Collisional Processes in Ionospheric Plasmas: Applications

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Abstract

Earth's ionosphere is an open laboratory that enable to study plasma in situ. Composed of a myriads of chemical species subject to ionization and recombination, it plays an important role for earth's environment. The ionosphere is stratified composed of many layers that is responsible of protecting earths from solar extreme radiations, meteorites and is crucial for communication by using satellites or wave reflection from ground.

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Why modeling?

main aim

giving the present predict the future by means of physical models

Important

- to find species density that are crucial for our life (ozone,....)
- study variations due to external perturbations such as solr wind, magnetic storms, cyclones and earthquake.
- ensure to satellite functionalities
- study space weather and its effect on our meteorology.

Focus on

Collision Theory in Earth's Ionosphere: From Fundamentals to Space Tech

Example: Blackout in the ionosphere

A disruption or complete loss of radio communication caused by disturbances in the Earth's ionosphere and have a direct impact on:

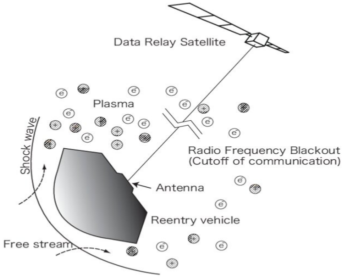
- HF (shortwave) signals are absorbed or scattered, affecting aviation, maritime, and military communications.
- GPS and GNSS signals experience errors due to ionospheric scintillation.
- Radar performance degrades.

Mainly due to

- Intense X-ray and ultraviolet (UV) radiation from solar flares ionize the D-layer.
- Charged particles from coronal mass ejections (CMEs) distort the ionosphere's structure, creating turbulence and density irregularities during Geomagnetic Storms.
- During auroral events, energetic particles collide with ionospheric gases, increasing ionization and absorption in the E-layer .

Satellites Blackout

Can be explained by



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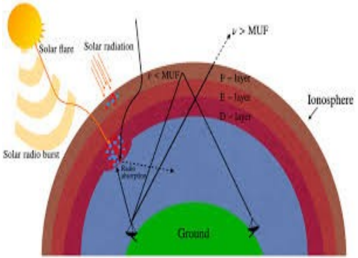

https://www.youtube.com/watch?v=kDCz5jBfJoc&t=141s

Altitude, km	Density, kg/m^3	T(K)	V(m/s)	AOA, Degree
85.0	8.18×10^{-6}	191.0	7577	20.0
80.0	1.85×10^{-5}	195.8	7609	20.0
75.0	4.07×10^{-5}	201.7	7593	19.2
70.0	8.83×10^{-5}	210.9	7542	19.2

Table: Atmospheric parameters versus altitude.

Boltzmann distribution

One of the greatest scientists who made significant contributions to statistical mechanics, which is:

- A pillar of modern physics.
- relates macroscopic parameters to microscopic ones.
- used in various fields, including medicine and stock markets.
- is based on probability theory.

Distribution

Boltzmann statistics is based on fundamental concepts

1. **Phase Space**
   For a system of N particles, each particles has 6 degrees of freedom: position $(x,\ y, z)$ noted $\mathbf{r}$ and momentum $(p_x,\ p_y,\ p_z)$ noted $\mathbf{p}$.
   The set: $(\mathbf{r}, \mathbf{p}) = (x,\ y,\ z,\ p_x,\ p_y,\ p_z)$ refers to Phase Space.
   The elementary volume in the phase space is

$$d^3 r d^3 p = dxdydzdp_x dp_y dp_z$$

2. **Indistinguishable particles**:
   The number of these particles is immense (Avogadro's number), and all particles within the same type are identical. Consequently, it is impossible to track the movement of individual particles.

Equi-partition

3. **Density**

   For any particle to have coordinates $(\mathbf{r},\ \mathbf{p})$ the probability is

$$dN = f(\mathbf{r}, \mathbf{p}, t)\, d^3\mathbf{r}\, d^3\mathbf{p}$$

   where $f((\mathbf{r},\ \mathbf{p}), t)$ is a probability density function such as the number of particles in the elementary volume of coordinates $(\mathbf{r},\ \mathbf{p})$ is

$$dN = f(\mathbf{r}, \mathbf{p}, t)\, d^3\mathbf{r}\, d^3\mathbf{p}$$

   the number of particles in a specific volume is

$$N = \int \int_{Domain} d^3\mathbf{p} d^3\mathbf{r}\, f(\mathbf{r}, \mathbf{p}, t)$$

4. Domains of the same volume have the same number of particles.
5. In the phase space the minimum volume is h^3 (Planck const.)

Boltzmann Equation

Under a force $\mathbf{F}$ at t, the change in a volume of coordinates $((\mathbf{r},\ \mathbf{p}))$ after a time period Δt are $\mathbf{r} + d\mathbf{p}$ and $\mathbf{p} + d\mathbf{p}$ such as,

$$f\left(\mathbf{r} + \frac{\mathbf{p}}{m}\Delta t, \mathbf{p} + \mathbf{F}\,\Delta t, t + \Delta t\right)\, d^3\mathbf{r}\, d^3\mathbf{p} = f(\mathbf{r}, \mathbf{p}, t)\, d^3\mathbf{r}\, d^3\mathbf{p}$$

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The total number of particle is conserved
Taylor development

$$f\left(\mathbf{r} + \frac{\mathbf{p}}{m}\Delta t, \mathbf{p} + \mathbf{F}\Delta t, t + \Delta t\right) d^3\mathbf{r}\, d^3\mathbf{p} - f(\mathbf{r}, \mathbf{p}, t)\, d^3\mathbf{r}\, d^3\mathbf{p} = \Delta f\, d^3\mathbf{r}\, d^3\mathbf{p}$$

There are two possibilities

$$\frac{df}{dt} = 0, \qquad \text{or} \qquad \frac{df}{dt} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

General case

Corresponds to

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} \qquad (*)$$

These equation can be solved to find the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ and. All macroscopic parameters can be obtained using,

$$< x >= \int_{Domain} xf(\mathbf{r}, \mathbf{p}, t) d^3 r d^3 p \qquad (*)$$

or obtain conservation equation by integrating Boltzmann equation.
For example by using momentum of order 0 (relative to V), we multiply by $v^0 = 1$ and integrating over the phase space

$$\int_{-\infty}^{\infty} \left\{ \frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} \right\} d^3 r d^3 p = \int_{-\infty}^{\infty} \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} d^3 r d^3 p$$

Fluid Equations

From the last equation and when the collision term vanishes, after some mathematical simplification one obtains,

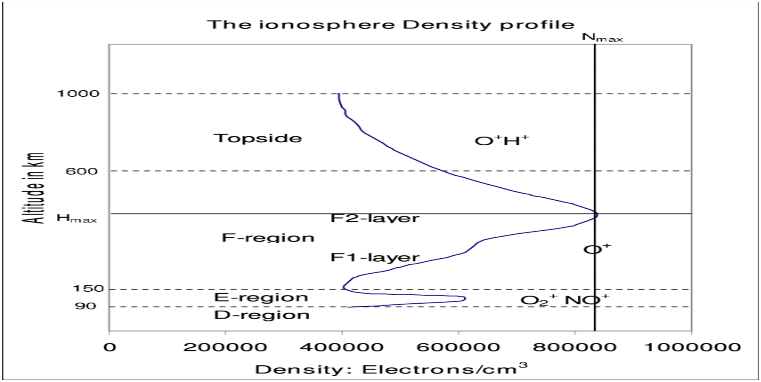
$$\frac{\partial n_j}{\partial t} + \nabla.(n_j \mathbf{v_j}) = 0$$

We want to find the density but this Equation gives a new unknown v. additional equation is needed to handle order 1 momentum mv. This procedure is endless.

The set of fluid equations has to be closed by an extra equation such us: ideal gas equation, or energy transfer process,...

What we want?

Let for example look after the density profile versus altitude in the Earth's ionosphere



Based on what we want (wave, energy transfer,..) we can focus on a certain region of the ionosphere.

Electron density

In principal, by solving equation (*) and using (**) we can find the density of electrons. However, we have to consider

1. The available data: If the main source of electrons is ionization by solar radiations (UV, EUV) then Boltzmann equations have to be written in terms of energy flux, $\phi_e(\mathbf{r}, E, \Omega, t)$

$$f_e(\mathbf{r}_e, \mathbf{v}_e, t) = \frac{m_e}{v^2}\phi_e(\mathbf{r}, E, \Omega, t) \tag{1}$$

To obtain

$$\begin{aligned}\frac{1}{v}\frac{\partial\phi_e(\mathbf{r}, E, \Omega, t)}{\partial t} + \frac{\mathbf{v}_s}{v}\nabla_r(\phi_e(\mathbf{r}, E, \Omega, t)) - n_e\frac{\partial}{\partial E}(L(E).\phi_e(\mathbf{r}, E, \Omega, t)) \\ = \frac{1}{v}Q(\mathbf{r}, E, \Omega, t)\end{aligned} \tag{2}$$

In this equation we have to consider all input/output

Supra-thermal electrons

2. Electrons flowing from higher layers have higher energy called suprathermal electron and all other electron sources have to be included in electron "budget",

$$P_\nu = n(z) \int_0^\infty \sigma_\nu(E)\phi(E)dE \tag{3}$$

3. Depending to the altitude range of the studied ionospheric region, sink/source terms of electron must be considered

$$X + Z \rightleftarrows XZ^+ + e$$

but the species X can also be involved in chemical reactions

$$X^+ + Y \longrightarrow X + Y^+$$

So we have to find $$I_{X^+Y}.n_Y = K_{X^+Y}.n_{X^+}.n_Y$$

This is a rough task to obtain reaction rates (reactions cross sections).

Model validation

For each species continuity equation must be added and if the transport is considered also momentum equation must be added.

Code

Solve numerically the set of Boltzmann and fluid equations.
We need inputs to start the Code

The model is validate

- Comparison with available data:
  - Satellite web site depository
  - Convert the data in appropriate Format
- Compare with other reference Model IRI, MSIS,..
  https://irimodel.org/
  The data on which this article is based are available in
  https://ccmc.gsfc.nasa.gov/modelweb/models/iri2016_vitmo.php
  https://ccmc.gsfc.nasa.gov/modelweb/models/msis_vitmo.php
- Absence of models or date, be the reference one.