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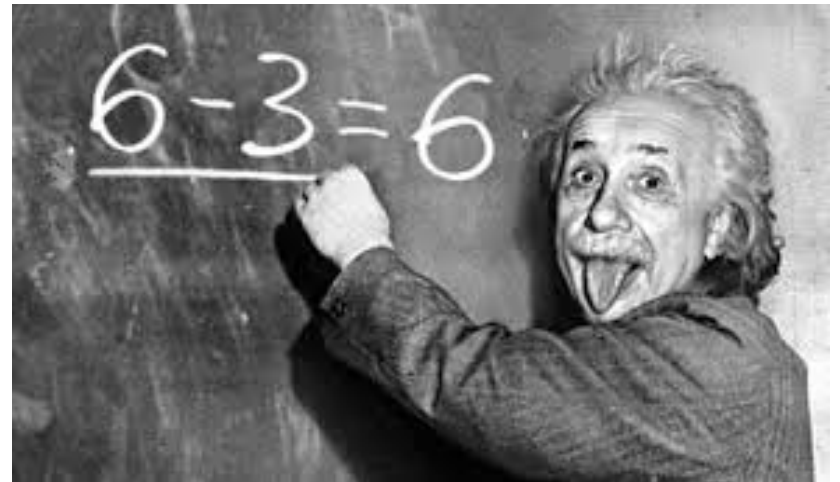
# Dynamical effect of laser bumped electron-hole semiconductor

By

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**I' m Mathematician  
not Physicist**



# Outline

Introduction

Electromagnetic wave effect

Objective of the Paper

The wave Equation

Dispersion Relation

Sagdeev Potential

Modified NLSE

Appendix

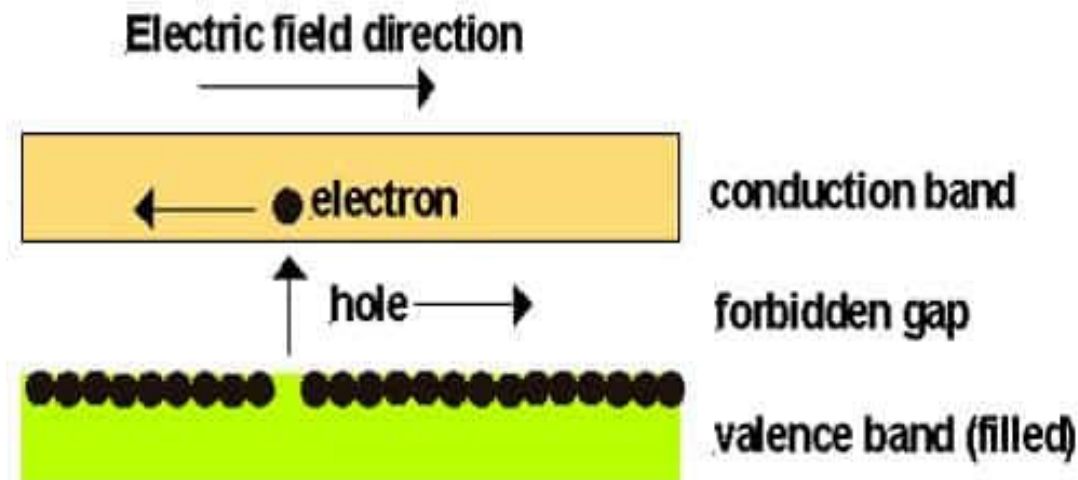
# Introduction

## An Electron

is defined as a negative charge or negative atomic particle.

## A Hole

as a vacancy left in the valence band because of the lifting of an electron from the valence band to a conduction band.



## Introduction

- **Hole** is a positive-charge, positive-mass quasiparticle.
- **Holes** can move from atom to atom in semiconducting materials as electrons leave their positions.
- **The mobility** of electrons is higher than that of the holes, because the effective mass of electron is less than a hole.

### An electron-hole pair

For every electron raised to the conduction band by external energy, there is one hole left in the valence band, creating what is called an electron-hole pair.

## Introduction

### Recombination

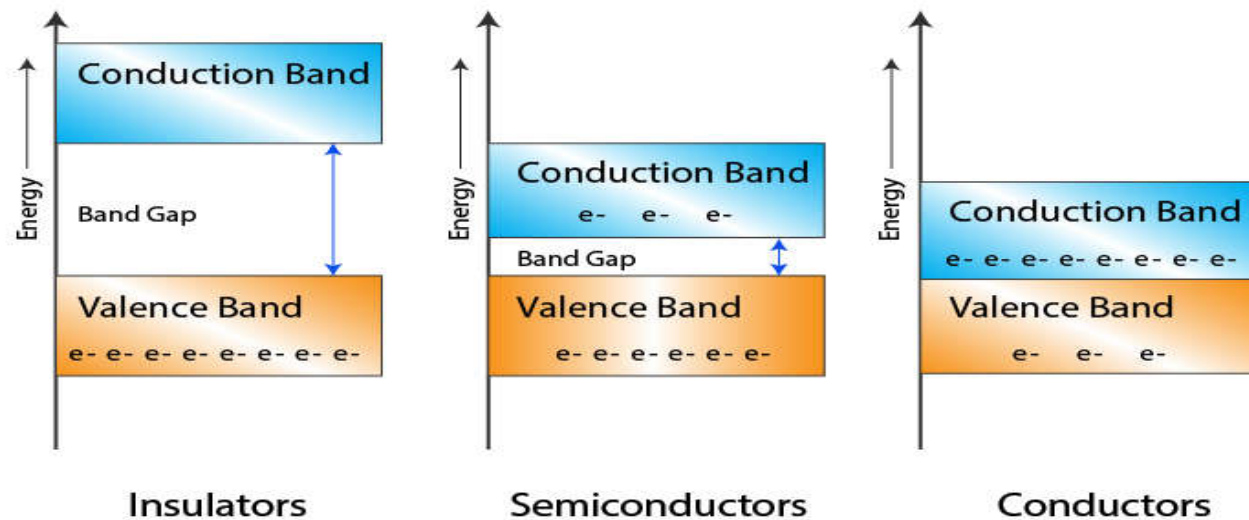
Occurs when a conduction-band electron loses energy and falls back into a hole in the valence band.

- **Recombination** rate is controlled by the minority carrier lifetime.
- **Recombination** mechanisms for materials is highly important for the optimization of semiconductor devices such as solar cells and light emitting diodes.

# Introduction

## Semiconductors

are the materials which have a conductivity between conductors (generally metals) and non-conductors or insulators (such ceramics).

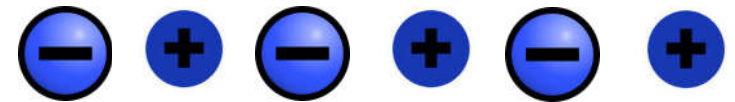


# Introduction

## □ Types of Semiconductors

- Intrinsic Semiconductor

( holes = electrons )



- Extrinsic Semiconductor

( excess or shortage of electrons )

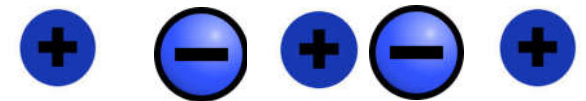
- ❖ N-Type Semiconductor

( Mainly due to electrons )



- ❖ P-Type Semiconductor

( Mainly due to holes )





# Semi-conductor materials

Material	Symbol	Usage
Germanium	Ge	radar detection diodes - first transistors
Silicon	S	integrated circuits -insulation layers
Gallium arsenide	GaAs	high performance RF devices
Silicon carbide	SiC	yellow and blue LEDs.

# Semi-conductor materials

Material	Symbol	Usage
Gallium Nitride	GeN	microwave transistors -microwave ICs-blue LEDs
Gallium phosphide	GaP	produce a green light-(+N ) yellow-green- (+ZnO) red.
Cadmium sulphide	CdS	photoresistors - solar cells

**EMW effect on  
Semi-Conductor  
E-H particles**

- Recombination Process
- Increase in current flow
- Diffusion of charges
- Decrease the energy gap
- Decrease the mobility of carriers
- Increase the collision rate
- Increase the conductivity (Intrinsic)
- Decrease the conductivity ( Extrinsic)

## Objectives

- This study introduces the mathematical model of the interaction between electromagnetic field and electron hole particles.
- Using Maxwell's equations along with e-h fluid equations that contain laser field effect, we derive an evolution wave equation describing the system, which is called a modified nonlinear Schrödinger equation (mNLSE).
- The mNLSE is reduced to an energy equation containing the Sagdeev potential describing the localized propagating pulses in semiconductor.

## The wave equation for homogeneous plasma

- From Maxwell's Eqs. , we can extract the wave equation , which describe the propagation of laser field in the plasma.

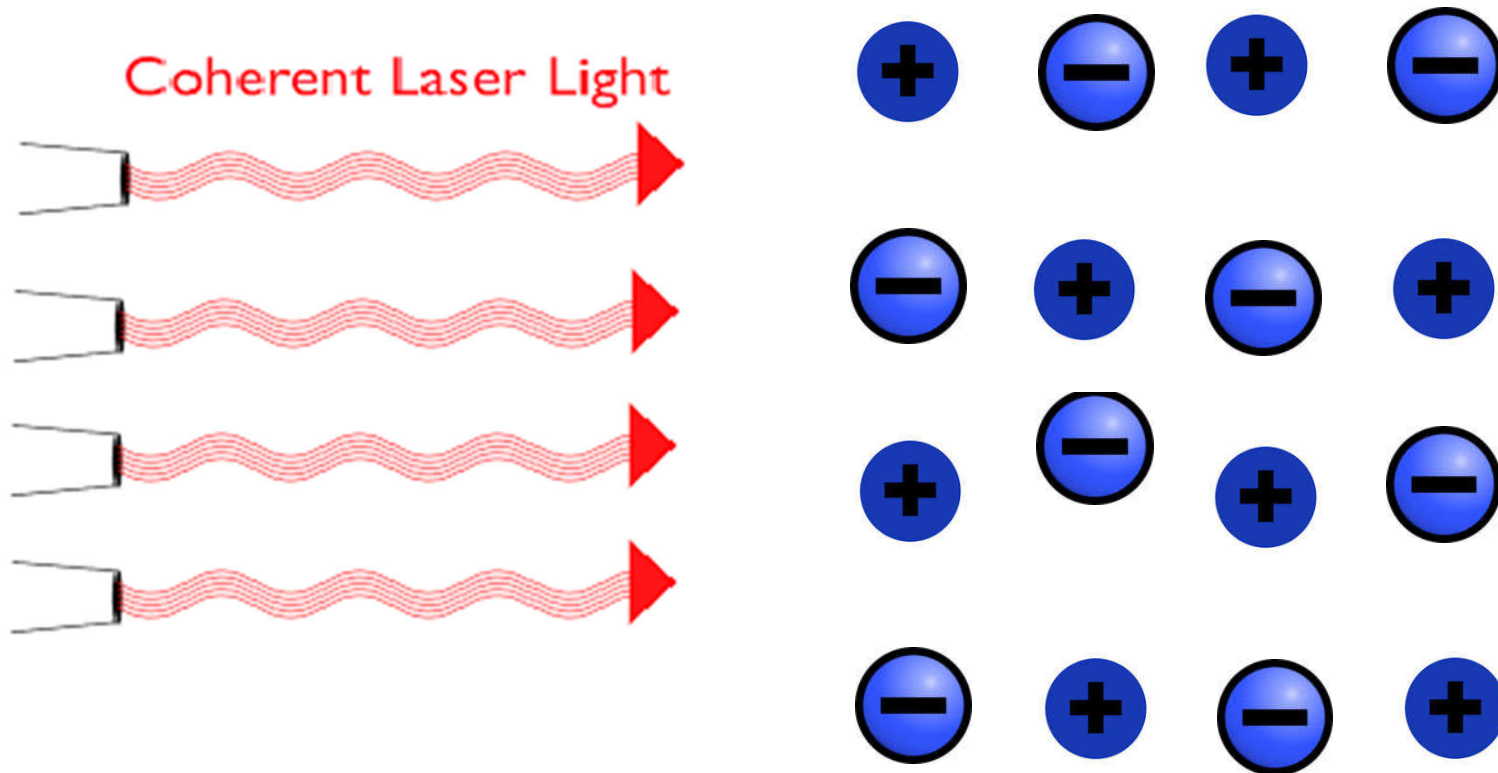
$$\nabla \times \nabla \times A + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = \frac{4\pi J}{c} \quad (1)$$

$$\nabla^2 \Phi = -4\pi \rho_e \quad (2)$$

when

$$J = -en_e v_e + \sum_k q_k n_k v_k \quad , \text{and} \quad \rho_e = -en_e + \sum_k q_k n_k \quad (3)$$

# Laser - Electron – Hole Interaction



## Laser - Electron – Hole Interaction

- We can extract the wave equation , which describe the propagation of laser field in the electron-hole plasma.

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = -\frac{4\pi e}{c} (\mathbf{n}_h \mathbf{v}_h - \mathbf{n}_e \mathbf{v}_e) \quad (4)$$

- And the Poisson eq.

$$\nabla^2 \phi = -4\pi e (\mathbf{n}_e - \mathbf{n}_h) \quad (5)$$

- Where the velocity with relativistic effect

$$\mathbf{v}_e = \frac{e \mathbf{A}}{m_e c \gamma_e} , \quad \mathbf{v}_h = -\frac{e \mathbf{A}}{m_h c \gamma_h} \quad (6)$$

## Laser - Electron – Hole Interaction

- Also from the relativistic fluid equation

$$\partial_t \mathbf{p} + (\mathbf{v}_e \cdot \nabla) \mathbf{p} = e \left( \mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} \right) - \frac{1}{n_e} \nabla P \quad (7)$$

at

$$\mathbf{p} = -e \mathbf{A} / c \quad (\text{momentum}), \quad P = n_e k T_e \quad (\text{pressure}) \quad (8)$$

,we derive the densities

$$(a) \text{ for electron} \quad \longrightarrow \quad n_e = n_{e0} \exp \left\{ -\frac{m_e c^2}{T_e} (\gamma_e - 1) + \frac{q_e}{T_e} \phi \right\} \quad (9)$$

$$(b) \text{ for hole} \quad \longrightarrow \quad n_h = n_{h0} \exp \left\{ -\frac{m_h c^2}{T_h} (\gamma_h - 1) + \frac{q_h}{T_h} \phi \right\} \quad (10)$$



## Laser - Electron – Hole Interaction

- The vector potential of the light takes the form

$$\mathbf{A} = \frac{1}{2} \chi(\mathbf{r}, t) (\mathbf{x} + i \mathbf{y}) \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega t) \quad (11)$$

- The differential equation after substituting in eq. (4) takes the form

$$i \frac{\partial \chi}{\partial t} + \frac{c^2}{2\omega} \nabla^2 \chi + i \frac{k c^2}{\omega} \nabla \chi + \frac{\omega_{pe}^2}{2\omega} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + \mathbf{M} \left[ 1 - \frac{N_h}{\sqrt{1 + \mathbf{M}^2 |\mathbf{a}|^2}} \right] \right) \chi = 0 \quad (12)$$

## Laser - Electron – Hole Interaction

- The resulting form from the wave equation after the scaling, the nondimensional form can be reduced to a modified NLSE

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \frac{1}{2} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + M \left[ 1 - \frac{N_h}{\sqrt{1 + M^2 |\mathbf{a}|^2}} \right] \right) \mathbf{a} = \mathbf{0} \quad (13)$$

- Where

$$\chi = \frac{m_0 c^2 \mathbf{a}}{e}, \quad t = \left( \tau \omega_0 / \omega_{pe}^2 \right), \quad (14)$$

$$r = \left\{ c \left( \xi - u_g \tau \right) / \omega_{pe} \right\}, \quad u_g = c k / \omega_{pe}$$

# Modified NLSE

- NL dispersion relation
- The modulational instability growth rate

- The Sagdeev potential.
- The light amplitude.

- Instability of system
- Solution of NLSE by **HPM**

## The nonlinear dispersion relation - Growth rate

- At the potential takes the form

$$\mathbf{a} = (a_0 + a_1) \exp(i \delta \tau) \quad , \text{where } a_0 \gg |a_1| \quad (15)$$

Substituting in the differential eq.,

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \frac{1}{2} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + M \left[ 1 - \frac{N_h}{\sqrt{1 + M^2 |\mathbf{a}|^2}} \right] \right) \mathbf{a} = 0 \quad (16)$$

- The nonlinear frequency shift take the form

$$\delta = \frac{1}{2} \left( 1 - \frac{N_e(a_0)}{\sqrt{1 + |a_0|^2}} + M \left[ 1 - \frac{N_h(a_0)}{\sqrt{1 + M^2 |a_0|^2}} \right] \right) \quad (17)$$

## The nonlinear dispersion relation - Growth rate

- We linearize the differential eq. (16) At  $a_1 = (X + iY) \exp(i k \cdot \xi - i \Omega \tau)$  with respect to  $a_1$  where  $\Omega(k)$  is the frequency of the low frequency modulations, and  $X$ ,  $Y$  are real constants.
- After Taylor series, and linearization

$$\frac{N_e(a)}{\sqrt{1 + |a|^2}} = \alpha + \beta (a_1 + a_1^*), \quad \frac{N_h(a)}{\sqrt{1 + M^2 |a|^2}} = \alpha^* + \beta^* (a_1 + a_1^*) \quad (18)$$

## The nonlinear dispersion relation - Growth rate

- After substituting in Eq. (16) with respect to  $a_1$ , the differential eq. takes the form

$$i \frac{\partial a_1}{\partial \tau} + \frac{1}{2} \nabla^2 a_1 - \frac{a_0}{2} (\beta + \beta^* M) (a_1 + a_1^*) = 0 \quad (19)$$

- Let

$$a_1 = U + i V \quad \Rightarrow \quad a_1^* = U - i V$$

- Assume the plane wave

$$U = u_0 e^{i k \cdot \xi - i \Omega \tau} \quad , \text{ and } \quad V = v_0 e^{i k \cdot \xi - i \Omega \tau} \quad (20)$$

## The nonlinear dispersion relation - Growth rate

- The nonlinear dispersion relation is

$$\Omega^2 = \frac{k^2}{2} \left( \frac{k^2}{2} + a_0 (\beta + \beta^* \mathbf{M}) \right) \quad (21)$$

- The modulational instability growth rate

$$\Gamma = -i\Omega = \frac{k}{\sqrt{2}} \sqrt{\left( \frac{k^2}{2} + a_0 (\beta + \beta^* \mathbf{M}) \right)} \quad (22)$$

## The Sagdeev potential

- The nonlinear system at

$$\mathbf{a}(z, \tau) = w(z) \exp(-i \Omega \tau) \quad (23)$$

Substituting in the differential eq.,

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \frac{1}{2} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + M \left[ 1 - \frac{N_h}{\sqrt{1 + M^2 |\mathbf{a}|^2}} \right] \right) \mathbf{a} = 0$$

- The integration of a modified NLSE gives this form

$$\frac{1}{2} (w'(z))^2 + \psi(w) = 0 \quad (24)$$



## The Sagdeev potential

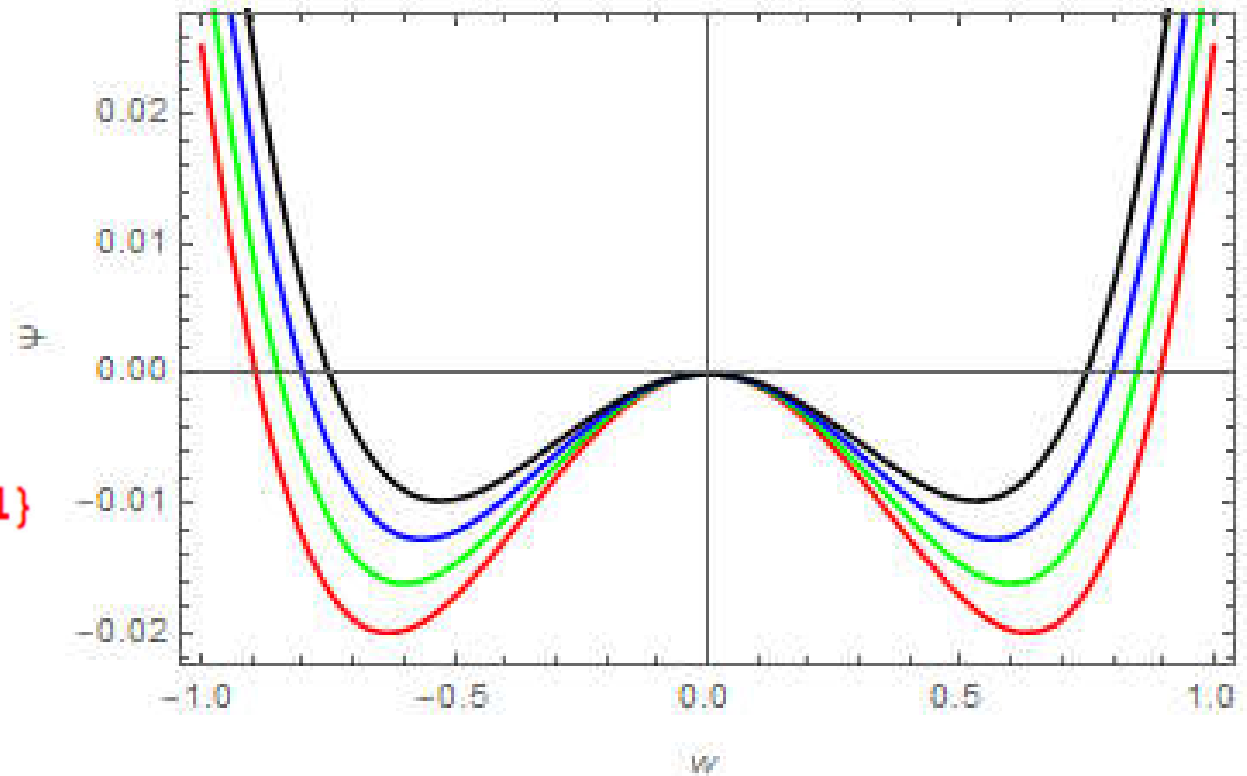
- Where  $\psi(w)$  is the Sagdeev potential

$$\begin{aligned} \psi(w) = & \Omega w^2 + \frac{1}{8} \left( 1 + M^3 - (1 + E + H M^2) \beta_e \right) w^4 \\ & - \frac{1}{8} \left( (E M + H M^3 + M^3) \beta_h \right) w^4 + O(w^6) \end{aligned} \quad (25)$$

### Fig. (1)

$\psi(w)$  is a function of light amplitude  $w$ . It has the figure of Sagdeev potential in the (+ve) and (-ve) values of amplitude. At the values ;

$$\Omega = \{-0.07, -0.08, -0.09, -0.1\}$$



## The wave amplitude (EMW)

- Then eq. (24) takes the form

$$\frac{1}{2}(w'(z))^2 + \Omega w^2 + \frac{1}{8}(1 + M^3 - (1 + E + H M^2)\beta_e)w^4 - \frac{1}{8}((E M + H M^3 + M^3)\beta_h)w^4 + O(w^6) = 0 \quad (25)$$

- The light amplitude  $w$  is

$$\int \frac{dw}{w \sqrt{1 - \frac{A}{8\Omega} w^2}} = \sqrt{-2\Omega} \int dz \quad (26)$$

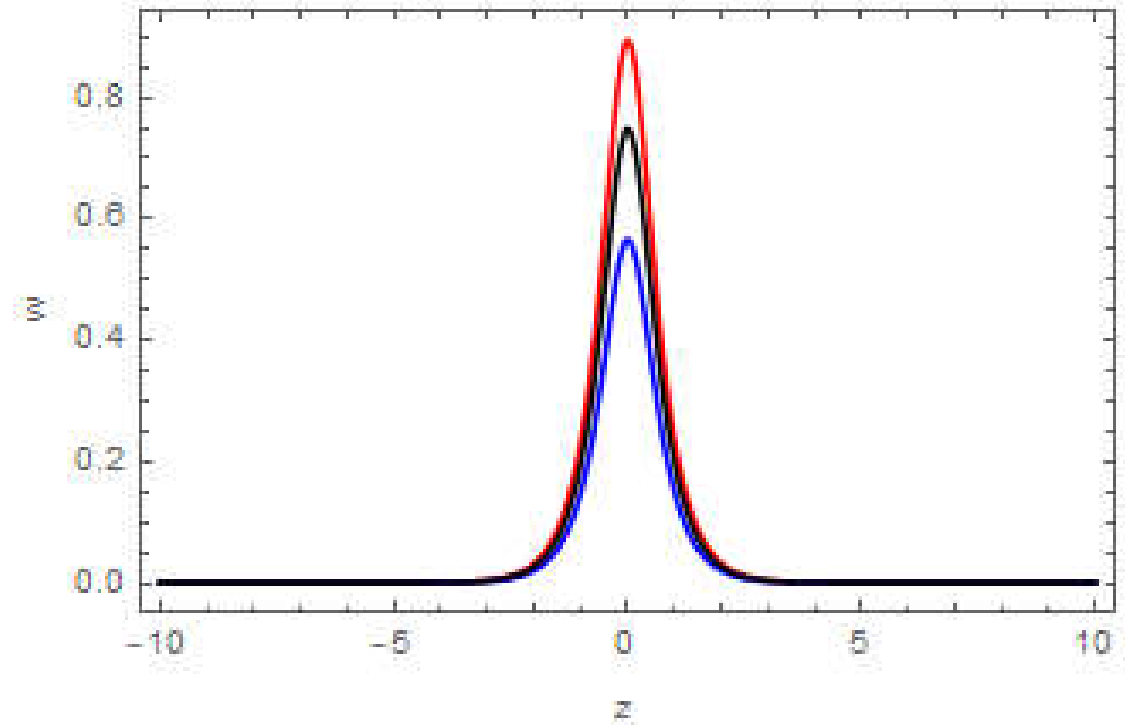
$$\text{at } A = (1 + M^3 - (1 + E + H M^2)\beta_e - (E M + H M^3 + M^3)\beta_h)$$

## Fig. (2)

The light amplitude  $w(z)$  is a function of the distance  $z$ , which gives a soliton wave at all values of the distance.

At the values ;

$$\Omega = \{-0.07, -0.08, -0.1\}$$



## Modified NLSE

- The last differential eq. takes the form

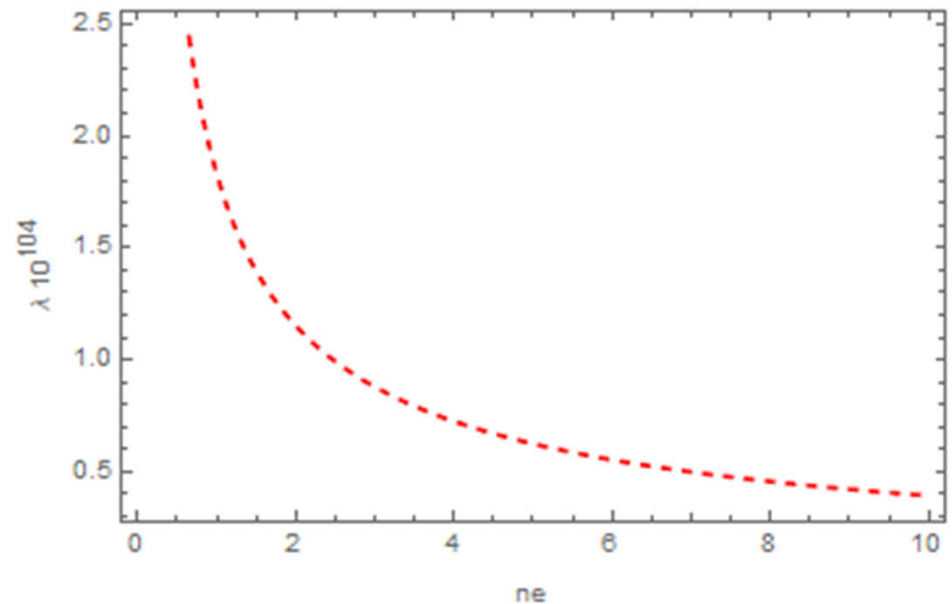
$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \frac{1}{2} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + M \left[ 1 - \frac{N_h}{\sqrt{1 + M^2 |\mathbf{a}|^2}} \right] \right) \mathbf{a} = \mathbf{0} \quad (27)$$

- After Taylor series, the differential eq. takes the form

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \lambda |\mathbf{a}|^2 \mathbf{a} = \mathbf{0} \quad (28)$$

### Fig. (3)

This case, with  $\lambda$  positive, gives unstable envelope soliton, which has Bright soliton solutions.



## HPM for solving mNLSE

- The last differential eq. takes the form

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \frac{1}{2} \left( 1 - \frac{N_e}{\sqrt{1 + |\mathbf{a}|^2}} + \mathbf{M} \left[ 1 - \frac{N_h}{\sqrt{1 + \mathbf{M}^2 |\mathbf{a}|^2}} \right] \right) \mathbf{a} = 0 \quad (27)$$

- After Taylor series takes the form

$$i \frac{\partial \mathbf{a}}{\partial \tau} + \frac{1}{2} \nabla^2 \mathbf{a} + \lambda |\mathbf{a}|^2 \mathbf{a} = 0 \quad (28)$$

- Using HPM

$$i \frac{\partial \mathbf{a}}{\partial \tau} + p \left( \frac{1}{2} \nabla^2 \mathbf{a} + \lambda |\mathbf{a}|^2 \mathbf{a} \right) = 0 \quad (29)$$

## HPM for solving mNLSE

- The solution take the formula

$$\mathbf{a} = \mathbf{a}_0 + p \mathbf{a}_1 + p^2 \mathbf{a}_2 + p^3 \mathbf{a}_3 + \dots \quad (30)$$

,where  $p$  is embedding parameter  $0 \leq p \leq 1$

- Substituting from the formula (30) in the eq. (29) ,and compare the coefficients  $p$  , we find



## HPM for solving mNLSE

$$p^0 : i D_\tau a_0 = 0 \quad \Rightarrow \quad a_0 = a(z, 0) = e^{iz}$$

$$p^1 : i D_\tau a_1 = -\frac{1}{2} \nabla^2 a_0 - \lambda (\bar{a}_0 a_0^2)$$

$$p^2 : i D_\tau a_2 = -\frac{1}{2} \nabla^2 a_1 - \lambda (\bar{a}_0 a_0 a_1 + \bar{a}_0 a_1 a_0 + \bar{a}_1 a_0 a_0)$$

$$p^3 : i D_\tau a_3 = -\frac{1}{2} \nabla^2 a_2 - \\ \lambda (\bar{a}_2 a_0 a_0 + \bar{a}_0 a_2 a_0 + \bar{a}_0 a_0 a_2 + \\ \bar{a}_0 a_1 a_1 + \bar{a}_1 a_1 a_0 + \bar{a}_1 a_0 a_1)$$

## HPM for solving mNLSE

then,

$$a_j = i\tau \left\{ \frac{1}{2} \nabla^2 a_{j-1} + \sum_{i=0}^{j-1} \sum_{k=0}^{j-i-1} a_i a_k \bar{a}_{j-i-k-1} \right\} \quad (31)$$

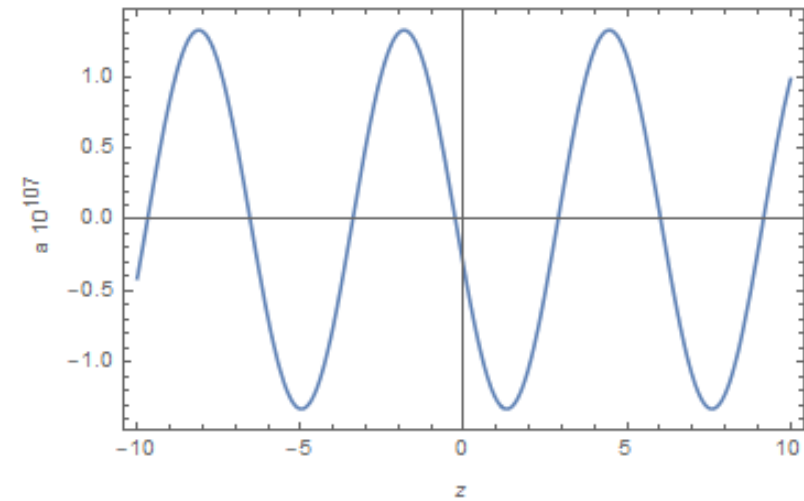
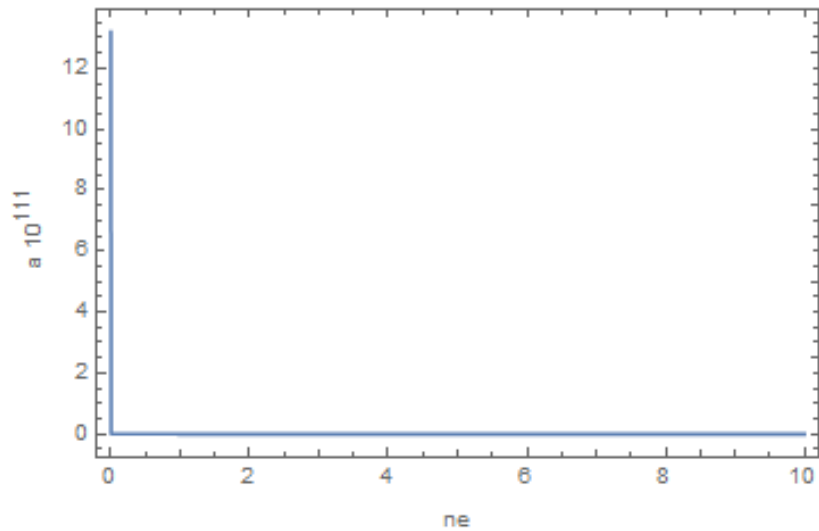
Then the series takes the four terms

$$a_0 = e^{iz}, \quad a_1 = i\tau \left( \lambda - \frac{1}{2} \right) e^{iz}, \quad a_2 = \left( \lambda - \frac{1}{2} \right) \left( \frac{3}{2} (i\tau)^2 - i\tau \right) e^{iz},$$
$$a_3 = \left( \lambda - \frac{1}{2} \right) \left( -\frac{3}{4} (i\tau)^3 + \frac{1}{2} i\tau \right) e^{iz}$$

## HPM for solving mNLSE

The potential by HPM is

$$\lim_{p \rightarrow 1} a = a_0 + a_1 + a_2 + a_3 + \dots \quad (32)$$



## Appendix

$$M = \frac{m_e}{m_h}, \quad \omega_{pe}^2 = \frac{4\pi e^2 n_{eo}}{m_e}, \quad \omega_{ph}^2 = \frac{4\pi e^2 n_{ho}}{m_h}$$

$$E = \frac{\beta_e}{\beta_e + M\beta_h}, \quad H = \frac{\beta_h}{\beta_e + M\beta_h}$$

$$N_e = \exp[\beta_e(1 - \sqrt{1 + |a|^2} + \Phi)]$$

$$N_h = \exp[\beta_h(1 - \sqrt{1 + M^2|a|^2} - M\Phi)]$$

$$\Phi = \frac{\beta_e}{\beta_e + M\beta_h} \left(1 - \sqrt{1 + |a|^2}\right) - \frac{\beta_h}{\beta_e + M\beta_h} \left(1 - \sqrt{1 + M^2|a|^2}\right)$$

$$\lambda = \frac{1}{4} \left(1 + \beta_e(1 - E + H M^2) + M^3 + M\beta_h(1 + M E - H M^3)\right)$$

## Appendix

$$\alpha = \left\{ \frac{(1 + \beta_e(1 - E + H))}{2(1 + a_0^2)^{1/2}} - \beta_e \left( 1 + E - H \frac{(1 + M^2 a_0^2)^{1/2}}{(1 + a_0^2)^{1/2}} \right) \right\}$$

$$\alpha^* = \left\{ \frac{(1 + \beta_h(1 + ME - MH))}{2(1 + M^2 a_0^2)^{1/2}} - \beta_h \left( 1 - MH + ME \frac{(1 + a_0^2)^{1/2}}{(1 + M^2 a_0^2)^{1/2}} \right) \right\}$$

$$\beta = \left\{ \frac{-a_0(1 + \beta_e(1 - E + H))}{2(1 + a_0^2)^{3/2}} + \frac{\beta_e H a_0(1 + M^2 a_0^2)^{1/2}}{2(1 + a_0^2)^{3/2}} - \left( \frac{\beta_e H a_0^2 M^2}{2(1 + M^2 a_0^2)^{1/2}(1 + a_0^2)^{1/2}} \right) \right\}$$

$$\beta^* = \left\{ \frac{-a_0 M^2(1 + \beta_h(1 + ME - MH))}{2(1 + M^2 a_0^2)^{3/2}} + \frac{\beta_h E a_0 M^3(1 + a_0^2)^{1/2}}{2(1 + M^2 a_0^2)^{3/2}} - \left( \frac{\beta_h E a_0 M}{2(1 + M^2 a_0^2)^{1/2}(1 + a_0^2)^{1/2}} \right) \right\}$$

**THANK U FOR  
LISTENING  
ANY QUESTION**

