \frac{1}{\sqrt{12\sigma_1}}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] + \frac{\gamma}{\sqrt{12\mu\sigma_2}}\left[\frac{1}{R_3} - \frac{1}{R_4}\right] \leq 0$$

Min Mach number

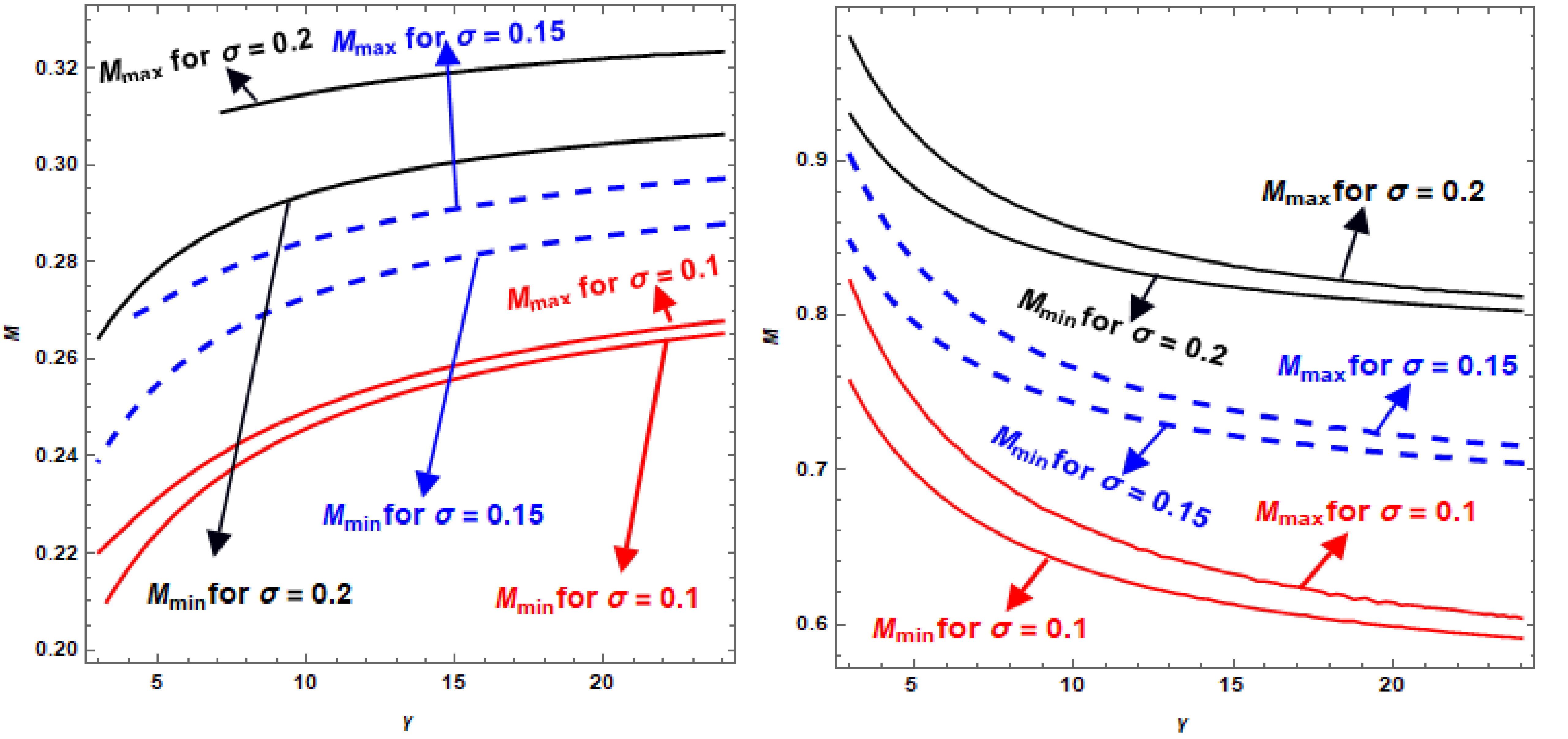
(ii) $V(\phi_{max}) \geq 0$ where $\phi_{max}(\phi_{max} \approx R_2^2/2)$. This implies that the inequality

$$\frac{1}{\sqrt{108\sigma_1}}\left\{R_1^3\left[1-\left(1-\frac{R_2^2}{R_1^2}\right)^{3/2}\right]\right\} + \frac{\gamma\mu}{\sqrt{\frac{108\sigma_1}{\mu}}}\{[R_3^3\left[1-\left(1-\frac{R_2^2}{\mu R_3^2}\right)^{3/2}\right] - R_4^3[1-\left(1-\frac{R_2^2}{\mu R_4^2}\right)^{3/2}]$$
$$-\alpha\{e^{R_2^2/2} - 1\} \geq 0$$

Max Mach number

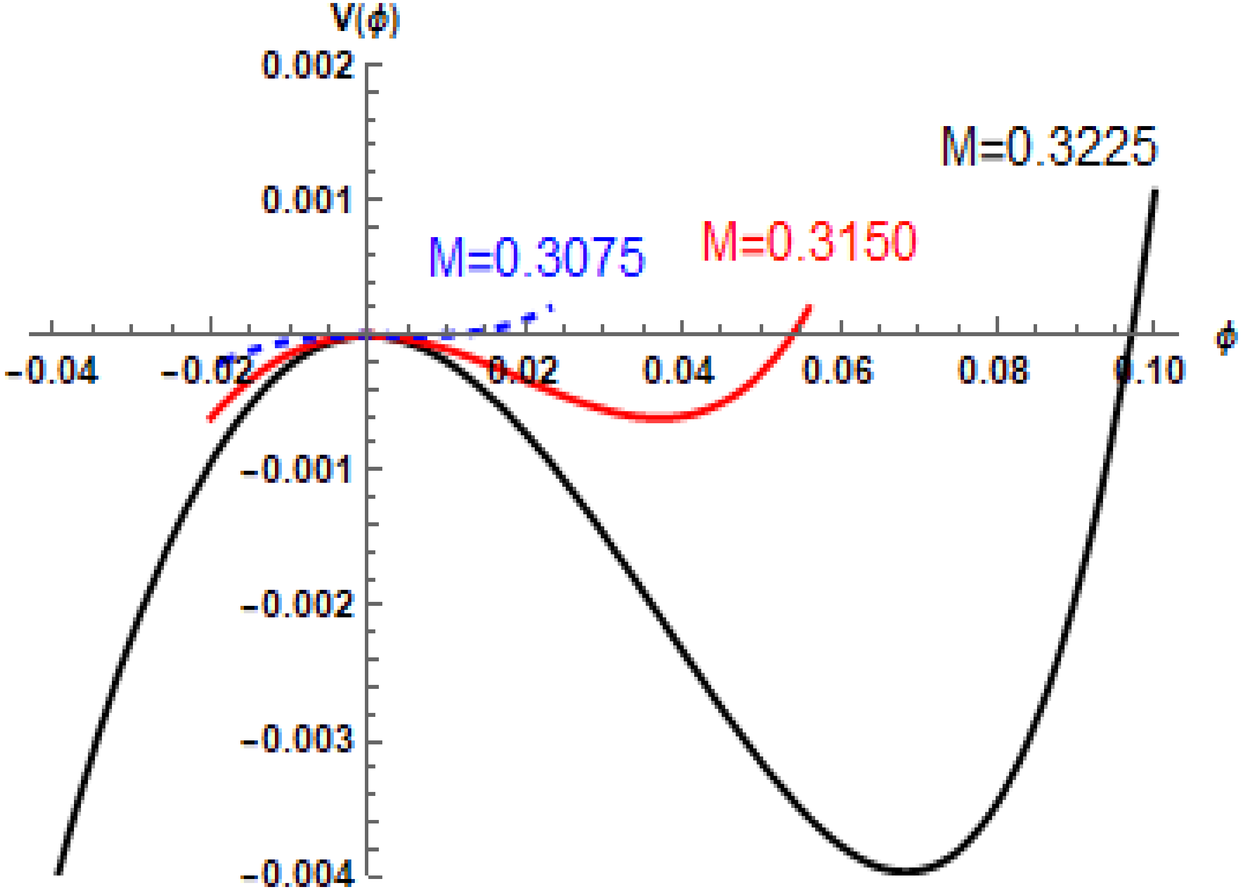
(iii) $V(\phi) < 0$ where $0 < \phi < \phi_{max}$.

large amplitude IAWs



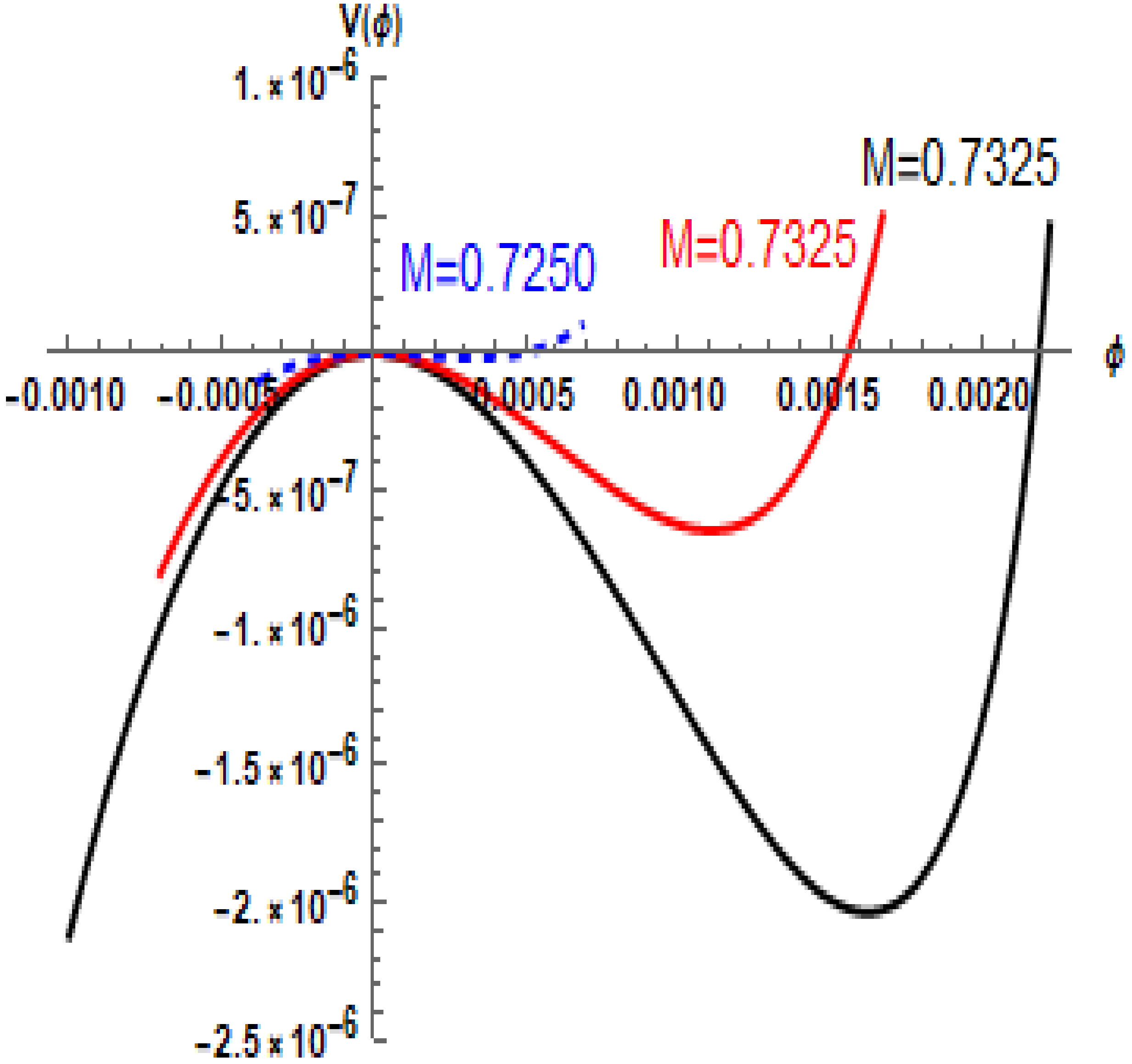
large amplitude IAWs

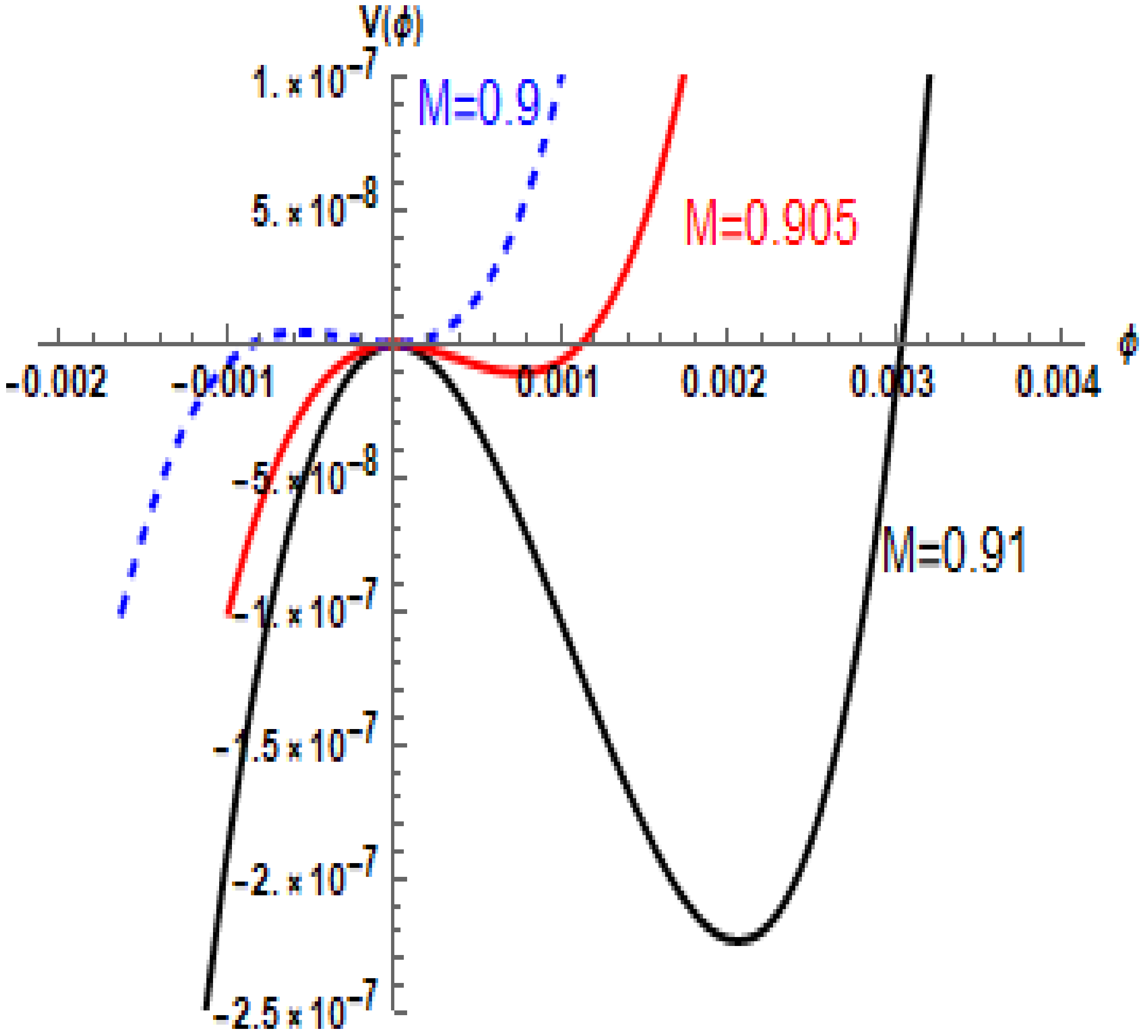
The dependence of the Sagdeev potential on the Mach number M is shown in following figures.



It is seen that the potential profile becomes spiky (i.e., taller and narrower) for fast pulses (higher Mach number) while slower ones (lower Mach number) will be shorter and wider in agreement with the soliton phenomenology.

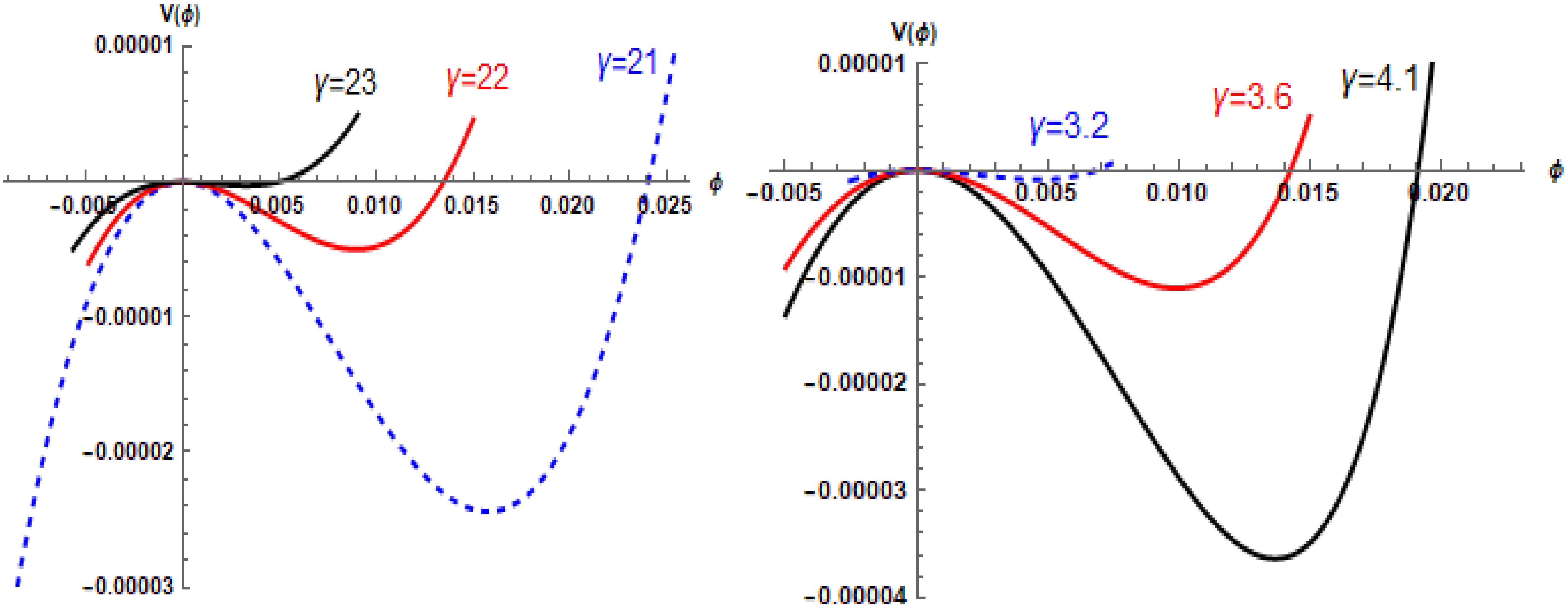
large amplitude IAWs





large amplitude IAWs

The potential profiles are investigated for different values of γ and we get two different behaviors



Summary

Analytical and numerical calculations of the wave amplitude reveals only a positive potential, i.e., only compressive solitons are formed in the core of Venus ionosphere at altitude 100 ~1000 km and they are subsonic pulses

The existence regions of large amplitude solitary pulses are determined and defined by lower and upper limits of the Mach number. It is seen that these limits become wider for higher σ at $0.2 < M < 0.3$ but the soliton existence region becomes narrower for higher σ at $M > 0.6$.

Numerical solution of the Sagdeev potential shows that only positive solitary pulses exist, which have the property faster pulses are taller and narrower.

Summary

It is emphasized that the amplitude and width of the soliton profiles are very sensitive to parameters γ and σ. It has been observed that they decreases with increase of σ. The parameter γ has two different effects depending on the used phase velocity root λ_1 or λ_2. When γ increases, the pulses become taller and narrower for λ_1. However, when we use λ_2, as γ increases, both the amplitude and the width decreases.

The amplitude of the electric field is strongly dependent on parameters γ and σ. It is shown that the amplitude increases with the increase of γ and the electric field becomes high at lower altitude. On the other hand, when σ increases the amplitude becomes shorter and narrower.

Thank you

for your attention