Quantum plasma

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* Contents:

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1- Scale:

- Defination: the size or scope at which physical phenomena are observed or analyzed.
- Fundemental Quantities: Length (L) & Time (T) & Mass (M)
- Types:

Scale

- Spatial (Length)
  - Small → QM — $L \sim \lambda_B$
  - Large → CM — $L \gg \lambda_B$
- Temporal (Time)
  - Slow → NRM — $\tau \gg \frac{L}{c}$
  - Fast → RM — $\tau \sim \frac{L}{c}$
- Energy (mass)
  - Low → ~~SRM~~ Not Standard model — $E \ll E_c$
  - High → ~~QRM~~ ~~SRM~~ Standard model — $E \sim E_c$

$E_c = \frac{m_e c^3}{e\hbar} \sim 10^{18}\ V/m$ → Schwinger limit

QED effects

Pair Production

oncepts & principles: Classical Vs Quantum

BC / Aspects		Classical	Quantum
1	Nature	Deterministic	Probabalistic
2	Identity	Distinct	Neither & Duality
3	Trajectory	Definite	Jumping
4	Space	Physical	Hilbert
5	State representation	$\vec{r}, \vec{p}$	$\psi(\vec{r}, t)$
6	State evolution	Newton	Schrodinger
7	Superposition	Distinct State	Mixed State
8	observables	measured values Continuous	Operator Quantized / Discrete
9	observer	Passive	Active
10	Measurements	Exact / Simultaneous / certain	~~uncertain~~ Approximate / ordered / uncertain.
11	Statistics	Maxwell-Boltzman	Fermi-Dirac Bose - Einstein
12	Causality	Absolute / well-defined	Relative / ill-defined
13	Exclustion	N Applicable	Applicable
14	Entanglement / locality	Local	Nonlocal
15	tunneling	Not possible	Possible

III- Quantum Plasma:

* Defination: Classical plasma + Quantum effects (for

↳ it can be called "Semi-classical Plasma".

* Parameters:

	Classical	Quantum
Length scale	L	(Wigner-Seitz radius)
time:	ω_p	ω_p.
speed:	V_{th}	V_F
temperature:	T	T_F
Energy:	E_{th}	E_F
shielding:	λ_D (Debye)	λ_F (Fermi)
Collision:	ν_c (kT)	ν_φ (kT_F)

* Existence Conditions:

	Classical	Quantum
①	$L \gg \lambda_D$	$r \sim \lambda_{dB}$
②	$n\lambda_D \gg 1$	$n\lambda_{dB} \gg 1$
③	$\nu_c \ll \omega_p$	$\nu_\varphi \ll \omega_p$

* Conclusion:

① $n > n_Q = \left(\frac{2\pi m k_B T}{\hbar}\right) \sim 10^{29}\ cm^{-3}$

Exp.: Gold. $n \sim 10^{22}\ cm^{-3}$

$T_F \sim 10^4\ K$.

② $T_F \gg T$

③ Degeneracy $\xi = \frac{P_F}{mc} \gg 1$

* Classical plasma: low density + Higher temperature + MB statistic

* Quantum plasma: High " + Low " + FD "

④

Plasma Models: Classical vs Quantum

Note that: the quantization appears only in particles not fields (E & B)

Model	Classical	Quantum
i-PM	<u>Newton-~~Poisson~~ Maxwell</u> $\frac{d\vec{P}_i}{dt} = \sum_i \vec{F}_i$ + Maxwell	<u>Schrodinger-Maxwell</u> $\hat{E}\,\Psi(\vec{r},t) = H\,\Psi(\vec{r},t)$ + Maxwell
ii-KM Moments	<u>Vlasov-Maxwell</u> $\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f + \frac{\vec{F}}{m}\cdot\nabla_v f = 0$ $f \approx MB$ $f \geq 0$ + Maxwell	<u>Wigner-Maxwell</u> $\frac{\partial f}{\partial t} + \frac{\vec{P}}{m}\cdot\vec{\nabla} f + \frac{1}{\hbar}[\hat{H}, f] = 0$ f: Wigner DF $f \gtrless 0$ + Maxwell Moments Madelung transformation
iii-FM	<u>Euler-Maxwell</u> Continuity momentum + Maxwell	<u>Euler-Maxwell</u> Continuity Momentum + Quantum effects + Maxwell $mn\left[\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right] = CF + QF$
	ii $\xrightarrow{\text{Moments}}$ iii	i $\xrightarrow[\text{transfer}]{\text{Madelung}}$ iii ii $\xrightarrow{\text{Moments}}$ iii

V- Quantum effects:

① Bohm potential:

- It is a quantum hydrodynamic effect.
- It includes: Wave-particle duality & quantum tunneling & quantum diffraction & quantum Diffusion.
- It emerges naturally when transferring Schrodinger to fluid:
- It arises due to wave function nature (spreading)
- Bohm potential: $V_B = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}}$

$$h \to 0 , \quad V_B \to 0$$
$$m \to \infty , \quad V_B \to 0$$
$$\frac{1}{\nabla} \to \infty , \quad V_B \to 0$$

- Schrodinger eq:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi$$

- Madelung transformation:

let: $\psi(\vec{r},t) = \sqrt{n(\vec{r},t)}\, e^{\frac{i S(\vec{r},t)}{\hbar}}$

$$\vec{\nabla} S = \vec{P}$$
$$n = |\psi|^2$$

∴ the quantum Hydrodynamic:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$$
$$mn\left[\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}\right] = -\nabla V_c - \nabla V_B$$
$$V_B = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}}$$

$F_B = -\vec{\nabla} V_B$;
↳ Bohm force.

Quantum tunneling: the total Energy

$$E = V_C + V_B$$

So, even if the classical energy of particle is small compared to the potential, the Bohm potential increases the potential of the particle and help him tunnel.

- Quantum diffraction / Diffusion: The Particle spreads up when it passes through small slit instead of moving in a straight line.

  due to ∇^2 operator.

- Wave - particle duality: the number density is connected with the wave nature

$$n = |\psi|^2.$$

2) Degeneracy pressure:

- Due to Pauli exclusion principle: when the system ~~become~~ compressed and become so dense, this pressure arises to prevent more compression, because fermion can't occupy the same quantum state.

- It is discovered in astrophysics objects.
- There are 3 possibles of implosion (Gravity > Nuclear force) Thermal
  - i- If $M_{star} < 1.4 M_\odot$: light star ⟶ White Dwarfs stars (ordinary matter & stable)
    ↳ chandrasekhar limit ~~stable~~ s.
  - ii- $M_{star} > 1.4 M_\odot$: white Dwarfs stars ⟶ stable Neutron stars ↳ neutrons:
  - iii- $M_{star} > 2.5 M_\odot$: neutron stars ⟶ stable Black hole
    ↳ Tolman-Oppenhimer Volkoff limit (TOV).

- Chandrasekhar equation of state:

$$P = A f(\xi)$$

$$A = \frac{\pi m^4 c^5}{3h^3}$$

Pressure: thermal: $\propto T$. Degeneracy: $\propto n$.

$$f(\xi) = \left[\xi(2\xi^2-3)\sqrt{1+\xi^2} + 3\sinh^{-1}\xi\right]$$

Degeneracy Parameter ξ:

$$\xi = \frac{P_F}{mc} = \frac{V_F}{c} = \left(\frac{n}{n_Q}\right)^{\frac{1}{3}} \quad ; n_Q \sim 10^{29}\,\text{cm}^{-3}$$

$$= \frac{E_f}{\hbar\omega_p} = \frac{\text{Fermi energy}}{\text{plasmon energy}} \qquad E_f > E_p$$

$$P_s \begin{cases} \xi \ll 1 : NR : \propto n^{5/3} \\ \xi \gg 1 : UR : \propto n^{\frac{4}{3}} \end{cases}$$

* Note: Explosion: thermal > gravity. Implosion: thermal < gravity.

Correlation - Exchange:

- Electrons don't interact through Coloumb potential; but through.

(i) Exchange: - Due to Pauli exclusion principle, identical electrons Can't occupy the same quantum state.

- When the electrons exchange OR swapping its position with each other leads to

$$\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$$

- ~~Parall~~ Electrons with parallel spin doesn't like to stay close to each other, so they repel each other → Repulsion force.

(ii) Correlation: - Due to spatial distribution nature of the electrons, the electrons motion correlated to each other.

- When the electrons moves, they adjust its position to become close to each other
  ↳ Attractive force.

- $E_{xc} = -\nabla(V_x + V_c)$

$V_{xc} = a_1 n^{\frac{1}{3}} + a_2 n^{\frac{2}{3}}$.