Basic Waves in Plasma

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Outline

- Why waves is important in plasma?
- Plasma models/theories
- Plasma components & Waves
- ESWs & EMWs
- Linear & Nonlinear theory
- Idea of perturbation
- Electrons oscillation & wave
- Ion wave

(1)

$$m a = F$$

$$m \frac{d^2 x}{dt^2} = qE = -eE$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot E = \frac{en}{\varepsilon_0}$$

$$\frac{E}{x} = \frac{en}{\varepsilon_0}$$

$$m \frac{d^2 x}{dt^2} = -e \frac{en}{\varepsilon_0} x$$

$$\frac{d^2 x}{dt^2} = - \frac{e^2 n}{\varepsilon_0 m} x$$

$$\frac{d^2 x}{dt^2} + \left(\frac{e^2 n}{\varepsilon_0 m}\right) x = 0$$

Show that the equation of motion (Newton's Second law) for each electrons is given by:

What is the physics behind this equation?

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} n_e V_e = 0$$

$$m\left(\frac{\partial V_e}{\partial t} + V_e \frac{\partial V_e}{\partial x}\right) = -eE$$

$$\varepsilon_0 \frac{\partial E}{\partial x} = \rho = e(n_i - n_e)$$

$$\frac{\partial E_1}{\partial x} = -\frac{e}{\varepsilon_0} n_{e1} \quad \text{— ①}$$

$$m \frac{\partial V_{e1}}{\partial t} = -eE_1$$

$$\frac{\partial^2 V_{e1}}{\partial t \partial x} = -\frac{e}{m} \frac{\partial E_1}{\partial x} \quad \text{— ②}$$

$$\frac{\partial^2 V_{e1}}{\partial t \partial x} = -\frac{e}{m} * -\frac{e}{\varepsilon_0} n_{e1}$$

$$\frac{\partial^2 V_{e1}}{\partial t \partial x} = \frac{e^2}{m \varepsilon_0} n_{e1} \quad \text{— ③}$$

$$\frac{\partial n_{e1}}{\partial t} + n_{e0} \frac{\partial V_{e1}}{\partial x} = 0$$

$$\frac{\partial V_{e1}}{\partial x} = -\frac{1}{n_{e0}} \frac{\partial n_{e1}}{\partial t} \quad \text{— ④ in ③}$$

$$\frac{\partial n_{e1}}{\partial t^2} = -\left(\frac{e^2 n_{e0}}{m \varepsilon_0}\right) n_{e1}$$

Electron plasma waves

$$m_e\left(\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial x}\right) = -eE - \frac{\partial P_e}{\partial x}$$

(m → n_e, E → n_e)

→ divided by ne

$$m_e\left(\frac{\partial v_e}{\partial t} + v_e\frac{\partial v_e}{\partial x}\right) = -eE - \frac{1}{n_e}\frac{\partial P_e}{\partial x}$$

$$P_e = 3k_B T_e n_e$$

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 V_{th}^2$$

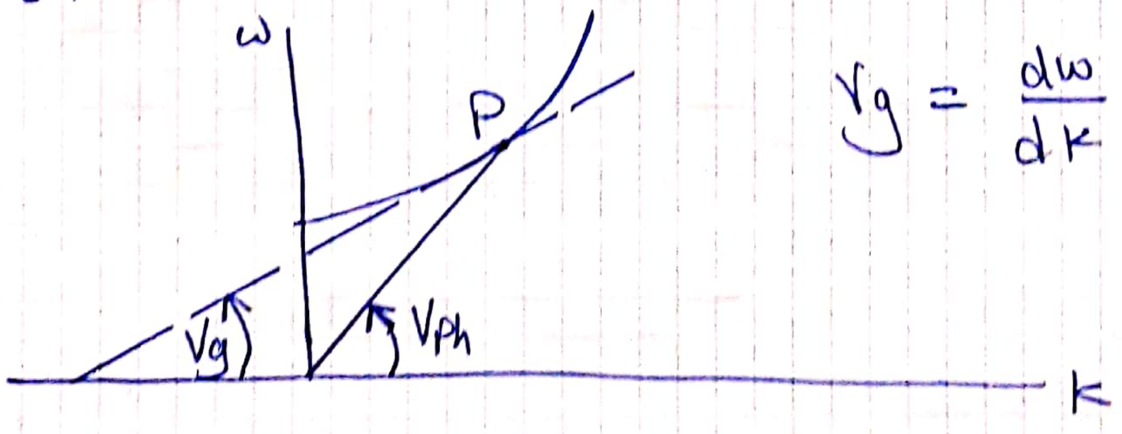
$$V_{th}^2 \to \frac{2k_B T_e}{m_e}$$

we use plan wave analysis

$$V_e = V_{e1} = v_{e1}\exp(ikx - i\omega t)$$

$$n_e = n_{e0} + n_{e1} \Rightarrow n_{e1} = n_1 \exp(ikx - i\omega t)$$

$$\frac{\partial}{\partial t} \longrightarrow -i\omega \quad , \quad \frac{\partial}{\partial x} \longrightarrow ik$$



$$V_g = \frac{d\omega}{dk} = \frac{3}{2}\frac{k}{\omega}V_{th}^2$$

Ion Waves

* ion time scale $\longrightarrow$ what about the electrons?

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} n_i u_i = 0$$

$$m_i n_i \left(\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} \right) = e n_i E - \frac{\partial P_i}{\partial x}$$

$$E = - \frac{\partial \phi}{\partial x}$$

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}(n_i - n_e) \quad \text{Gauss}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_i) \quad \text{Poisson}$$

$$n_e = n_{e0} \exp\left(\frac{e\phi}{k_B T_e} \right)$$

Two cases

$n_e = n_i$

Neutrality cond.

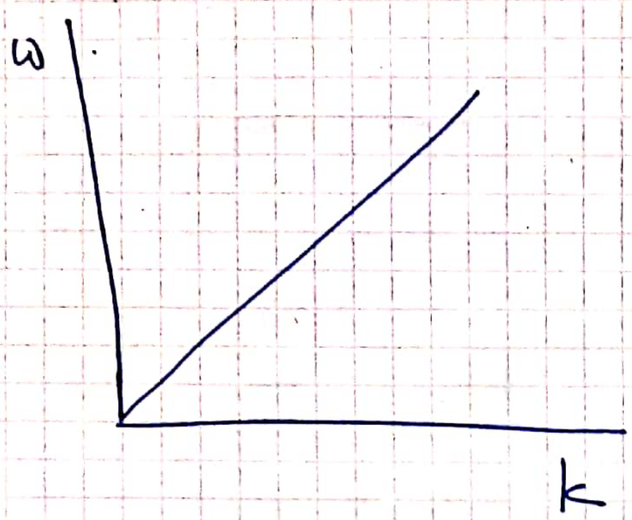
at ~~...~~ small wave number

$$\frac{\omega}{k} = \left(\frac{k_B T_e}{m_i} \right)^{1/2} = C_s$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_i)$$

Poisson eq

$$\frac{\omega}{k} = \left(\frac{k_B T_e}{m_i(1 + k^2 \lambda_D^2)} \right)^{1/2}$$

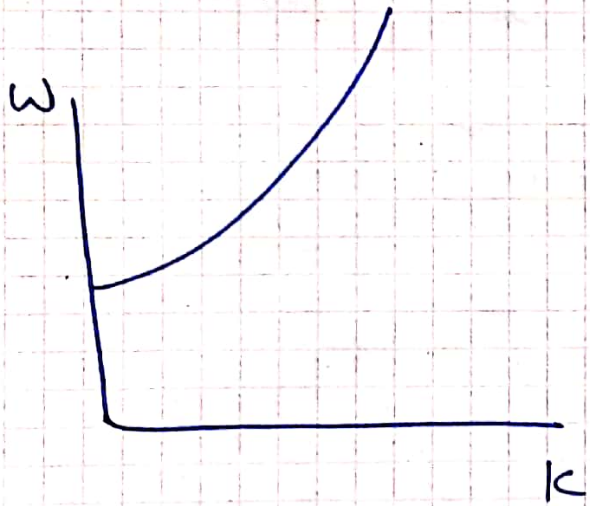


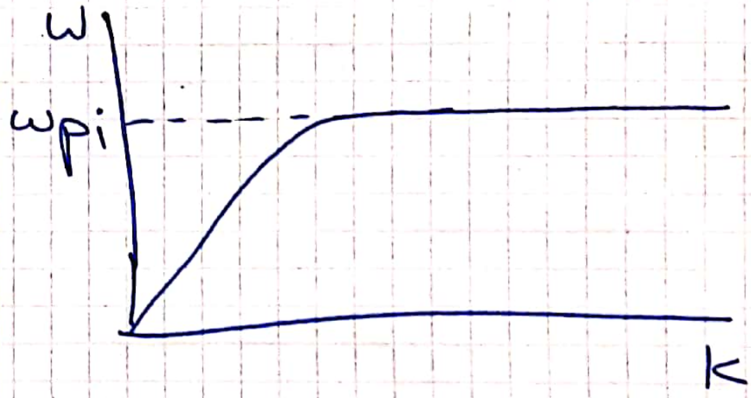
for $k^2\lambda_D^2 \ll 1$

$$\frac{\omega}{k} = C_s$$

for $k^2\lambda_D \gg 1$

$$\frac{\omega}{k} = \frac{C_s}{k\lambda_D} \Rightarrow \omega = \frac{C_s}{\lambda_D} = \omega_p$$





Comparison between e & i plasma waves

e	i
at $k=0 \Rightarrow \omega = \omega_{pe}$	at $k=0$ $\omega = 0$
at larg $k \Rightarrow \frac{\omega}{k} \rightarrow$ const	$\omega = \omega_{pi}$
∇P_e should be presented	∇P_i can be cancelled ∇P_e produce the prusser to wave formation

Thanks