

*The Standard Model  
of Particles and Interactions  
III- Towards The Standard Model*

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# The gauge symmetries of the Standard Model

The (Yang-Mills) action  $\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$  is invariant under  $\Psi(x) \rightarrow U(x)\Psi(x)$

Abelian U(1) symmetry

$$U(x) = e^{-iq\theta(x)}$$

Non-abelian SU(N)

$$U(x) = e^{-ig\theta^a(x)T^a}$$

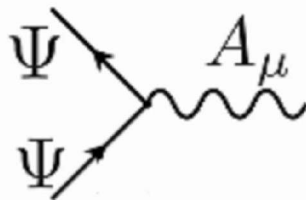
$T^a$ :  $N^2-1$  generators ( $N \times N$  matrices) acting on

$$A_\mu(x) = A_\mu^a T^a$$

$$A_\mu(x) \rightarrow A_\mu + \frac{i}{q}(\partial_\mu U)U^\dagger$$

$$A_\mu(x) \rightarrow U A_\mu U^\dagger + \frac{i}{g}(\partial_\mu U)U^\dagger$$

coupling constants



infinitesimal transformation  $U(x) = 1 - ig\theta^a(x)T^a + \mathcal{O}(\theta^2)$

$$A_\mu^a(x) \rightarrow A_\mu^a + \partial_\mu\theta^a - gf^{abc}\theta^b A_\mu^c$$

$$D_\mu\Psi = (\partial_\mu + iqA_\mu)$$

$$D_\mu\Psi = (\partial_\mu + igA_\mu^a T^a)$$

# More about Matter and Higgs fields

Nature is symmetric under the group of Lorentz transformations, rotations, and translations which all together form the Poincaré group.

Particles are classified by spin: scalars, fermionic spinors, vector bosons. They correspond to irreducible representations of the Poincaré group

Spinors are of two types: the fundamental (left-handed) and the anti-fundamental (right-handed). **The chirality of a spin 1/2 field refers to whether it is in the fundamental or the anti-fundamental and is therefore a label associated with a representation of the Lorentz group**

$$\begin{array}{l} \text{Weyl spinors} \\ \Psi_L : \left(\frac{1}{2}, 0\right) \\ \Psi_R : \left(0, \frac{1}{2}\right) \end{array} \quad \text{Dirac spinor} \quad \Psi = \begin{bmatrix} \Psi_L \\ \Psi_R \end{bmatrix}$$

helicity is a physical quantity: it is the projection of the spin onto the direction of motion



for a massless particle: chirality= helicity

$SU(2)_L$

(thus we call the fundamental spinors the left-handed spinors and the anti-fundamental spinors the right-handed spinors)

The Standard model is a chiral theory: the left-handed and right-handed spinors not only transform differently under the Lorentz group but also under the EW gauge group

$SU(2)_L * U(1)$

The left-handed fields are denoted  $Q=(u_L, d_L)$  and  $L=(\nu_L, e_L)$  while the right-handed fields are denoted  $u_R, d_R$  and  $e_R$

# Fermi Model

- Current-current interaction of 4 fermions

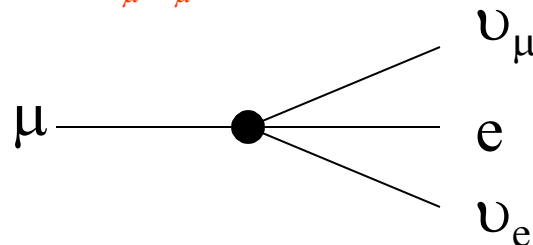
$$L_{FERMI} = -2\sqrt{2}G_F J_\rho^+ J^\rho$$

- Consider just leptonic current

$$J_\rho^{lept} = \bar{\nu}_e \gamma_\rho \left( \frac{1-\gamma_5}{2} \right) e + \bar{\nu}_\mu \gamma_\rho \left( \frac{1-\gamma_5}{2} \right) \mu + hc$$

- Only left-handed fermions feel charged current weak interactions (maximal P violation)
- This induces muon decay

$$\Gamma \approx G_F^2 m_\mu^5$$



$$G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

This structure known since Fermi

# Fermion Multiplet Structure

- $\Psi_L$  couples to  $W^\pm$  (cf Fermi theory)
  - Put in SU(2) doublets with weak isospin  $I_3 = \pm 1/2$
- $\Psi_R$  doesn't couple to  $W^\pm$ 
  - Put in SU(2) singlet with weak isospin  $I = I_3 = 0$

## What about fermion masses?

- Fermion mass term:

$$L = m\bar{\Psi}\Psi = m(\bar{\Psi}_L\Psi_L + \bar{\Psi}_R\Psi_R)$$



Forbidden by  
SU(2)xU(1) gauge  
invariance

- Left-handed fermions are SU(2) doublets

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

- Scalar couplings to fermions:

$$L_d = -\lambda_d\bar{Q}_L\Phi d_R + h.c.$$

- Effective Higgs-fermion coupling

$$L_d = -\lambda_d\frac{1}{\sqrt{2}}(\bar{u}_L, \bar{d}_L)\begin{pmatrix} 0 \\ v+h \end{pmatrix}d_R + h.c.$$

- Mass term for down quark:

$$\lambda_d = -\frac{M_d\sqrt{2}}{v}$$

## Fermion Masses, 2

- $M_u$  from  $\Phi_c = i\tau_2 \Phi^*$

$$\Phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$$

$$\lambda_u = -\frac{M_u \sqrt{2}}{v}$$

$$L = -\lambda_u \bar{Q}_L \Phi^* u_R + hc$$

- For 3 generations,  $\alpha, \beta=1,2,3$  (flavor indices)

$$L_Y = -\frac{(v+h)}{\sqrt{2}} \sum_{\alpha, \beta} (\lambda_u^{\alpha\beta} \bar{u}_L^\alpha u_R^\beta + \lambda_d^{\alpha\beta} \bar{d}_L^\alpha d_R^\beta) + h.c.$$



## Fermion masses, 3

- Unitary matrices diagonalize mass matrices

$$\begin{aligned} u_L^\alpha &= U_u^{\alpha\beta} u_L^{m\beta} & d_L^\alpha &= U_d^{\alpha\beta} d_L^{m\beta} \\ u_R^\alpha &= V_u^{\alpha\beta} u_R^{m\beta} & d_R^\alpha &= V_d^{\alpha\beta} d_R^{m\beta} \end{aligned}$$

- Yukawa couplings are *diagonal* in mass basis
- Neutral currents remain flavor diagonal

- Charged current:

$$J^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L^\alpha \gamma^\mu d_L^\alpha = \frac{1}{\sqrt{2}} \bar{u}_L^{m\alpha} \gamma^\mu (U_u^\dagger V_d)_{\alpha\beta} d_L^{\beta m}$$

CKM matrix

# Abelian Higgs Model

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1 gauge field,  $A_\mu$

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A would look like:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why  $M_A = 0$

Gauge invariance is guiding principle

## *Abelian Higgs Model, 2*

- Add complex scalar field,  $\phi$ , with charge  $-e$ :

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

- Where

$$D_\mu = \partial_\mu - ieA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2$$

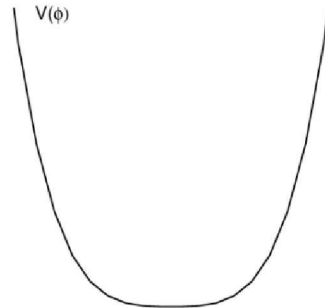
- L is invariant under local U(1) transformations:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$$

$$\phi(x) \rightarrow e^{-ie\eta(x)} \phi(x)$$

## *Abelian Higgs Model, 3*

- Case 1:  $\mu^2 > 0$ 
  - QED with  $M_A=0$  and  $m_\phi=\mu$
  - Unique minimum at  $\phi=0$



$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2$$

$$\lambda > 0$$

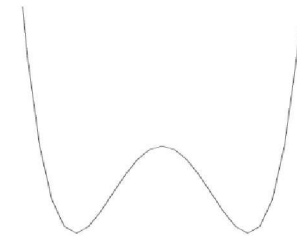
## Abelian Higgs Model, 4

- Case 2:  $\mu^2 < 0$

$$V(\phi) = -|\mu^2||\phi|^2 + \lambda(|\phi|^2)^2$$

- Minimum energy state at:

$$\langle \phi \rangle = \sqrt{-\frac{\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$



Vacuum breaks U(1) symmetry

Aside: What fixes sign ( $\mu^2$ )?

## Abelian Higgs Model, 5

- Rewrite  $\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}} (v+h)$   $\chi$  and  $h$  are the 2 degrees of freedom of the complex Higgs field
- L becomes:
 

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - ev A_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} (\partial_\mu h \partial^\mu h + 2\mu^2 h^2) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \cdot \text{int interaction})$$
- Theory now has:
  - Photon of mass  $M_A = ev$
  - Scalar field  $h$  with mass-squared  $-2\mu^2 > 0$
  - Massless scalar field  $\chi$  (*Goldstone Boson*)

## *Abelian Higgs Model, 6*

- What about mixed  $\chi$ -A propagator?
  - Remove by gauge transformation

$$A'_\mu \equiv A_\mu - \frac{1}{ev} \partial_\mu \chi$$

- $\chi$  field disappears
  - We say that it has been *eaten* to give the photon mass
  - $\chi$  field called Goldstone boson
  - *This is Abelian Higgs Mechanism*
  - This gauge (unitary) contains only physical particles

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^2 v^2}{2} A'^\mu A'_\mu + \frac{1}{2} (\partial_\mu h \partial^\mu h) - V(h)$$

## *Higgs Mechanism summarized*

*Spontaneous breaking of a gauge theory by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson*



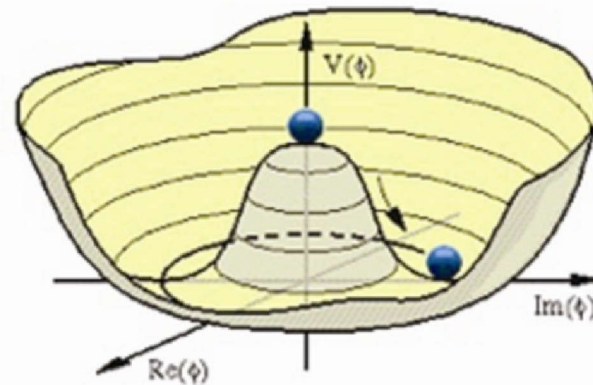
## The (ad hoc) Higgs Mechanism (a model without dynamics)

EW symmetry breaking is described by the condensation of a scalar field

$$\Phi = \frac{v + \underbrace{H}_{\text{Higgs boson: excitation of the higgs medium}} + i\phi_2}{\sqrt{2}}$$

↑ Background value, Higgs medium  
↑ Higgs boson: excitation of the higgs medium

The Higgs selects a vacuum state by developing a non zero background value. When it does so, it gives mass to SM particles it couples to.



$$V(\Phi) = \frac{\mu^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$

Why is  $\mu^2$  negative ?

the puzzle:

We do not know what makes the Higgs condensate.

We ARRANGE the Higgs potential so that the Higgs condensates but this is just a parametrization that we are unable to explain dynamically.<sup>17</sup>

# The gauge symmetries of the Standard Model

Gauge Group  $U(1)_Y$  (abelian)

$$\psi' = e^{-iY g' \alpha_Y} \psi,$$

$$B'_\mu = B_\mu + \partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi = (\partial_\mu + i g' Y B_\mu) \psi$$

Gauge Group  $SU(2)_L$  acts on the two components of a doublet  $\Psi_L = (u_L, d_L)$  or  $(\nu_L, e_L)$

$$\Psi_L \rightarrow e^{-i g T^a \alpha^a} \psi_L \quad U = e^{-i g T^a \alpha^a} \quad T^a = \sigma^a / 2 \quad \text{Pauli matrices}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g \epsilon^{abc} W_\mu^b W_\nu^c, \quad a = 1, \dots, 3 \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = -i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \psi_L = (\partial_\mu + i g W_\mu^a T^a) \psi_L$$

Gauge Group  $SU(3)_c$   $q = (q_1, q_2, q_3)$  (the three color degrees of freedom)

$$q \rightarrow e^{-i g_s T^a \alpha^a} q \quad U = e^{-i g_s T^a \alpha^a} \quad [T^a, T^b] = i f^{abc} T^c \quad (3 \times 3) \text{ Gell-Mann matrices}$$

$$G_\mu^a T^a \rightarrow U G_\mu^a T^a U^{-1} + \frac{i}{g_s} \partial_\mu U U^{-1}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \quad a = 1, \dots, 8$$

$$D_\mu q = (\partial_\mu + i g_s G_\mu^a T^a) q$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

# The gauge symmetries of the Standard Model

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$$\psi' = e^{-iY g' \alpha_Y} \psi,$$

$$B'_\mu = B_\mu + \partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi = (\partial_\mu + i g' Y B_\mu) \psi$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

all Standard Model fermions  
carry  $U(1)$  charge

## Gauge Group $SU(2)_L$

$$\Psi_L \rightarrow e^{-i g T^a \alpha^a} \psi_L \quad U = e^{-i g T^a \alpha^a}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc}W_\mu^b W_\nu^c, \quad a = 1, \dots, 3$$

$$D_\mu \psi_L = (\partial_\mu + i g W_\mu^a T^a) \psi_L$$

$$\Psi_{L=(u_L, d_L)} \text{ or } (\nu_L, e_L)$$

only left-handed fermions charged  
under it  $\rightarrow$  chiral interactions

## Gauge Group $SU(3)_c$

$$q \rightarrow e^{-i g_s T^a \alpha^a} q \quad U = e^{-i g_s T^a \alpha^a}$$

$$G_\mu^a T^a \rightarrow U G_\mu^a T^a U^{-1} + \frac{i}{g_s} \partial_\mu U U^{-1}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \quad a = 1, \dots, 8$$

$$D_\mu q = (\partial_\mu + i g_s G_\mu^a T^a) q$$

$$q=(q_1, q_2, q_3)$$

all quarks transform under it  
 $\rightarrow$  vector-like interactions

# The Lagrangian of the Standard Model

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \quad \text{describe massless gauge bosons}$$

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\bar{\psi}_L\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\bar{\psi}_R\gamma^\mu D_\mu \psi_R \quad \text{describe massless fermions and their interactions with gauge bosons}$$

$$D_\mu \psi_R = [\partial_\mu + ig'Y B_\mu] \psi_R$$

only left-handed fermions

all fermions carrying a  $U(1)_Y$  charge  
i.e. all Standard Model fermions

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \quad \longrightarrow \quad \text{gives mass to EW gauge bosons} \quad \frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu}$$

$$D_\mu \Phi = \left[ \partial_\mu + i\frac{g}{\sqrt{2}}(\tau^+ W_\mu^+ + \tau^- W_\mu^-) + i\frac{g}{2}\tau_3 W_\mu^3 + i\frac{g'}{2}B_\mu \right] \Phi \quad : \text{covariant derivative of the Higgs}$$

H charged under  $SU(2) \times U(1)_Y$

responsible for electroweak symmetry breaking!

$$\mathcal{L}_{\text{Yukawa}} = -Y_l \bar{L} \Phi \ell_R - Y_d \bar{Q} \Phi d_R - Y_u \bar{Q} \tilde{\Phi} u_R + \text{h.c.} \quad \longrightarrow \quad \text{gives mass to fermions}$$

$$SU(3) \times SU(2)_L \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$$

8 massless gluons      3 massive gauge bosons  $W^+ W^- Z_0$       8 massless gluons      1 massless photon  $\gamma$

remaining unbroken symmetry

The W and Z bosons interact with the Higgs medium, the  $\gamma$  doesn't.



$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$$

$SU(3)_c$

$SU(2)_L$

$U(1)_Y$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf^{abc}G_\mu^b G_\nu^c$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

in mass eigen state basis

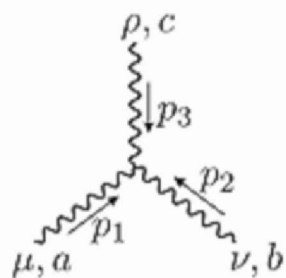
$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}$$

$$Z_\mu = W_\mu^3 \cos \theta_W + B_\mu \sin \theta_W$$

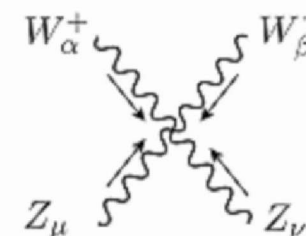
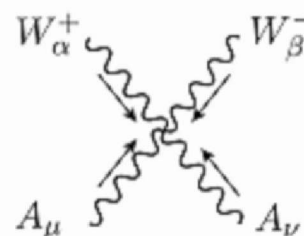
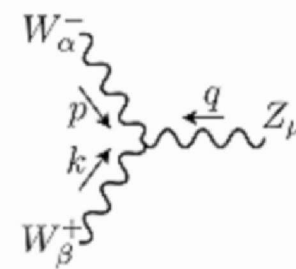
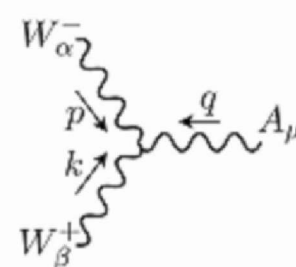
$$A_\mu = -W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W$$

$$\cos \theta_W = g/\sqrt{g^2 + g'^2}$$

$$\sin \theta_W = g'/\sqrt{g^2 + g'^2}$$

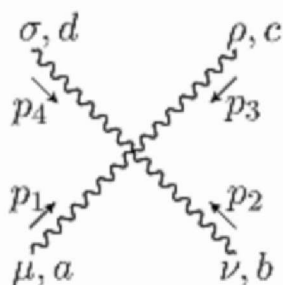


three gauge boson vertex



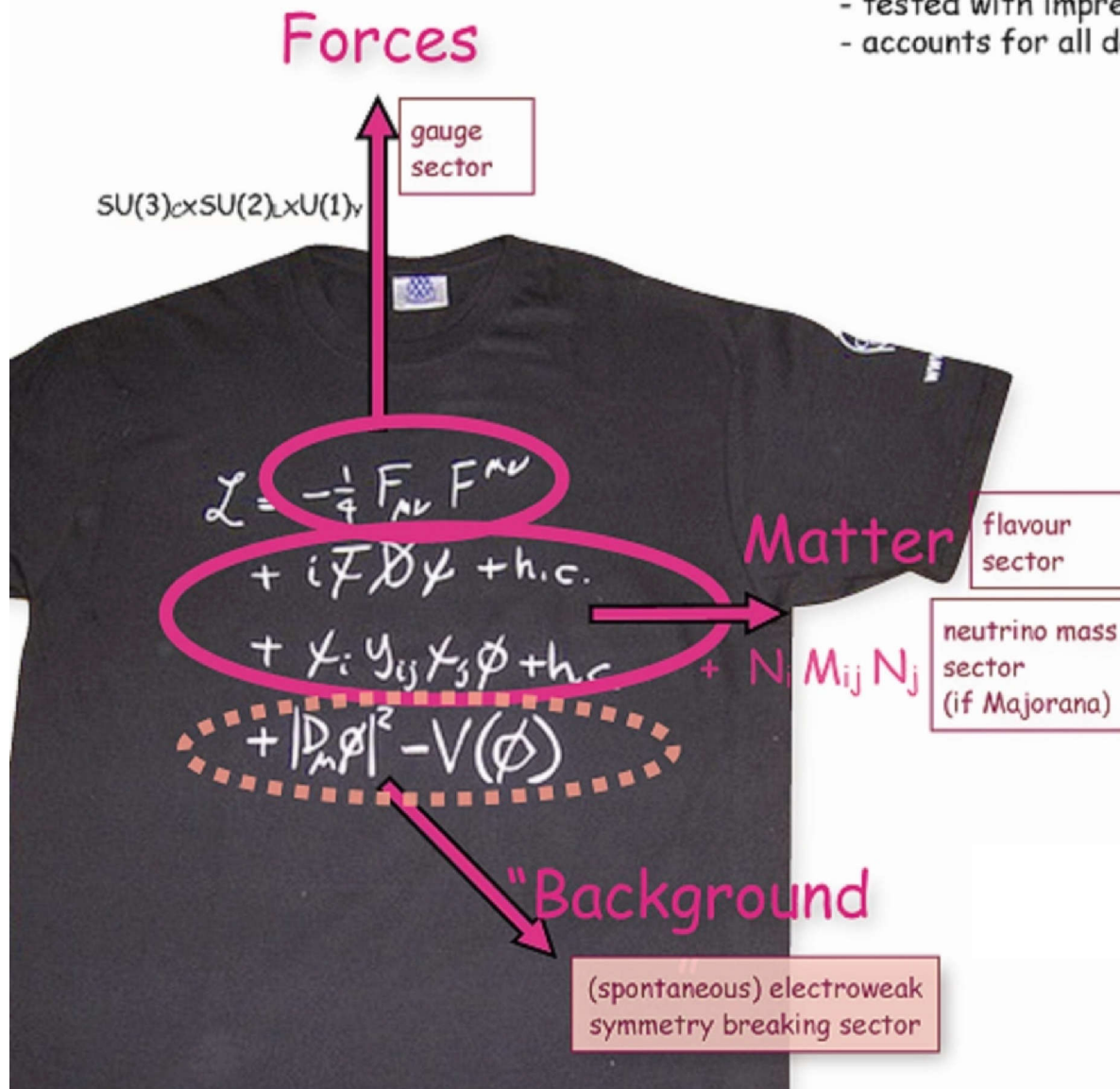
four gauge boson vertex

no such interactions for photon!



# The Standard Model of Particle Physics

- one century to develop it
- tested with impressive precision
- accounts for all data in experimental particle physics



Finally the Higgs has been found in 2012, which is the last missing piece in the Standard Model.

Field	$SU(3)$	$SU(2)_L$	$T^3$	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
$g_\mu^a$ (gluons)	<b>8</b>	<b>1</b>	0	0	0
$(W_\mu^\pm, W_\mu^0)$	<b>1</b>	<b>3</b>	$(\pm 1, 0)$	0	$(\pm 1, 0)$
$B_\mu^0$	<b>1</b>	<b>1</b>	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	<b>3</b>	<b>2</b>	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
$u_R$	<b>3</b>	<b>1</b>	0	$\frac{2}{3}$	$\frac{2}{3}$
$d_R$	<b>3</b>	<b>1</b>	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	<b>1</b>	<b>2</b>	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_R$	<b>1</b>	<b>1</b>	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	<b>1</b>	<b>2</b>	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\Phi^c = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$	<b>1</b>	<b>2</b>	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

# Lots still not understood!

- How to calculate predictions for the hard questions in QCD?
  - What happens at nearby energies to allow the force couplings to unify at much higher energy? SUSY?
  - What causes the fermions to have the observed mass pattern?
  - What about neutrinos



# Lots still not understood!

- What gives the universe matter excess over antimatter?
- What particles make up most of the (dark) mass of the universe?
- Where did the "dark energy" come from?
- What about gravity?

# References

In preparing this presentation I used the following lectures and presentations

1. Four Lectures Leading to the Standard Model of Particle Physics, Frank Sciulli, 2001.
2. The future of particle physics, S. F. King, 2004.
3. Sumer school lectures by Geraldine Servant , 2012.
4. Introduction to the Standard Model, Sally Dawson, TASI, 2006