Plasma Waves

* Outlines:

i- Why Waves?

ii- Terminology & Concepts.

iii- Classifications

iv- Wave equation & Dispersion Relation (DR)

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د/ إبراهيم العتاس
علوم - المنصورة

i- Why Waves?

~~1- It describes~~

1- Because everything in physics oscillates or propogates in wave-like form.

From smallest particles in quantum mechanics to the largest structure in the universe, "From QM to Cosmology".

2- Because it describes how the matter & energy & momentum & information transport/transfere through space and time.

3- Because it describes how nature communicates, through sound, light, ---.

Waves are nature massengers.

4- Because it ~~provide~~ can employed as an indirect diagonstic tool.

5- Because it can describes the nature as a mathematical framework.

Thus: Waves are the backbone of modern physics and technology - without them, there would be no light, no sound, no communication, and no quantum mechanics.

erminology & Concepts

system: is any well-defined set of physical components that interact according to known laws and it has a clear boundaries separating it from the surrounding environment.

Exp: Particle & Field.

- **state:** a complete description of the system at a given moment, defined by a set of physical properties (state variables or state functions) that determine its behaviour and evolution over time.
  - The state provides all necessary information that define its present condition and to predict the system's future behaviour using the laws of physics.
  - state variables: measurable physical quantities
  - " equations: mathematical expressions that describe how the system evolves.

Exp: CM: $\vec{r}, \vec{p}$

TM: T, P, V

QM: $\psi(\vec{r}, t)$

SM: $f(\vec{r}, \vec{v}, t)$

- **Equilibrium state:** The state in which, No net change over time for the State Variables.
  - i- Static equilibrium: No force → $\vec{F} = 0$
  - ii- Dynamic " : Balanced force → $\sum_j \vec{F}_j = 0$

- **Perturbation:** a small external disturbance or deviation from equilibrium state.

- **Response:** How the system reacts to the Perturbation
  - i- Shielding
  - ii- oscillation
  - iii- Wave.

- Shielding: The ability to reduce, absorb, counteract the exter. Perturbations to maintain its Equilibrium State.

- oscillation: a repetitive motion around the equilibrium position due to the effect of
  - i- Inertia force
  - ii- Restoring force.

- Wave: a disturbance that Propagates through Space and time, transferring energy without a net tranport of matter.

...ve Classifications

based on:

1- Medium: i- Mechanical: Needs medium
ii- Electromagnetic: No medium.

2- Propagation:
i- Longitudinal: $\vec{k} \parallel \vec{u}$.
ii- Transversal: $\vec{k} \perp \vec{u}$.

3- Magnetic field: i- Parallel: $\vec{k} \parallel \vec{B_0}$
ii- Perpenduclar: $\vec{k} \perp \vec{B_0}$
iii- Oplique: $\vec{k} \wedge \vec{B_0}$.

4- Electric field:
i- Electrostatic: $\vec{k} \parallel \vec{E}$, $\vec{k} \times \vec{E} = 0$, $\frac{\partial \vec{B}}{\partial t} = 0$
ii- Electromagnetic: $\vec{k} \perp \vec{E}$, $\vec{k} \times \vec{E} \neq 0$, $\frac{\partial \vec{B}}{\partial t} \neq 0$

5- Speed: i- Standing: $V_{ph} = 0$
ii- Travelling: $V_{ph} \neq 0$

6- Wavefront: i- plane
ii- spherical
iii- cylindrical

7- time & space: i- Pulse: $\lim_{\substack{x \to \infty \\ t \to \infty}} f \Rightarrow 0$
ii- Periodic: $f(t+T) = f(t)$, $f(x+L) = f(x)$
iii- Aperiodic: ...

iv- Wave equation & Dispersion relation:

- The spatial propagation and temporal evolution of any wave is governed by the generalized wave equation:

$$\frac{\partial^2 f(\vec{r},t)}{\partial t} - v^2(\vec{r},t)\, \nabla^2 f(\vec{r},t) = \mathcal{H}(f,\vec{r},\vec{v},t)$$

- $f(\vec{r},t)$: the amplitude of the wave.
- $v(\vec{r},t)$: the speed of the wave.
- $\mathcal{H}(f,\vec{r},\vec{v},t)$: a function that includes the properties of the medium.

(i) $\mathcal{H} = \mathcal{H}(f)$: Nonlinear medium $\rightarrow \vec{v}(f)$

(ii) $\mathcal{H} = \mathcal{H}(\vec{r})$: ~~In~~ Non homogenous " $\rightarrow \vec{v}(\vec{r})$

(iii) $\mathcal{H} = \mathcal{H}(t)$: Dynamical " $\rightarrow \vec{v}(t)$

(iv) $\mathcal{H} = \mathcal{H}(\vec{v})$: INISotropic " $\rightarrow \vec{v}(\vec{v})$

(v) : Non Dispersive : $\vec{v}(\vec{k})$.

- the speed of the wave:

$$V = \frac{\text{Restoring force}}{\text{Inertial force}}.$$

Exp:

1- Electromagnetic wave:

$$\frac{\partial^2 \vec{E}}{\partial t} - c^2 \nabla^2 \vec{E} = 0$$

$$V = C$$

2- Sound wave:

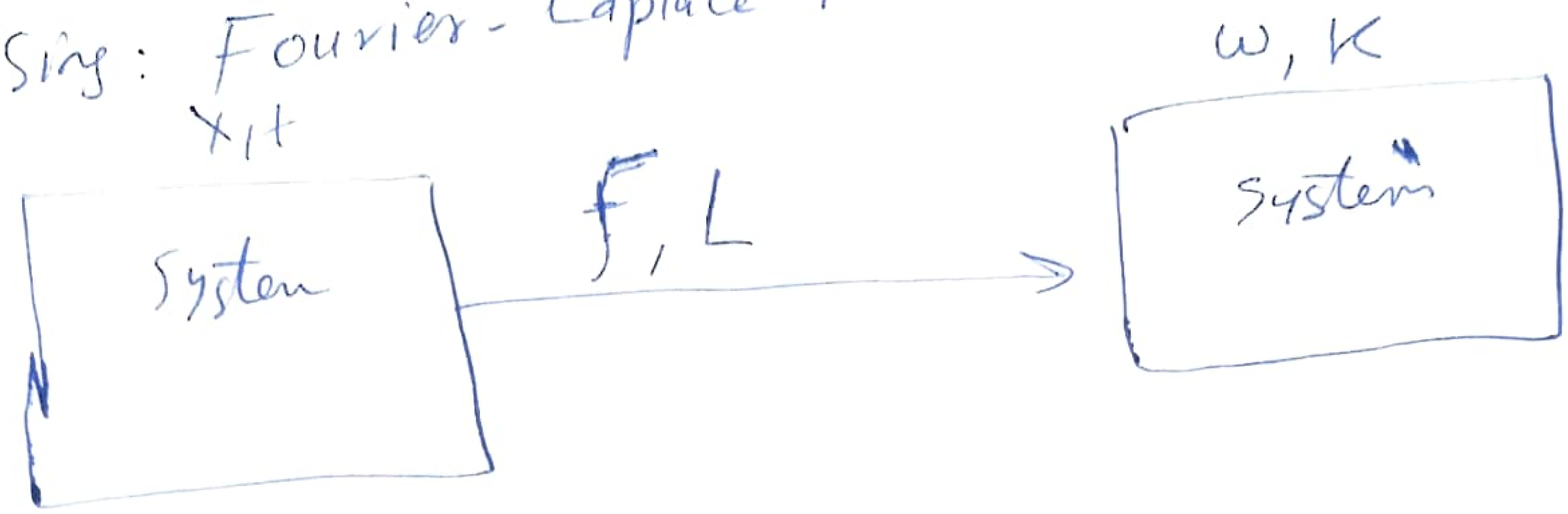
$$\frac{\partial^2 \rho}{\partial t} - C_s \nabla^2 \rho = 0$$

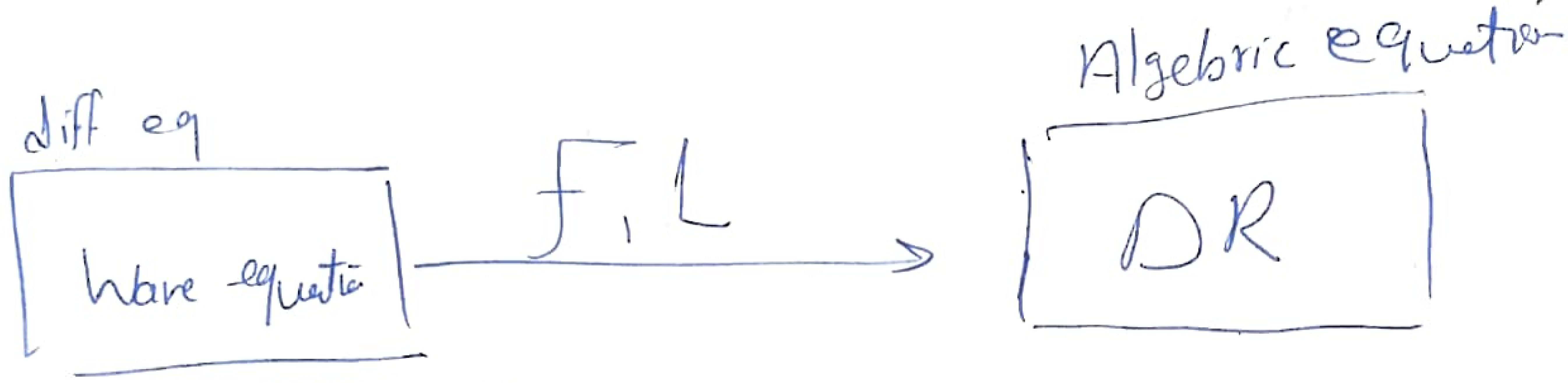
$$C_s = \sqrt{\frac{P}{\rho_0}}$$

Persion relation:

Also, ~~the~~ the wave characteristics can be determined ~~by the~~ without solving wave equation.

- By using: Fourier - Laplace transformation:





$$f = f_0 \, e^{i(Kx - wt)} \longrightarrow \omega = \omega(k)$$

- **Wave characteristics**

1- Wave speed
2- " width
3- " Amplitude
4- Wavelength & frequency.
5- Wave energy.

V- Plasma Waves:

$\vec{E}$ ← i- Electrostatic wave: $\frac{\partial \vec{B}}{\partial t} = 0$, $\tilde{E} \neq 0$

$\vec{E}, \vec{B}$ ← ii- Electromagnetic: $\frac{\partial \vec{E}}{\partial t} \neq 0$, $\frac{\partial \vec{B}}{\partial t} \neq 0$

$\vec{B}$ ← iii Hydromagnetic wave: $\frac{\partial \vec{E}}{\partial t} = 0$, $\frac{\partial \vec{B}}{\partial t} \neq 0$.

i- Electrostatic Waves:

electron OR Ion

a- Parallel waves: $k \parallel B_0$

1- Thermal wave: $\omega^2 = \omega_p^2 + \frac{3}{2} k^2 V_{th}^2$

2. Acoustic wave: $\omega^2 = V_s^2 k^2$

b- Perpendicular waves: $k \perp B_0$

1- Cyclotron wave: $\omega^2 = \Omega_c^2 + V_s^2 k^2$.

2- Hybrid wave: $\omega^2 = \omega_p^2 + \Omega_c^2 + V_s^2 k^2$

$= \omega_H^2 + V_s^2 k^2$

↳ Hybrid frequency

ii- Electromagnetic wave

Electrons only

a- Parallel waves: $k \parallel B_0$

1- Right wave: $\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1-(\Omega_c^2/\omega^2)}$

2- Left wave: $\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1+(\Omega_c^2/\omega^2)}$

b- Perpendicular wave: $k \perp B_0$:

1- Ordinary (O) wave: $\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$

2- Extraordinary (X) wave: $\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_H^2}$

Hydromagnetic wave

Plasma as a whole (Ions + electrons)

a- Parallel wave: $K \parallel B_0$

1- Alfven wave: $\omega^2 = K^2 V_A^2$

b- Perpendicular wave: $K \perp B_0$

1- Magnetosonic wave: $\omega^2 = C^2 \frac{V_s^2 + V_A^2}{C^2 + V_A^2} K^2$

* Note that: