

Introduction to Plasma Physics

* Content:

① Historical background

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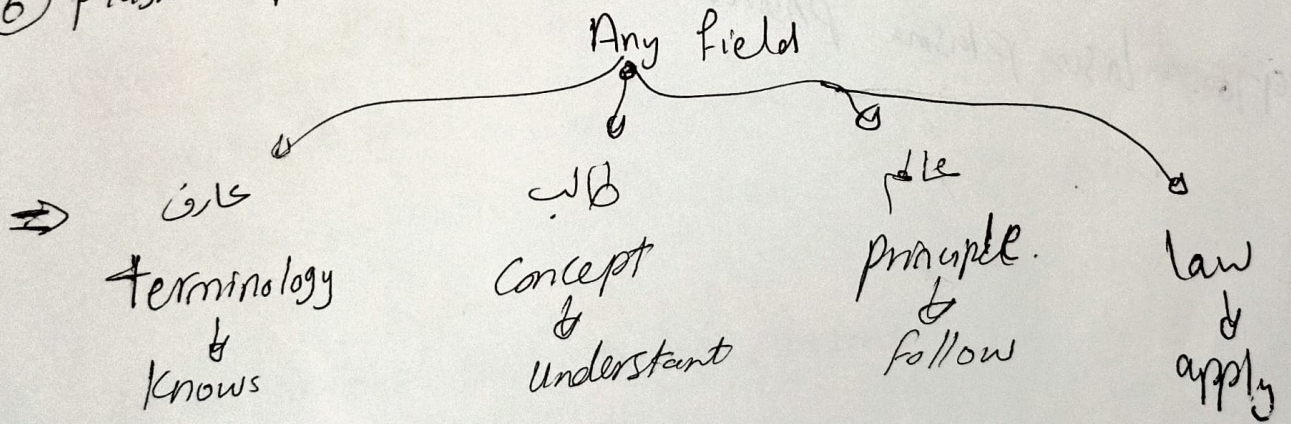
② plasma definition.

③ plasma parameters.

④ plasma criteria.

⑤ plasma classification.

⑥ plasma applications.



* Plasma history

- when the blood is cleared of its various corpuscles there remains a transparent liquid which was named plasma (In Greek: jelly or moldable substance)

1937: Langmuir used it to describe the ionized gas.
 blood plasma carries white and red corpuscles
 $\left\{ \begin{array}{l} \text{swells} \\ \text{positive} \\ \text{negative} \end{array} \right.$ charges.
 $\left\{ \begin{array}{l} \rightarrow \text{radio transmission} \\ \rightarrow \text{current rectification} \end{array} \right.$
 Ionospheric Oscillation tube. Gasous electron tube.

1940: Alfven and MHD for Sverker Flores and Wiesel.
 MHD $\left\{ \begin{array}{l} \rightarrow \text{magnetic reconnection} \\ \rightarrow \text{dynamo theory} \end{array} \right.$

1952: Controlled thermonuclear (hydrogen bomb).

1958: Van Allen radiation belts (space plasma).

1970: laser plasma physics \rightarrow plasma processing

plasma definition: (Qualitatively)

Plasma: It is an ionized medium which is

① Quasineutral

② exhibits a collective behaviour

that is - plasma is a state of a very matter
 ① Ionized
 ② Quasineutral
 ③ Behave collectively.

State: Condensed
 - plasma gas of
 - liquid of
 - solid of

* Ionization: net charge $\neq 0$

neutral gas

+ + + +
- - - -

$Q = 0$: (net charge $\Delta Q = 0$)

Ionized gas

+ + + + +
- - - - -

$Q \neq 0$ (net charge $\Delta Q \neq 0$)

$Q_+ > Q_-$
 $Q_- > Q_+$

- It is a process by which an atom or molecule acquires a negative or positive charge by gaining or losing energy.

$E_{ion} = 13.6 \text{ eV/atom}$
 $E_0 = 12.5 \text{ eV/atom}$

* Quasineutrality: (locally)

neutral gas

$\Delta Q = 0$

Ionized gas

$\Delta Q \neq 0$

Plasma gas

$\Delta Q \approx 0$

↳ charge imbalance locally

- For an ionized gas to be plasma

charge imbalance globally

$E \approx 0 \Rightarrow$

$Q_+ \approx Q_-$
 $\Delta Q \neq 0$

→ Saha eq:

$\frac{n_i}{n_n} \sim 2.4 \times 10^{-21} \frac{T^{3/2}}{n_i} e^{-U_i/k_B T}$

→ for ordinary air: $\frac{n_i}{n_n} \approx 10^{-122}$; U_i : ionization energy

③

⊕

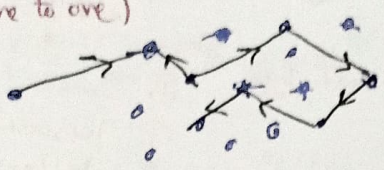
Plasma free
The linear
 $n_e = n$
 $\frac{dn_e}{dt}$
 $\frac{d}{dt}$

* Collective behaviour:

(i) Individual behaviour:

neutral gas

- Short-range interaction.
- Contact force.
- Mutual collision (one to one)
- Straight line trajectory
- one time interaction



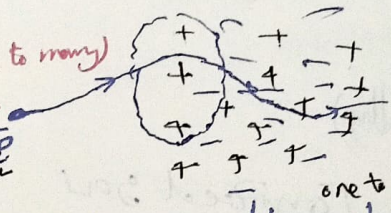
Hard sphere collisions
Short-range interaction

- The particles interact only with one at a time.
- The ν doesn't feel the other.
- The orbit trajectory of the particle is a straight line.
- The motion is controlled by short-range force such as collisions force or contact force.

(ii) Collective behaviour:

Plasma

- long-range interaction
- field force
- Coulomb collision (one to many)
- Curvilinear trajectory
- Simultaneously interact



Coulomb collision
long-range interaction

inter-particle interaction
short collision
long & electromagnetic
one to many

- The particles interact simultaneously with many other charged particles (not just mutual collision).
- The net force on the charge due to many other charges in certain range.
- The trajectory of the particle is curvy.
- The motion is controlled by long-range force or field force such as EM force.

Neutral state

- net charge = 0
- Individual behavior
- Short range interaction: ~~Coulomb force~~ hard sphere
- contact force
- mutual collision or binary collision
- straight line trajectory

Plasma

- net charge $\neq 0$
- collective behaviour
- long-range interaction: Lorentz force
- field force
- Coulomb collision
- curvy trajectory

Plasma frequency:

Plasma parameters → time scale → length
 (natural frequency)

The linearized eqs:

$$n_e = n_0 + n_e(r,t) \quad (1)$$

$$\frac{\partial n_e(r,t)}{\partial t} + n_0 \nabla \cdot u_e(r,t) = 0 \quad (2)$$

$$\frac{\partial u_e}{\partial t} = -\frac{e}{m_e} E(r,t) \quad (3)$$

$$f = -e[n_0 + n_e] + e n_0 = -e n_e(r,t) \quad (4)$$

$$\nabla \cdot E = \frac{f}{\epsilon_0} = -\frac{e}{\epsilon_0} n_e(r,t) \quad (5)$$

$$\nabla \cdot (3) \Rightarrow \frac{\partial}{\partial t} \nabla \cdot u_e = -\frac{e}{m_e} \nabla \cdot E$$

$$(2) \Rightarrow \nabla \cdot u_e = \frac{1}{n_0} \frac{\partial n}{\partial t}$$

$$\frac{1}{n_0} \frac{\partial}{\partial t} \frac{\partial n}{\partial t} = -\frac{e}{m_e} \nabla \cdot E \rightarrow m_e \frac{\partial^2 n}{\partial t^2} = -\frac{e n_0}{m_e} \nabla \cdot E$$

from (5) $\Rightarrow \frac{\partial^2 n_e}{\partial t^2} = -\frac{n_0 e}{m_e} \cdot -\frac{e}{\epsilon_0} n_e(r,t)$

- Restoring force: $-e^2 n_0 / \epsilon_0 n_e \equiv -kx$

- Inertia: $m_e \frac{\partial^2 n_e}{\partial t^2} = -\left(\frac{n_0 e^2}{\epsilon_0 m_e}\right) n_e(r,t)$

$$\frac{\partial^2 n_e}{\partial t^2} + \omega_p^2 n_e(r,t) = 0$$

where $\omega_p^2 = \frac{n_0 e^2}{\epsilon_0 m_e} \Rightarrow \omega_p = \sqrt{\frac{\epsilon_0 n_0 e^2}{m_e}}$

$$\omega_p = \left(\frac{e^2 n_0}{\epsilon_0 m_e}\right)^{1/2} \approx C \sqrt{n_0} \text{ rad/sec}$$

$n_0 (\text{cm}^{-3})$ $\omega_p \approx 58 \sqrt{n_0} \text{ rad/sec}$
 $C = 5.64 \times 10^4$
 $f_p \approx 9 \sqrt{n_0} \text{ Hz}$
 $\omega_p \approx 1.88 \times 10^{11} \text{ rad/sec}$
 $f_p \approx 2880 \text{ Hz}$

(5) $n_e(r,t) = n_e(r) e^{-i\omega_p t}$
 $n_e(r,t) = n_e(r) \cos(\omega_p t)$

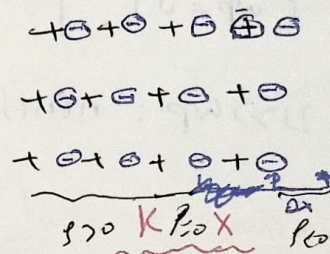
Assume a plasma consists of electrons with charge (-e) and mass m_e and same number of ions (+e) and mass m_i : $n_i = n_e = n_0$
 $T_i = T_e = T$



- Restoring force: $E \propto n_0$
 - Inertia: m_e

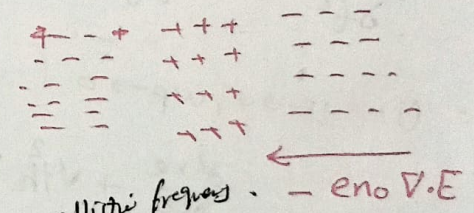
$$F = K \nabla \cdot E$$

$$K \propto \frac{e^2 n_0}{\epsilon_0}$$



$$F = -Kx$$

$$K \propto \frac{e^2 n_0}{\epsilon_0} \nabla \cdot E$$



→ plasma frequency
 $\omega_p \sim \frac{N e e^2}{\epsilon_0 m}$
 electrical conductivity

$$\omega_p = \sqrt{\frac{K}{m}}$$

$$K \equiv \frac{e^2 n_0}{\epsilon_0}$$

medium const

$\tau_p = \frac{1}{\omega_p}$: plasma time response to external perturbation
 $\sim 5.6 \text{ ps}$
 microwave

Debye Length

1) Test charge in Vacuum:

- The electrostatic potential due to Q (Coulomb's potential)

$$\nabla \cdot E = \frac{Q}{\epsilon_0}$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \nabla \times (\nabla \phi) = 0, E = -\nabla \phi$$

$$\nabla \times B = \mu_0 (J + \epsilon_0 \frac{\partial E}{\partial t})$$

$$E = -\nabla \phi$$

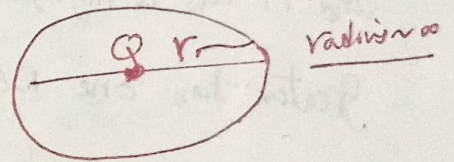
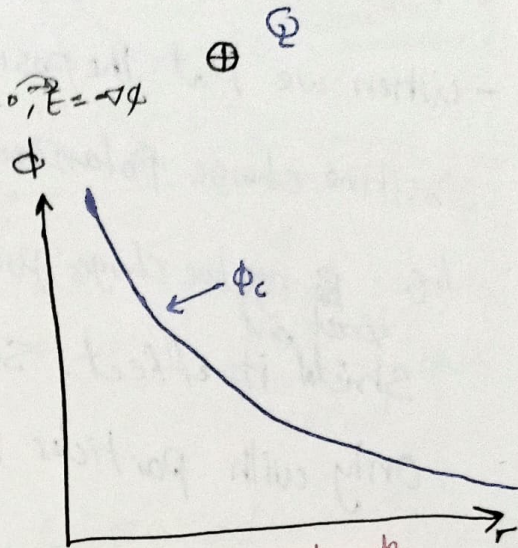
$$\nabla^2 \phi = -\frac{Q}{\epsilon_0} \Rightarrow \frac{d^2 \phi(r)}{dr^2} = -\frac{Q}{\epsilon_0}$$

$$\phi_c(r) = \frac{1}{\epsilon_0} \frac{1}{4\pi r} Q$$

\sim circumference of sphere.
 $\phi_c \sim \frac{1}{r}$

Limit $\phi_c(r) \rightarrow 0$

$r \rightarrow \infty$ radius of effect (influence)



(ii) Test charge in plasma:

$$\nabla^2 \phi = -\frac{Q}{\epsilon_0} + \frac{\rho}{\epsilon_0} ; \rho = en_0 e^{\frac{e\phi}{k_B T}} \approx en_0 (1 + \frac{e\phi}{k_B T} + \dots)$$

$$\rho \approx \frac{e^2 n_0}{\epsilon_0 k_B T} \phi$$

spatial simple harmonic motion \rightarrow oscillation in space.

$$\nabla^2 \phi_0 - \left(\frac{e^2 n_0}{\epsilon_0 k_B T} \right) \phi_0(r) = -\frac{Q}{\epsilon_0}$$

$$\nabla^2 \phi_0 + k_D^2 \phi_0(r) = -\frac{Q}{\epsilon_0} ; k_D^2 = \frac{e^2 n_0}{\epsilon_0 k_B T}$$

ω spatial frequency

$$k_D^2 = \frac{e^2 n_0}{\epsilon_0 k_B T}$$

\rightarrow create displacement

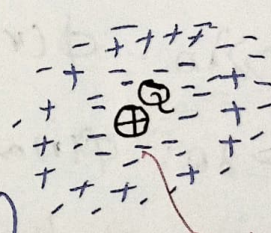
$$\lambda_D = \frac{\epsilon_0 k_B T}{e^2 n_0}$$

$$\phi_D(r) = \phi_c e^{-k_D r}$$

$$\phi_0(r) = \phi_c e^{-\frac{r}{\lambda_D}}$$

$$\phi_D \sim \frac{1}{r} e^{-r}$$

at $r \rightarrow \lambda_D$ $\phi_0 = 0.37 \phi_c =$
at $r \rightarrow 10\lambda_D$ $\phi_0 = 0.01 \phi_c \rightarrow 0$



polarization effect for negative charges. Restoring force.

- At $r \gg \lambda_D$

$\phi \rightarrow 0$

* Physical mechanisms

- When we put the positive test charge in the plasma, the positive charge polarizes (attracts) the negative charge, i.e. ~~the~~ negative charges polarize toward the positive charge and shield its effect. So, a charged in plasma interacts effectively only with particles situated at distance less than one Debye length and it has a negligible influence on particles lying at distances greater than one Debye length.

$\frac{e^2 n_0}{k_B T}$

$\phi(r) \propto \frac{1}{r} e^{-r/\lambda_D}$

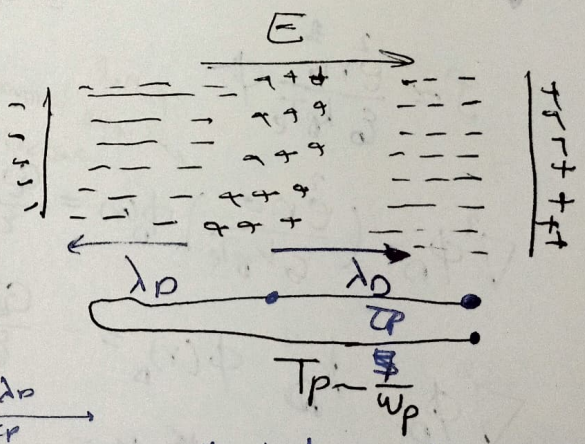
(i) $r \ll \lambda_D$: $\phi(r) \rightarrow \phi_{vac}(r)$: charge in vacuum.

(ii) $r \gg \lambda_D$: $\phi(r) \rightarrow 0$: charge in plasma.

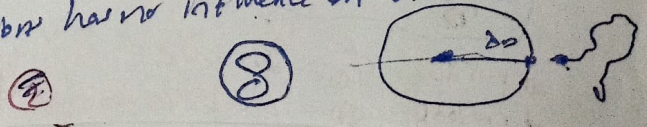
(iii) $r = \lambda_D$: $\phi(r) \sim \frac{\phi_{vac}}{e} \sim 0$

* Applying an Electric Field:

- When we apply an electric field with speed v_{th} , the plasma will displace a distance λ_D ~~in~~ then it will oscillate with frequency ω_p .



- ① λ_D : is the maximum distance over which the electrons can move with respect to the ions, against the electrostatic force with its thermal energy
- ② the distance at that electrons more it at λ_D with velocity v_{th} (maximum displacement of oscillation)
- ③ the distance beyond it, the ions has no influence on other charges



$$D = \sqrt{\frac{e^2 n_0}{\epsilon_0 k_B T}}$$

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{e^2 n_0}} \approx 6.9 \sqrt{\frac{T_e (K)}{n_0 (cm^{-3})}} \text{ cm}$$

$$\approx 7340 \text{ m} \sqrt{\frac{T_e (eV)}{n_e (m^{-3})}} \propto \sqrt{T_e} \propto \frac{1}{\sqrt{n_e}}$$

* Typical laboratory plasma:

$$T_e \sim 1 \text{ keV} = 10^3 \text{ eV}$$

$$n_e \sim 10^{19} \text{ m}^{-3} ; n = 10^{13} \text{ cm}^{-3} \text{ in vacuum.}$$

$$- \lambda_D \approx 10^{-4} \text{ m} \approx 10^{-2} \text{ cm} ; \omega_{pe} \approx 1.8 \times 10^{11} \text{ rad/sec} ; f_{pe} \approx 28 \text{ GHz (microwaves)}$$

$$- D_x \approx n^{-1/2} \approx 0.5 \times 10^{-6} \text{ m} \approx 0.5 \mu\text{m} \quad \text{U}_{th} \approx 3 \times 10^7 \text{ m/sec.}$$

$$- a_0 \text{ (Bohr radius)} \approx 10^{-10} \text{ m}$$

$$10^{-8} < 10^{-5} < 10^{-2} < 1$$

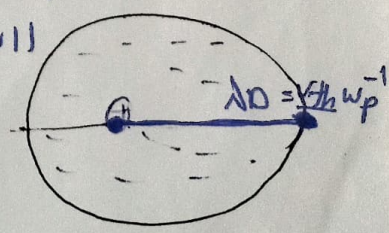
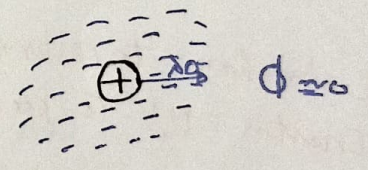
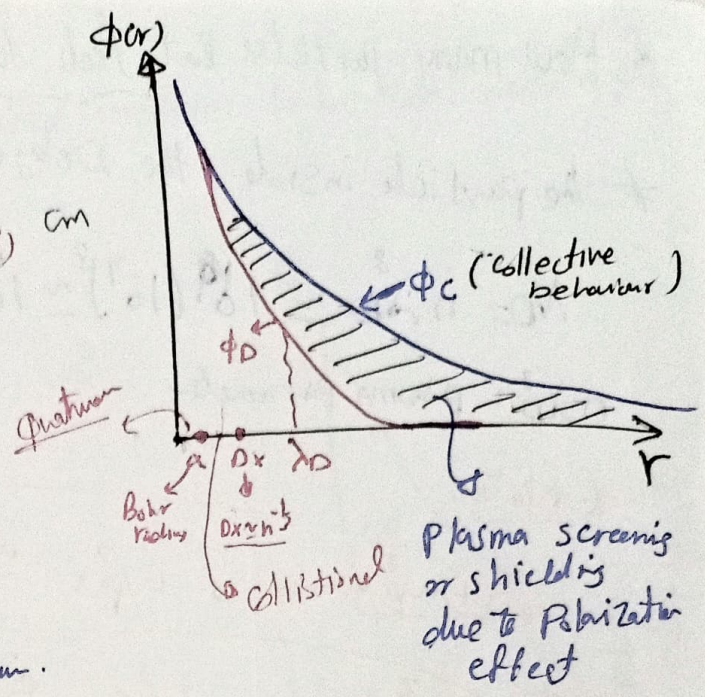
$$a_0 < D_x < \lambda < L$$

* Physical interpretation:

- $v_{the} = \omega_p \lambda_D$; thermal velocity:

- Debye length:

- ① The length at which the potential $\phi = 0$.
i.e: if there is a charge at $r > \lambda_D$, it will not affect by ϕ .
- ② The length of shielding a test charge.
- ③ $\sim \sim$ the test charge interacts collectively with other charges.
- ④ The distance that the charge moves at $t \approx \frac{1}{\omega_p}$ with velocity v_{th} .
as v_{th} increases, v_{th} increases, so the charge will move further distance λ_D until it stop.
- ⑤ The distance over which, the plasma is neutral.



⑨

* How many particles that feels the collective behaviour (Coulomb potential) Plasma

≠ the particle inside the Debye sphere.

$$N_D = n \lambda_D^3 \approx 10^9 (10^{-7})^3 \approx 10^6 \frac{\#}{m^3}$$

$n \lambda_D^3$: plasma parameter.

* Notes:

$$\frac{\delta n e}{\delta t^2} \oplus \omega_p^2 m = 0 ; \omega_p^2 = \frac{e^2 n_0}{\epsilon_0 m e} ; n \sim e^{-i\omega t} ; + \equiv -i$$

$$\frac{\delta \phi}{\delta x^2} \ominus k_D^2 \phi = 0 ; k_D^2 = \frac{e^2 n_0}{\epsilon_0 k_B T_e} ; \phi \sim e^{-k_D x} ; - \equiv -1 ;$$

→ temporal frequency:

$$\omega_p^2 = \frac{\text{Restoring force to neutral}}{\text{Perturbation (inertia)}} = \frac{e^2 n_0 / \epsilon_0}{m} ; \omega_p \sim \frac{1}{T_p} ; T_p: \text{Response time}$$

* Spatial frequency:

$$k_D^2 = \frac{\text{Restoring force to}}{\text{Inertia}} = \frac{e^2 n_0 / \epsilon_0}{k_B T_e} ; k_D \sim \frac{1}{\lambda_D} ; \lambda_D: \text{Response length}$$

* Restoring force: to restore the neutrality or electrical equilibrium

* Inertia: try to keep the non-equilibrium situation.

→ charge imbalances may exist only over a short distance (Debye length) or for a short period of time (inverse of plasma frequency).



Plasma criteria: (Quantitatively)

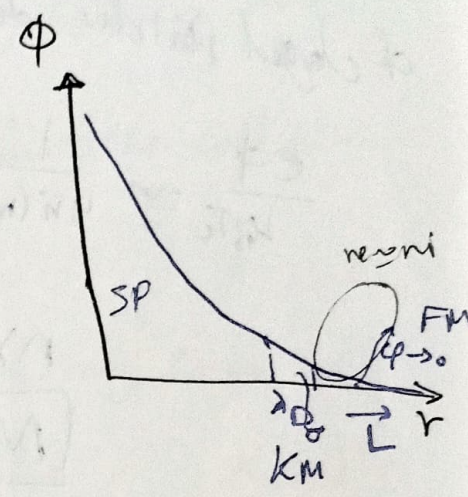
The conditions for any ionized medium to be considered a plasma

① Quasineutrality:

- From Gauss's law:

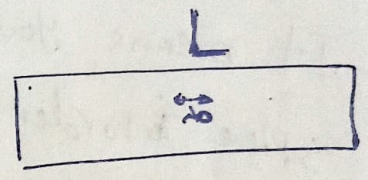
$$\nabla \cdot \mathbf{E} = \nabla^2 \phi = ne - ni$$

$$\frac{\lambda_D^2}{L^2} \phi = ne - ni$$



For $\lambda_D \ll L$: LHS = 0

$$\therefore ne - ni \approx 0 \rightarrow ne \approx ni$$



So, the condition for quasineutrality

$$L \gg \lambda_D$$

plasma size: L.
high temperature and low dense plasma

- Typical laboratory plasma

$$L \approx 10 \text{ cm} \approx 0.1 \text{ m} \approx 10^3 \lambda_D$$

* Conditions:

- ① collective behavior: $\Gamma \ll 1; N_D \gg 1$; necessary condition
- ② quasineutrality: sufficient \approx
- ③ $\omega_p \gg \nu_c$; sufficient

② Collective behaviour:

- that the mean interparticle distance, collective interaction of charged particles dominate over binary interaction.

$$\frac{e\phi}{k_B T_e} \approx \frac{1}{4\pi(n_e \lambda_D^3)^{2/3}} \ll 1$$

$$n \lambda_D^3 \gg 1$$

$$\boxed{N_D \gg 1}$$

$$\Gamma = \frac{e\phi}{k_B T_e} \ll 1 \rightarrow \text{one to one interaction}$$

$$\Gamma = \frac{e\phi}{k_B T_e} \ll 1 \text{ (weak coupling)}$$

↳ field interaction.

- It means you need so many particles inside the Debye sphere to interact collectively.

- Typical laboratory plasma:

~~$$n \sim 10^{19} \text{ m}^{-3}$$~~

$$N_D \sim 10^6 \gg 1$$

- there is a million particles inside the Debye sphere.

$$\therefore L \approx 10^3 \lambda_D \Rightarrow (N_D)^{1/3} \approx 10^2 \text{ m}^{-3}$$

- there is $10^{19} \frac{\text{m}^{-3}}$ in L .

→ Another meaning (Coupling parameter) Γ :

$$\Gamma = \frac{E_c}{E_{Th}} \ll 1$$

↳ plasma parameter

$$\frac{\text{Thermal fluctuation}}{\text{Collecting thermal energy density}} = \frac{\epsilon_0 |E|^2}{k_B T_e} \sim \frac{1}{n \lambda_D^3} \ll 1$$

$$\boxed{n \lambda_D^3 \gg 1}$$

- So the thermal fluctuation's level is so small compared to the thermal energy density

Weakly collision (weakly coupled):

- The long-range interaction force is dominant over the short-range interaction force or binary collision force.
- The plasma oscillate many times before, the collision happen.

- the time response of the plasma to the long-range force is smaller than the time response to the collision:

$$\nu_c \sim \frac{\omega_p}{\Lambda} \ln \Lambda \quad ; \quad \Lambda = \frac{4\pi n e^2}{3} \lambda_D^3$$

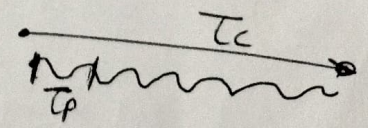
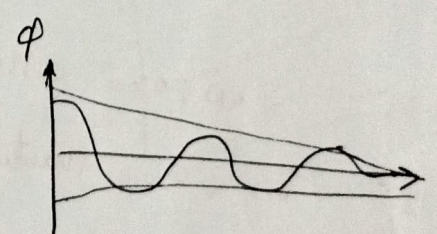
diffuse low dense plasma high temperature \leftarrow (i) $\omega_p \gg \nu_c$: weakly coupled plasma: $\Lambda \gg 1$
 collision do not interfere with plasma oscillation
 dense low temperature \leftarrow (ii) $\nu_c \gg \omega_p$: strongly coupled plasma: $\Lambda \ll 1$
 collisions effectively prevent plasma oscillation

$$\therefore (n \lambda_D)^3 \gg 1$$

(i) $\boxed{\nu_c \ll \omega_p}$ (light-damping)

(ii) $\boxed{\tau_c \gg \tau_p}$

(iii) $\lambda_{mfp} \gg L$: collisionless plasma.



$$\lambda_{mfp} = \frac{v_{th}}{\nu_c} \approx v_{th} \tau_c$$

ν_c : is the inverse of the typical time needed for enough collisions to occur that the particle trajectory is deviated through 90° or 90° scattering rate.

* Plasma in a tokamak:

$n_e \approx 10^{14} \text{ cm}^{-3}, T_e = 1 \text{ keV}, L = 20 \text{ cm}$

So, the plasma parameters

$v_{the} \approx 2 \times 10^9 \sqrt{\frac{2T_e}{m_e c}} \approx 2 \times 10^9 \text{ cm/sec}$

$\omega_{pe} \approx 5.6 \times 10^{11} \text{ sec}^{-1}$

$\lambda_{De} \approx 0.003 \text{ cm}$

$L = 20 \text{ cm}$

$ND = n_e d \approx 3 \times 10^6$

a_0 (Bohr radius) $\approx 10^{-8} \text{ cm}$
 $Dx \approx n^{-1/3} \approx 10^{-5} \text{ cm}$ --- Interparticle distance.

⇒ Plasma conditions:

① $a_0 \ll Dx \ll \lambda_D \ll L$
 ← No quantum effect
 $\text{cm } 10^{-8} \ll 10^{-5} \text{ cm} \ll 3 \times 10^3 \text{ cm} \ll \frac{1}{20} \text{ cm}$

② $ND \gg 1$

① $\lambda_D < Dx$: collisional

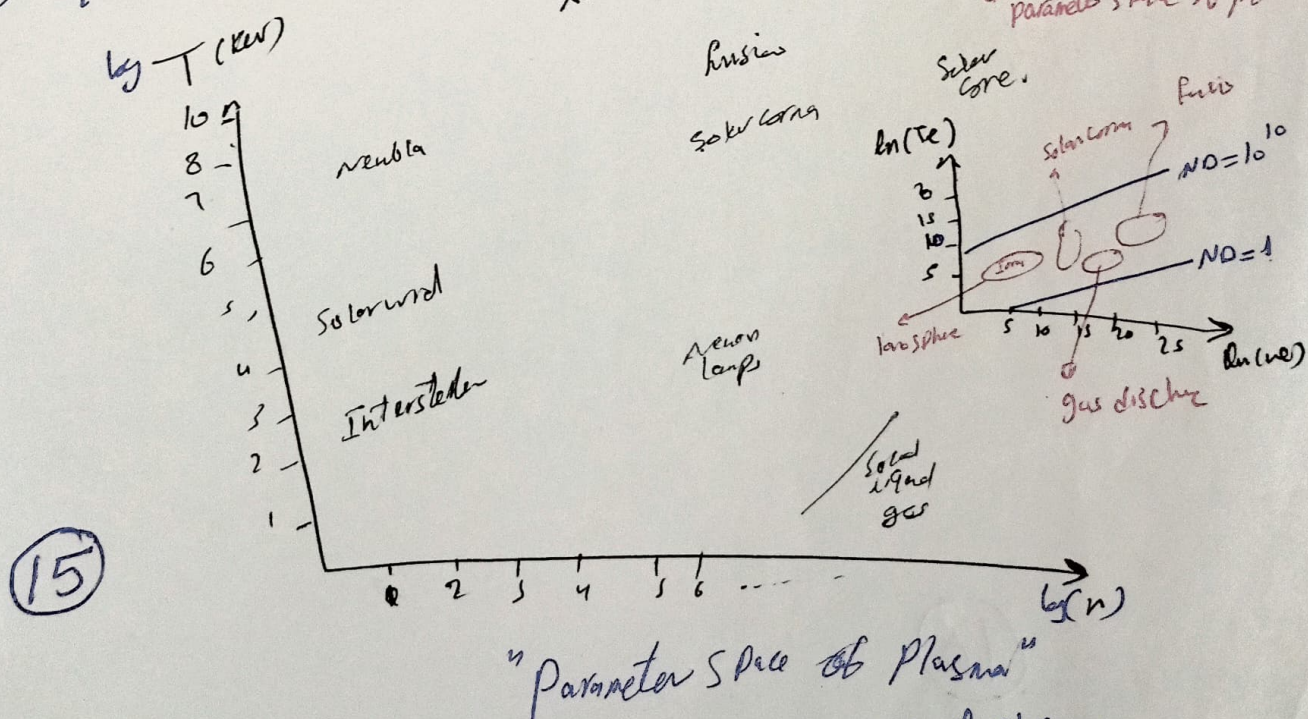
② $\lambda_D < a_0$: quantum

Plasma Classification (Scale): Category

- ① Laboratory plasma: (earth-scale length scale).
- ② Space plasma: (Solar-system length scale).
- ③ Astrophysics plasma: (Galaxy-length scale).
- ④ Cosmology plasma: (universe-length scale).

* Plasma Category: Classification by n_e, T_e
 $n^{-1/2} \ll \lambda_D \ll L$

- ① classical plasma: $L \gg \lambda_D$ OR $T_{th} \gg T_F$.
 $\lambda_D \ll d$; in gtd. $n_e^{-1/2}$
- ② Quantum plasma: $L \sim \lambda_D$ OR $T_F \gg T_{th}, E_T < 2.75 E_C$
 $\lambda_D \ll a_0$ ~ parameter space of plasma



(15)

- ③ relativistic plasma: $KT > m_e c^2$
- ④ collisional plasma: $\lambda_D > \lambda_{De}$, $\lambda_{De} < 1$
interparticle dist

* Plasma applications:

* Fusion:

Lawson criterion

$$n\tau > 1.7 \times 10^{14} \frac{\text{cc} \cdot \text{cm}^3}{\text{sec}}$$

low density

long confinement time

MCF: $n = 10^{14} / \text{cc}$

high density

$$\rho\tau = 1 \text{ sec}$$

short-confinement time

ICF: $n = 10^{25} / \text{cc}$

$$\rho\tau = 10^{-11} \text{ sec}$$