Plasma Models

*Outlines:

i- Introduction

ii- Particle Model (PM).

iii- Kinetic Model (KM)

iv- Fluid Model (FM)

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علوم - المنصورة

i- Introduction:

Topic Aspect

- Postulated Aspects
  - Postulates
    - theory
- Practical Aspects
  - Assumptions
    - Models

Particle mechanics

- theory
  - Classical Mechanics
- Models
  - Newtonian
    - F
  - Lagrangian
    - L
  - Hamiltonian
    - H

pes:

1- Nanoscope: QM.

2- Microscope: CM

3- Mesoscope: SM

4- Macroscope: FM

5- Mondoscope: TM

6- Cosmoscope: GR

ii- particle Model (PM):

- Theory : classical Mechanics.
- Scope : Microscope
- Space : Configuration ($\vec{r}_j$)
- system : Individual particles

- State Variables : $\vec{r}, \vec{P}$
- length scale : $L < \lambda_D$.
- Governing equations : Newtonian + Maxwell

$$\frac{d\vec{P}}{dt} = \vec{F}_L \quad ; \quad \vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$$

$$+$$

Maxwell's eq

Situations:

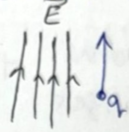
1- Static/Moving Charges in Vacuum:

→ No State change

2- Static / Moving in external Electric field:

→ the particle is accelerated in Electric field direction.

Accelerated translation Motion → Drift



3- static in external Magnetic field:

→ No state change

4- Moving charge parallel to external magnetic field:

→ No State change → Moves with Same velocity.

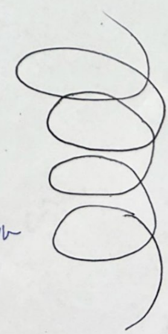
5- Moving charge perpendicular to external magnetic field:

→ Circulation motion in the plane perpendicular to & around the magnetic field

with radius $\vec{r}_c = \frac{m\vec{V}_\perp}{|q|\vec{B}}$ and Angular velocity $\vec{\Omega}_c = -\frac{|q|\vec{B}}{m}$

6- Moving charge oblique to external magnetic field:

→ Helical motion : circular motion normal to $\vec{B}$ + Linear motion Parallel to $\vec{B}$

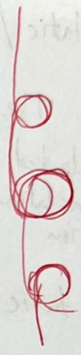


→ Same radius + Same velocity + linear motion with parallel velocity component.

7- Moving charge in external electric and magnetic field:

→ Cycloidal motion: Circular motion + Linear motion + Drift (Acceleration)

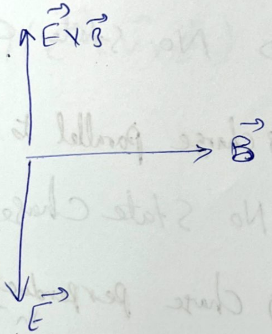
Circular motion ↓ $\perp \vec{B}$; Linear motion ↓ $\parallel \vec{B}$; Drift ↓ $\parallel \vec{B}$



→ radius changes + Linear velocity changes + Angular velocity changes.

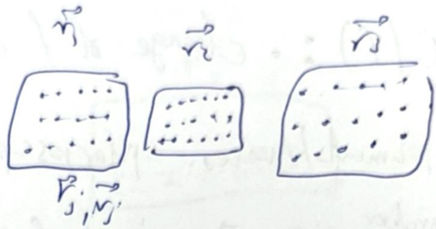
$\vec{r}_c = \frac{m\vec{v}_\perp}{q|\vec{B}|}$ $\cancel{\omega_c} =$ $\Omega_c = \frac{q|\vec{B}|}{m}$

→ Cycloidal motion in $\vec{E} \times \vec{B}$ direction



Kinetic Model (KM):

- Theory: Statistical Mechanics
- Scope: MesoScope
- Space: Phase $(\vec{r}_j, \vec{v}_j)$
- System: Parcel of particles



- State function: $f(\vec{r}, \vec{v}, t)$
- length scale: $L > \lambda_D$ ∴ $L \sim 10\lambda_D$
- Governing eq: Vlasov + Maxwell

$$\underbrace{\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f + \vec{a}\cdot\vec{\nabla}_v f}_{\text{Long-range interaction}} = \underbrace{C(f)}_{\text{Short-range interaction}}$$

- Phase space: Configuration space$(\vec{r})$ + velocity space$(\vec{v})$
- number density Distribution function: no of particles per unit Phase space Volume

$$f = \frac{d^6 N}{d^3\vec{r}\, d^3 v} = \frac{dN}{dV}$$

- $\frac{\partial f}{\partial t}$: time evolution of f.
- $\vec{v}\cdot\vec{\nabla}_r f$: - propagation in configuration space
  - Advect " " with velocity $\vec{v}$.

- $\vec{a} \cdot \vec{\nabla}_v f$: • Propagation in velocity space.
  - Advection ~ ~ ~ with velocity $\vec{a}$.

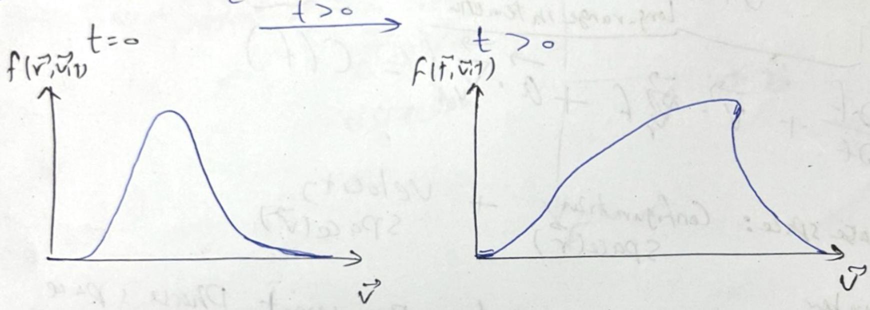
- $C(f)$: • change of f due to collision

- Moments/Averages : Macroscopic quantities

i. number density $\quad n(\vec{r},t) = \int f(\vec{r},\vec{v},t)\, d\vec{v}$

ii. momentum density $\quad \vec{p}(\vec{r},t) = m\, n(\vec{r},t)\, \vec{u}(\vec{r},t) = \int m\vec{v}\, f(\vec{r},\vec{v},t)\, d\vec{v}$.

iii. Kinetic Energy density $\quad E = \frac{1}{2} m n(\vec{r},t) u^2 = \int \frac{1}{2} m v^2 f(\vec{r},\vec{v},t)\, d\vec{v}$.

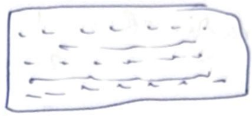


Fluid Model (FM):

Theory: Fluid Mechanics

- Scope: Macroscopic
- space: Physical ($\vec{r}$)
- System: Bulk
- state variables: $n, \vec{u}, p, E$.
- length scale: $L >> \lambda_D$: $L \sim 10^3 \lambda_D$.
- Governing eqs: Euler + Maxwell

$\vec{r}$,



$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{u}) = 0$$

$$mn\left[\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right] = q(\vec{E} + \vec{v}\times\vec{B}) + \vec{F}_{others}$$

OR

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix} \rho_m \\ \rho_p \\ \rho_E \end{pmatrix}}_{\text{time evolution}} + \underbrace{\vec{\nabla}\cdot\begin{pmatrix} \vec{J}_m \\ \vec{J}_p \\ \vec{J}_E \end{pmatrix}}_{\text{space propagation}} = \underbrace{\begin{pmatrix} F_m - S_m \\ F_p - S_p \\ F_E - S_E \end{pmatrix}}_{\text{Source}}$$

ρ: Quantity density

J: Quantity current density

F: Quantity Faucet

S: " Sink

$\vec{\nabla}\cdot\vec{J}$: Quantity Flux.

m: mass/matter

p: momentum

E: Energy

× [illegible]

- The Conservation of : 1- matter / mass
  2- Momentum
  3- Energy

- Conservation of : 1- matter
  2- momentum
  3- Energy

- transport of : 1- matter
  2- momentum
  3- Energy

- $\rho_m = mn$, $\vec{J}_m = mn\vec{u}$: matter
- $\rho_p = mn\vec{u}$, $\vec{J}_p = mn\vec{u}\vec{u}$: momentum
- $\rho_E = \frac{1}{2}mnu^2 + P$, $\vec{J}_E = (\frac{1}{2}mnu^2 + P)\vec{u}$: Energy

- Frame:

i- Euler: $\frac{\partial}{\partial t} \longrightarrow$ At specific position $\longrightarrow$ lab frame

ii- Lagrangian: $\frac{d}{dt} \longrightarrow$ Particle frame (moves with particle) $\longrightarrow$ Fluid frame

$$\boxed{\frac{d}{dt} - \frac{\partial}{\partial t} = \vec{u} \cdot \vec{\nabla}}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \longrightarrow \text{Advection Operator}$$