Beyond the Standard Model of Particle Physics: An Introduction to Grand Unification and Supersymmetry

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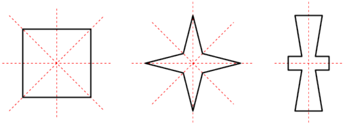
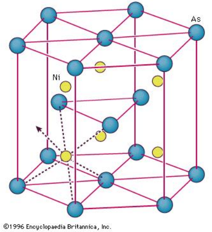
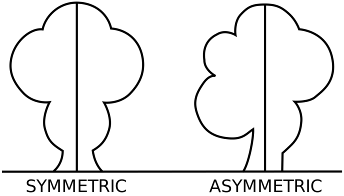
Outline

Symmetries in nature

- There is a lot of examples of symmetric shapes appears in natures

- In addition asymmetric shapes appear also

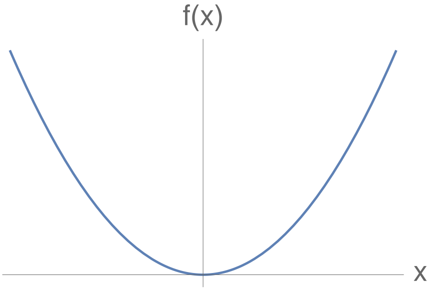


Symmetries: Physics and Mathematics

- We device a mathematical tool to describe the concept of symmetries: The transformation
- Example: $f(x) = x^2$,

  is **symmetric** or (**invariant**) under the transformation

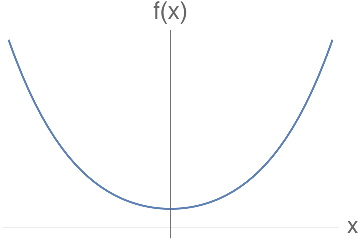
$$x \to -x$$



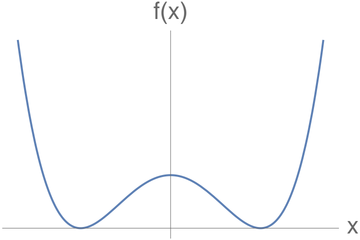
- In physics the above function represents.......?

Again $\mathbb{Z}_2$ symmetry : $x \to -x$

- $f(x) = \mu^2\, x^2 + \lambda x^4$



- $f(x) = -\mu^2\, x^2 + \lambda x^4$



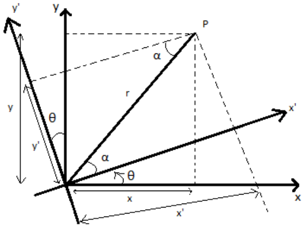
What about the symmetry of the minimum (vacuum)?

What are the corresponding phenomena in Physics?

Rotational symmetry and orthogonal transformations

The coordinate transformation is given by

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Or

$$\mathbf{r} \to r' = R(\theta)\, r$$

$\Rightarrow \mathbf{r'^T}\,\mathbf{r'} = \mathbf{r^T}\,\mathbf{r}$ (Length is invariant).

$$\mathbf{R}(\theta)\,\mathbf{R}(\theta)^{\mathbf{T}} = \mathbf{R}(\theta)^{\mathbf{T}}\,\mathbf{R}(\theta) = \mathbf{I} \;\; \text{(Orthogonal Transformation)}$$

$$det(R(\theta)) = 1 \;\; \text{(Proper Rotation)}$$

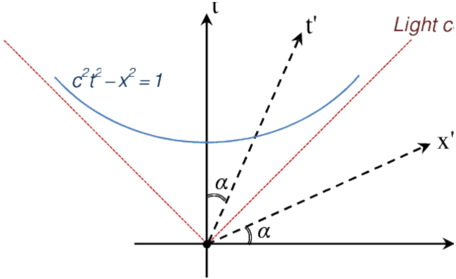
Rotation transformations $R(\theta)$ forms a group: $SO(2)$ (It is called Lie group).

Space-time Symmetries

- Spatial transformations
  - 2D Space rotations SO(2) group with one parameter θ.
  - 3D Space rotations SO(3) group with three parameter: Euler's angels θ, ϕ, ψ.
  - Invariance under space rotations implies that space is isotropic.
  - Invariance under space translations implies that space is homogenous.

- Space-time transformations
  - Lorenz transformation: The symmetry of the special theory relativity. Any relativistic theory of particles should respect Lorenz transformation. It is described by the group $SO(3,1)$. It acts on spacetime coordinates x^μ ; $\mu = 0, 1, 2, 3$

$$x^\mu \mapsto x'^\mu = \underbrace{\Lambda^\mu{}_\nu}_{\text{L.T.}} x^\nu$$



  - Fields (particles) are classified according to their transformations under Lorenz transforations: bosonic scalars (Higgs particle), Spinors (fermions such as the electron), vectors (gauge bosons like the photon)

Internal Symmetries symmetries

- It is a transformation on the field itself without affecting the space-time.
- Let's focus on $SU(N)$ transformations.
  1. Example: Complex scalar field ϕ and $U(1)$ global symmetry

     $$\mathcal{L} = \partial_\mu \phi^* \, \partial^\mu \phi - m^2 |\phi|^2 \,, \quad \text{(Leading to Klein Gordon Equation)}$$

     is invariant under $U(1)$ transformation $\quad \phi \to e^{-i\alpha} \phi$
  2. Example: Spinor field ψ and $U(1)$ global symmetry

     $$\mathcal{L} = i\overline{\psi}\gamma^\mu \partial_\mu \, \psi - m\overline{\psi}\psi \,, \quad \text{(Leading to Dirac Equation)}$$

     is invariant under $U(1)$ transformation $\quad \psi \to e^{-i\alpha} \psi$
- $U(1)$ symmetry group is Abelian group.

Gauge Symmetries (Local Symmetries)

- Example: $U(1)$ gauge symmetry (local symmetry) $\Rightarrow \alpha \equiv \alpha(x^\mu)$, $\psi \to \psi' = e^{i\alpha(x)}\psi$
- Let's call it $U(1)_{\mathrm{EM}}$
- Define the gauge field $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$

$$
\begin{aligned}
\mathcal{L} &= i\overline{\psi}\,\gamma^\mu D_\mu\,\psi - m\overline{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
D_\mu\,\psi &= (\partial_\mu + ieA_\mu)\psi && \text{(Covariant Derivative)} \\
F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu && \text{(Field Strength Tensor)}
\end{aligned}
$$

- This introduces the interactions $e(\,\overline{\psi}\,\gamma^\mu\psi)\,A_\mu$

$SU(2)$ Gauge Symmetry

- Example: Let's consider the isospin symmetry: $SU(2)$ gauge symmetry (Non-abelian gauge group)

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} , \quad \psi(x) \to \psi'(x) = \exp\left\{ \frac{-i\,\vec{\tau}\cdot\vec{\theta}(x)}{2} \right\} \psi(x)$$

- θ_i are thee independent parameters. The generators $T_i \equiv \frac{\tau_i}{2}$, τ_i are the pauli matrices:

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} , \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} , \left[\frac{\tau_k}{2}, \frac{\tau_l}{2} \right] = i\,\varepsilon_{klm} \frac{\tau_m}{2}$$

- Gauge field in adjoint representation:

$$A^k_\mu(x) \to A'^k_\mu(x) = A^k_\mu(x) - \frac{1}{g}\partial_\mu \theta^k(x) + i\varepsilon^{klm}\theta^l A^m_\mu$$

- The gauge invariant Lagrangian (Excersie!)

$$\mathcal{L} \;=\; i\overline{\psi}\,\gamma^\mu D_\mu\,\psi - m\overline{\psi}\psi - \frac{1}{4} F^k_{\mu\nu} F^{k\,\mu\nu}$$

$SU(2)$ Gauge Symmetry

- The Covariant Derivative $D_\mu \psi$ transforms in the same way as ψ

$$D_\mu \psi \to \exp\left\{\frac{-i\,\vec{\tau}\cdot\vec{\theta}(x)}{2}\right\} D_\mu \psi(x)$$

$$D_\mu \psi = \left(\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{A}_\mu}{2}\right)\psi$$

$$F^k_{\mu\nu} \equiv \partial_\mu A^k_\nu - \partial_\nu A^k_\mu + +g\varepsilon^{klm} A^l_\mu A^m_\nu$$

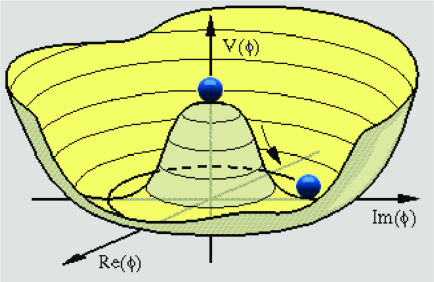
- This introduces the interactions between fermions and $SU(2)$ gauge bosons $g\,(\overline{\psi}\,\gamma^\mu\,(\vec{\tau}\cdot\vec{A}_\mu)\,\psi)$
- Moreover, gauge bosons self interactions appear in Non-abelian gauge symmetries.
- This formalism can be generalized to $SU(N)$ gauge symmetries

Spontaneous Symmetry Breaking (SSB) and Higgs Mechanism

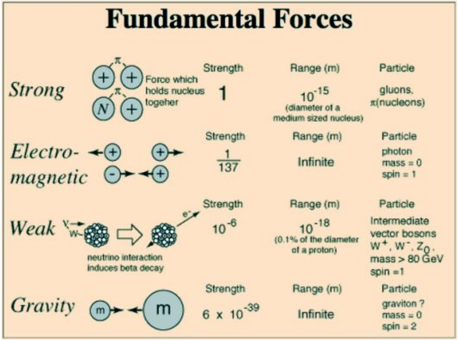
- $U(1)$ Abelian gauge symmetry

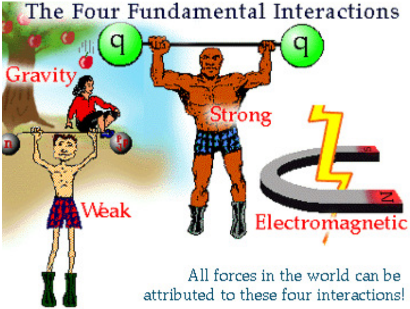
$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi, \phi^\dagger) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$
$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

- The global minimum of the potential at $|\phi| = \frac{v}{\sqrt{2}}$, with $v = \sqrt{\frac{\mu^2}{\lambda}}$



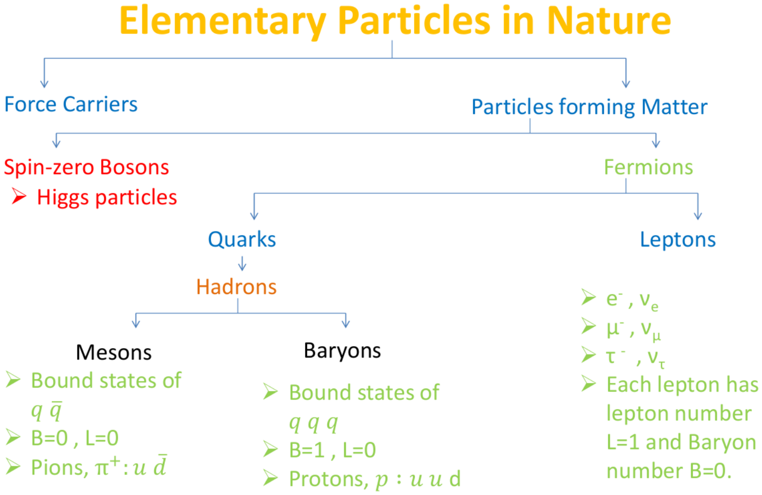
Fundamental Forces



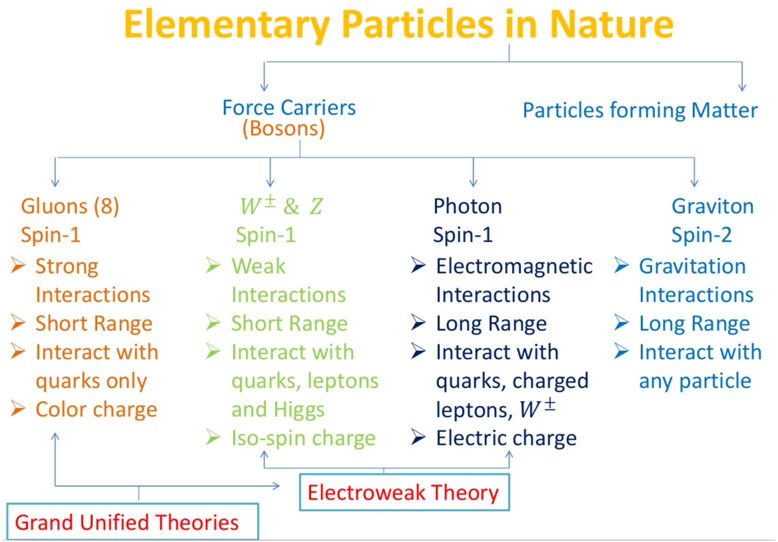


- The First Successful unification, was by Maxwell who unified the electric and magnetic forces in his known equations. This was shown manifestly after Einstein's special theory of relativity.
- Later attempts to unify electromagnetism and gravity by Einstein. In addition attempts by T. Kaluza and O. Klein by proposing the existence of an extra dimension.

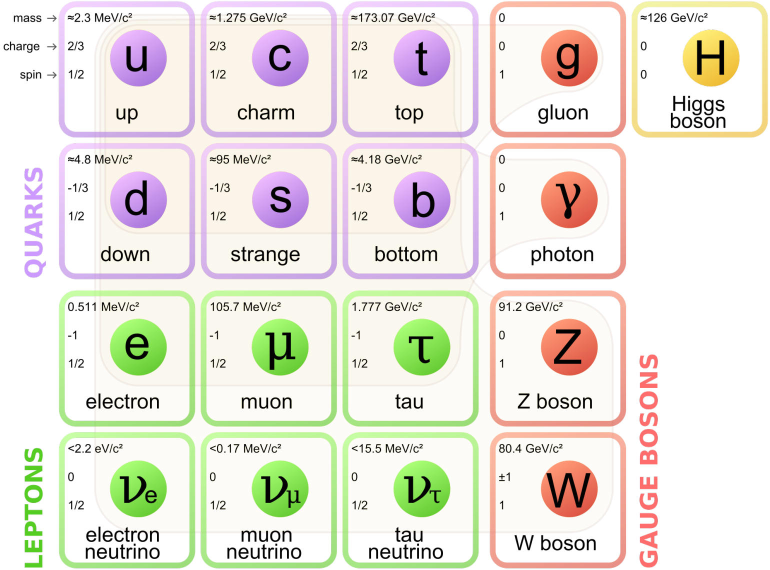
Elementary Particle and Force Carriers



Elementary Particle and Force Carriers



SM of elementary Particle Physics: Particle Content



SM: Gauge symmetry-Representations-Lagrangian

- The governing gauge symmetry is $SU(3)_c \times SU(2)_L \times U(1)_Y$
- Particle Representations with $Q_{\rm em} = T_{\rm 3L} + Y$

$$SU(3)_c \xrightarrow{gluons} \begin{bmatrix} u_r \\ u_g \\ u_b \end{bmatrix}, \begin{bmatrix} d_r \\ d_g \\ d_b \end{bmatrix}; SU(2)_L \xrightarrow{W^\pm} \underbrace{l_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L}_{Y=-1/2}, \underbrace{q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L}_{Y=1/6}, \underbrace{H = \begin{bmatrix} h^+ \\ h^0 \end{bmatrix}}_{Y=1/2}$$

- The gauge invariant Lagrangian consists of 4 pieces

$$\begin{aligned}
\mathcal{L}_{\rm SM} &= \mathcal{L}_{\rm GB} + \mathcal{L}_{\rm f} + \mathcal{L}_{\rm S} + \mathcal{L}_{\rm Y} \\
\mathcal{L}_{\rm GB} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\,\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} \\
\mathcal{L}_{\rm f} &= i\overline{\psi}_L \gamma^\mu D_\mu \psi_L + i\overline{\psi}_R \gamma^\mu D_\mu \psi_R \\
\mathcal{L}_{\rm S} &= (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 \\
\mathcal{L}_{\rm Y} &= -Y_e \overline{l_L} H\, e_R - Y_d \overline{q_L} H\, d_R - Y_u \overline{q_L} \tilde{H}\, u_R + h.c.
\end{aligned}$$

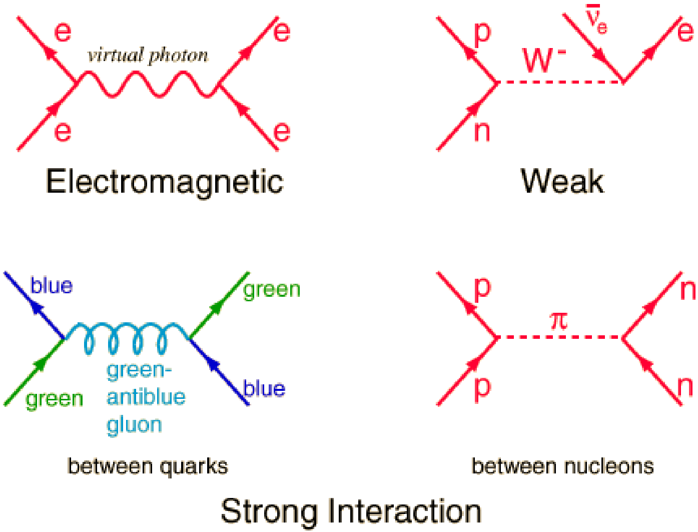
SM of elementary Particle Physics: Spontaneous Symmetry Breaking

- The SM gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ is spontaneously broken if the Higgs doublet H acquires a non-zero value [vacuum expectation value (vev)] at the ptential minimum, along the EM neutral direction h^0

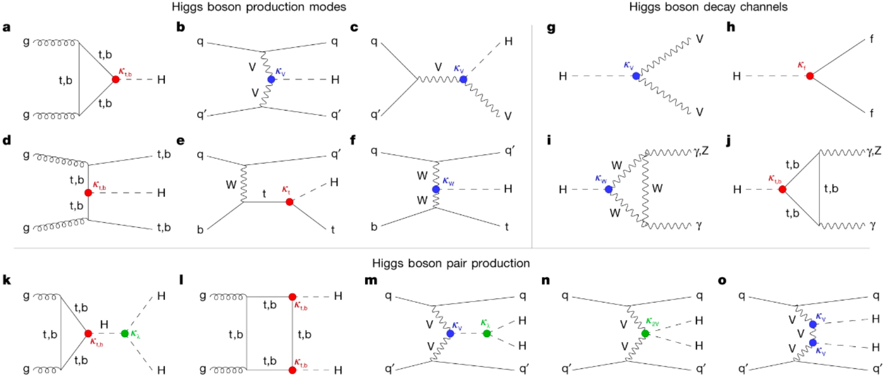
$$SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\langle H \rangle = \begin{bmatrix} 0 \\ v \end{bmatrix}} SU(3)_c \times U(1)_{\mathrm{EM}}$$

- 8 Glouns and Photons remain massless while the tree the weak gauge bosons $Z, W^{\pm}$ acquire masses of the same order of the EW symmetry breaing scale.
- Charged fermions acquire masses from the Yukawa Lagrangian.

SM Interactions



SM Successes: Higgs Bosons Discovery

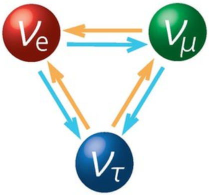


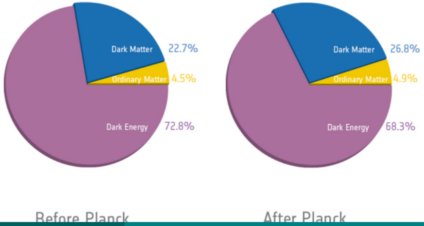
Reference: Nature **607**- 60-68 (2022).

Challenges facing particle physics

The Standard Model of particle physics (SM) is extremely successful in describing the Physics elementary particles. However there are experimental and theoretical evidences implying that the SM is not the ultimate fundamental theory:

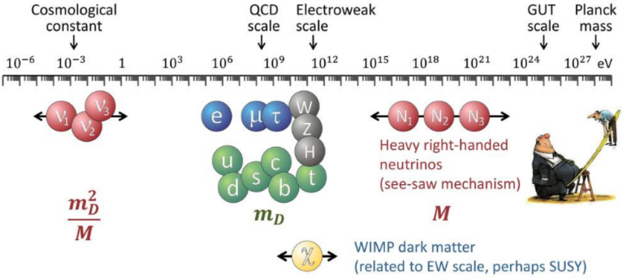
- Massive neutrinos: (nineties of the last century in Super-Kamiokande experiment).
- Dark matter: The SM suffers from lack of candidates for dark matter.
- Baryon asymmetry: Why there is excess of the matter compared to the antimatter in the universe?
- The hierarchy problem: The huge discrepancy between EW scale ($\mathcal{O}(10^3)$ GeV) and the Planck scale ($\mathcal{O}(10^{19})$ GeV).



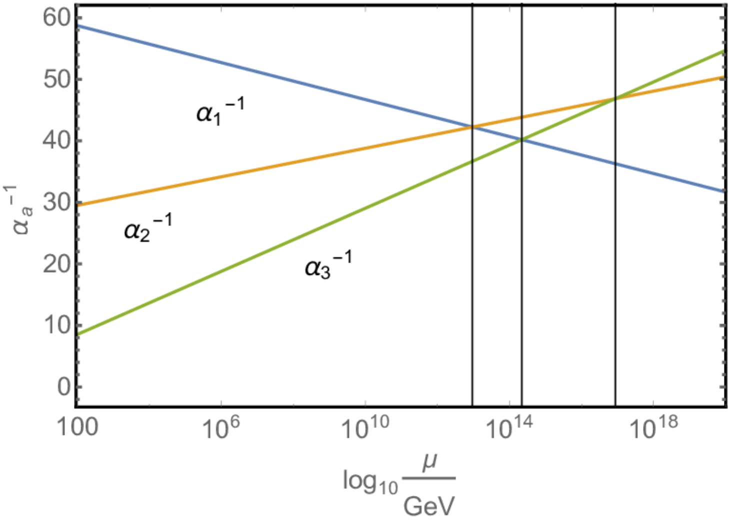


More Questions

- The Fundamental scale: Many scales in nature which are hierarchically different from each other like: cosmological constant scale ($\mathcal{O}(10^{-47})$ GeV^4), neutrino mass scale ($\mathcal{O}(10^{-11} - 10^{-9})$ GeV), EW scale ($\mathcal{O}(10^2)$ GeV), Grand Unification scale (GUT) ($\mathcal{O}(10^{15} - 10^{16})$ GeV) and the Planck scale (10^{19} GeV) at which the gravitation interactions on the quantum level become important. What is the deep origin of this differences? What are the possible relations between them? is there a fundamental scale from which these scales can be derived?



SM Forces and Unification



*Gives a hint to the existence of a unified theory with one gauge coupling

Georgi-Glashow SU(5) GUT

- Can we reduce all the gauge interactions to just one single group with one gauge coupling at a high energy scale?
- Can we reduce all different fermion multiplets into one or two representations under the unified theory?
- The GUT symmetry group should contain the SM group as a subgroup, $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U((1)_Y$. Rank($G_{\rm SM}$)=4.
- GUTs automatically quantize the electric charge, since we embed the U(1) into a non-abelian group!
- A good candidate is the *simple* group SU(5). This model was proposed by George and Glashow in 1974.

Fermion representations

$$\bar{5}_F = \psi_L = \begin{pmatrix} d_r^c \\ d_g^c \\ d_b^c \\ e \\ -\nu \end{pmatrix}_L ,$$

or equivalently we can have

$$5_F = \psi_R = \begin{pmatrix} d_r \\ d_g \\ d_b \\ e^+ \\ -\nu^C \end{pmatrix}_R ,$$

$$10 = \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_b^c & -u_g^c & -u_r & -d_r \\ -u_b^c & 0 & u_r^c & -u_g & -d_g \\ u_g^c & -u_r^c & 0 & -u_b & -d_b \\ u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \end{pmatrix}_L$$

Georgi-Glashow SU(5) GUT

SU(5) Generators. How many generators of G_{SM}

$$\lambda_{1\dots8} = \begin{pmatrix} & \lambda^C_{1\dots8} & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{9,10,11} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & T^i_L \end{pmatrix} , \quad \lambda_{12} = \frac{1}{2}\sqrt{\frac{3}{5}} \begin{pmatrix} \frac{-2}{3} & & & & \\ & \frac{-2}{3} & & & \\ & & \frac{-2}{3} & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

Georgi-Glashow SU(5) GUT

$$T_{3L} = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & \frac{1}{2} & \\ & & & & \frac{-1}{2} \end{pmatrix} = \lambda_{11},$$

and thus we have

$$\frac{Y}{2} = \begin{pmatrix} \frac{-1}{3} & & & & \\ & \frac{-1}{3} & & & \\ & & \frac{-1}{3} & & \\ & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix} = \sqrt{\frac{5}{3}}\lambda_{12}$$

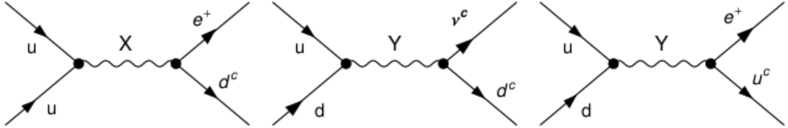
Georgi-Glashow SU(5) GUT

Gauge bosons

$$\mathbf{A}_\mu = A_\mu^a T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & \bar{X}_\mu^r & \bar{Y}_\mu^r \\ & G_\mu^8 & & \bar{X}_\mu^g & \bar{Y}_\mu^g \\ & & & \bar{X}_\mu^b & \bar{Y}_\mu^b \\ X_\mu^r & X_\mu^g & X_\mu^b & W_\mu^3 & W_\mu^+ \\ Y_\mu^r & Y_\mu^g & Y_\mu^b & W_\mu^- & -\frac{1}{\sqrt{2}} W_\mu^3 \end{pmatrix} + \sqrt{\frac{3}{5}} \begin{pmatrix} -1/3 & 0 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{pmatrix} B_\mu,$$

SU(5) GUT: Proton Decay

- After spontaneous symmetry breaking, 12 gauge bosons (X, Y) acquire heavy masses of the order of GUT symmetry breaking scale. The 12 SM gauge bosons remain massless till the EW scale.
- X, Y interactions with the leptons and quarks violate the baryon and lepton number conservation, hence allow for Proton decay



- Experimental limits from Super-Kamiokande and Hyper-Kamiokande: $\tau_p > 10^{34} - 10^{35}$ years, which put a limit on the GUT symmetry breaking scale $M_{\rm GUT} \gtrsim 5 \times 10^{15}$ GeV.
- Therefore the simple Georgi-Glasho is ruled out by experiment.

Introduction to Supersymmetry

Introduction to Supersymmetry

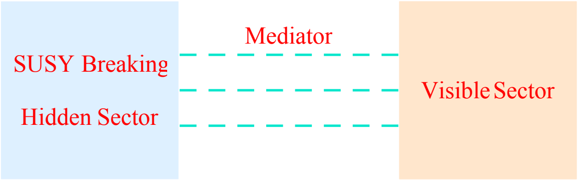
- Supersymmetry (SUSY) is a symmetry relating particles of integer spin, such as spin-0 and spin-1 bosons, and particles of spin 1/2, i.e. fermions [we ignore, for the moment, the graviton and its partner]

$$Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle\ , \quad Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle$$

- If SUSY is exact, the bosonic fields, i.e. the scalar and gauge fields of spin 0 and spin 1, respectively, and the fermionic fields of spin 2 1 have the same masses and quantum numbers, except for the spin.
- Supersymmetry is a symmetry that unifies fermions and bosons in one supermultiplet.
- Space-time will be extended to superspace represented by $(x^{\mu}, \theta_{\alpha}, \bar{\theta}^{\dot{\alpha}})$.

Supersymmetry breaking

- SUSY should be broken, since we didn't observe an s-electron of the same mass as the electron.
- SUSY is broken in a hidden sector via F or D-term SUSY breaking mechanisms for example.
- SUSY breaking effects are transmitted to the visible sector via messengers and appears in the low energy Lagrangian as soft terms.



Minimal Supersymmetric SM (MSSM)

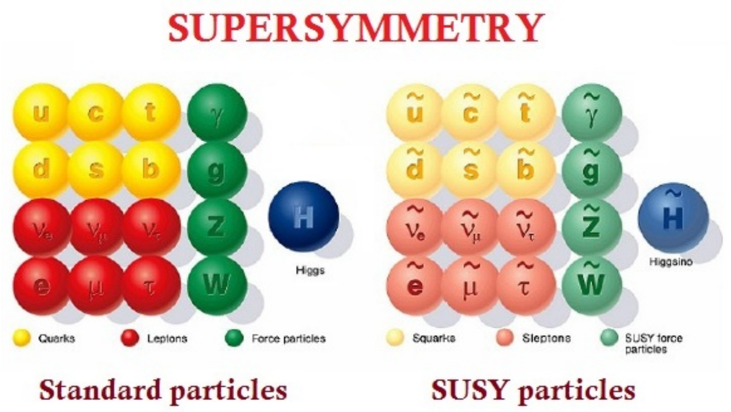
- The MSSM is based on the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{G}_a$	8	1	0	G_a^μ, $\tilde{G}_a$
$\hat{W}_a$	1	3	0	W_a^μ, $\tilde{W}_a$
$\hat{B}$	1	1	0	B^μ, $\tilde{B}$

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{G}_a$	8	1	0	G_a^μ, $\tilde{G}_a$
$\hat{W}_a$	1	3	0	W_a^μ, $\tilde{W}_a$
$\hat{B}$	1	1	0	B^μ, $\tilde{B}$

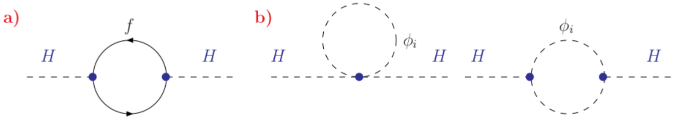
MSSM Predictions

- Copy of the SM particles called superpartners that have the same quantum numbers but different in spin. Since SUSY is broken, masses will be different.



Predictions

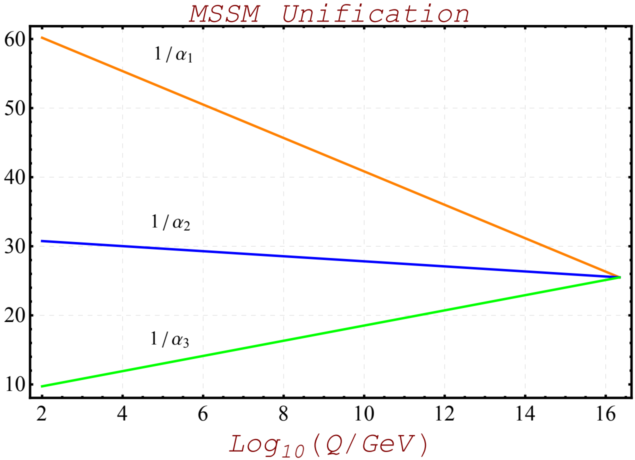
- If SUSY is broken at the TeV scale, then it provides a natural solution to the hierarchy problem of the SM



- It provides a candidate for DM, the neutralino.
- MSSM can be extended to explain the neutrino masses

Gauge Couping Unification and Supersymmetry

- SUSY provides a successful framework for gauge couplings unification at the Grand Unified (GUT) scale as in the MSSM and SUSY GUTs.



Thank you